Chapter 29

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With r = 20 ft = 6.10 m, we have

$$B = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(100\ \mathrm{A}\right)}{2\pi\left(6.10\ \mathrm{m}\right)} = 3.3\times 10^{-6}\ \mathrm{T} = 3.3\ \mu\mathrm{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. Equation 29-1 is maximized (with respect to angle) by setting θ = 90º (= π/2 rad). Its value in this case is

$$dB_{\max} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}.$$

From Fig. 29-34(b), we have $B_{\max} = 60\times 10^{-12}$ T. We can relate this $B_{\max}$ to our $dB_{\max}$ by setting "ds" equal to 1 × 10^{-6} m and R = 0.025 m. This allows us to solve for the current: i = 0.375 A. Plugging this into Eq. 29-4 (for the infinite wire) gives B_∞ = 3.0 μT.

3. (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Since $B = \mu_0 i/2\pi r$,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(0.080\ \mathrm{m}\right)\left(39\times 10^{-6}\ \mathrm{T}\right)}{4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}} = 16\ \mathrm{A}.$$

(b) The current must be from west to east to produce a field that is directed southward at points below it.

4. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current"). Also, the fields from the two semicircular loops cancel at C (by symmetry). Therefore, B_C = 0.

5. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 29-7) and a semicircular arc (Eq. 29-9 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 29-6(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(10\text{ A})}{2(0.0050\text{ m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\text{ T}$$

upon substituting i = 10 A and R = 0.0050 m.

(b) The direction of this field is out of the page, as Fig. 29-6(c) makes clear.

(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 29-4,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(10\text{ A})}{\pi(0.0050\text{ m})} = 8.0\times10^{-4}\,\text{T}.$$

(d) This field, too, points out of the page.

6. With the "usual" x and y coordinates used in Fig. 29-37, then the vector $\vec{r}$ pointing from a current element to P is $\vec{r} = -s\,\hat{\text{i}} + R\,\hat{\text{j}}$. Since $d\vec{s} = ds\,\hat{\text{i}}$, then $|d\vec{s}\times\vec{r}| = Rds$. Therefore, with $r=\sqrt{s^2+R^2}$, Eq. 29-3 gives

$$dB = \frac{\mu_0}{4\pi}\frac{iR\,ds}{(s^2+R^2)^{3/2}}\,.$$

(a) Clearly, considered as a function of s (but thinking of "ds" as some finite-sized constant value), the above expression is maximum for s = 0. Its value in this case is $dB_{\max} = \mu_0 i\,ds/4\pi R^2$.

(b) We want to find the s value such that $dB = dB_{\max}/10$. This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is $s = \sqrt{10^{2/3}-1}\,R$. If we set $R = 2.00$ cm, then we obtain s = 3.82 cm.

7. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with P do not contribute to the field at that point. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$B = \frac{\mu_0 i\theta}{4}\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.411\text{A})(74°\cdot\pi/180°)}{4\pi}\left(\frac{1}{0.107\text{m}}-\frac{1}{0.135\text{m}}\right)$$
$$=1.02\times10^{-7}\,\text{T}.$$

(b) The direction is out of the page.

8. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.281\mathrm{A})}{4}\left(\frac{1}{0.0315\mathrm{m}} - \frac{1}{0.0780\mathrm{m}}\right) = 1.67\times10^{-6}\,\mathrm{T}.$$

(b) The direction of the field is into the page.

9. (a) The currents must be opposite or antiparallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi\left(0.040\text{ m}\right)\left(300\times10^{-6}\,\mathrm{T}\right)}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 30\,\mathrm{A}.$$

10. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *C* do not contribute to the field at that point.

Equation 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at *C*. Thus, the magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.0348\mathrm{A})}{4(0.0926\mathrm{m})} = 1.18\times10^{-7}\,\mathrm{T}.$$

(b) The right-hand rule shows that this field is into the page.

11. (a) $B_{P_1} = \mu_0 i_1 / 2\pi r_1$ where i_1 = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2} = \mu_0 i_2 / 2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = \left(6.5\,\mathrm{A}\right)\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right) = 4.3\,\mathrm{A}.$$

(b) Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page.

12. (a) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current 3.00i, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad \Rightarrow \quad r = \frac{d}{4} = \frac{16.0 \text{ cm}}{4} = 4.0 \text{ cm}.$$

(b) Doubling the currents does not change the location where the magnetic field is zero.

13. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi} \frac{\sin\theta}{r^2} dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2 + R^2}$ and sin θ with $R/r = R/\sqrt{x^2 + R^2}$, we integrate from x = –L/2 to x = L/2. The total field is

$$B = \frac{\mu_0 iR}{4\pi} \int_{-L/2}^{L/2} \frac{dx}{\left(x^2 + R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi} \frac{1}{R^2} \frac{x}{\left(x^2 + R^2\right)^{1/2}} \Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2 + 4R^2}}$$

$$= \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)\left(0.0582 \text{ A}\right)}{2\pi\left(0.131 \text{ m}\right)} \frac{0.180\text{m}}{\sqrt{(0.180\text{m})^2 + 4(0.131\text{m})^2}} = 5.03 \times 10^{-8} \text{ T}.$$

14. We consider Eq. 29-6 but with a finite upper limit (L/2 instead of ∞). This leads to

$$B = \frac{\mu_0 i}{2\pi R} \frac{L/2}{\sqrt{(L/2)^2 + R^2}}.$$

In terms of this expression, the problem asks us to see how large L must be (compared with R) such that the infinite wire expression B_∞ (Eq. 29-4) can be used with no more than a 1% error. Thus we must solve

$$\frac{B_\infty - B}{B} = 0.01.$$

This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is

$$L = \frac{200R}{\sqrt{201}} \approx 14.1R \quad \Rightarrow \quad \frac{L}{R} \approx 14.1.$$

15. (a) As discussed in Sample Problem — "Magnetic field at the center of a circular arc of current," the radial segments do not contribute to $\vec{B}_P$ and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0 (0.40\,\mathrm{A})(\pi\,\mathrm{rad})}{4\pi(0.050\,\mathrm{m})}\hat{\mathrm{k}} - \frac{\mu_0 (0.80\,\mathrm{A})(2\pi/3\,\mathrm{rad})}{4\pi(0.040\,\mathrm{m})}\hat{\mathrm{k}} = -(1.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}| = 1.7\times10^{-6}\,\mathrm{T}$.

(b) The direction is $-\hat{\mathrm{k}}$, or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B} = -\frac{\mu_0 (0.40\,\mathrm{A})(\pi\,\mathrm{rad})}{4\pi(0.050\,\mathrm{m})}\hat{\mathrm{k}} - \frac{\mu_0 (0.80\,\mathrm{A})(2\pi/3\,\mathrm{rad})}{4\pi(0.040\,\mathrm{m})}\hat{\mathrm{k}} = -(6.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}| = 6.7\times10^{-6}\mathrm{T}$.

(d) The direction is $-\hat{\mathrm{k}}$, or into the page.

16. Using the law of cosines and the requirement that B = 100 nT, we have

$$\theta = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1B_2}\right) = 144^\circ,$$

where Eq. 29-10 has been used to determine B_1 (168 nT) and B_2 (151 nT).

17. Our x axis is along the wire with the origin at the right endpoint, and the current is in the positive x direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from x = –L to x = 0. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$

$$= \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(0.693\ \mathrm{A}\right)}{4\pi\left(0.251\ \mathrm{m}\right)}\frac{0.136\mathrm{m}}{\sqrt{(0.136\mathrm{m})^2+(0.251\mathrm{m})^2}} = 1.32\times10^{-7}\ \mathrm{T}.$$

18. In the one case we have $B_{\text{small}} + B_{\text{big}} = 47.25\ \mu$T, and the other case gives $B_{\text{small}} - B_{\text{big}} = 15.75\ \mu$T (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{\text{small}})+(1/r_{\text{big}})}{(1/r_{\text{small}})-(1/r_{\text{big}})} = \frac{1+(r_{\text{small}}/r_{\text{big}})}{1-(r_{\text{small}}/r_{\text{big}})} = 3\ .$$

The solution of this is straightforward: $r_{\text{small}} = r_{\text{big}}/2$. Using the given fact that the $r_{\text{big}} = 4.00$ cm, then we conclude that the small radius is $r_{\text{small}} = 2.00$ cm.

19. The contribution to $\vec{B}_{\text{net}}$ from the first wire is (using Eq. 29-4)

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r_1}\hat{\mathrm{k}} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(30\ \mathrm{A})}{2\pi(2.0\ \mathrm{m})}\hat{\mathrm{k}} = (3.0\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}.$$

The distance from the second wire to the point where we are evaluating $\vec{B}_{\text{net}}$ is r_2 = 4 m – 2 m = 2 m. Thus,

$$\vec{B}_2 = \frac{\mu_0 i_2}{2\pi r_2}\hat{\mathrm{i}} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(40\ \mathrm{A})}{2\pi(2.0\ \mathrm{m})}\hat{\mathrm{i}} = (4.0\times10^{-6}\ \mathrm{T})\hat{\mathrm{i}}.$$

and consequently is perpendicular to $\vec{B}_1$. The magnitude of $\vec{B}_{\text{net}}$ is therefore

$$|\vec{B}_{\text{net}}| = \sqrt{(3.0\times10^{-6}\ \mathrm{T})^2+(4.0\times10^{-6}\ \mathrm{T})^2} = 5.0\times10^{-6}\ \mathrm{T}\ .$$

20. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2} = \dfrac{\mu_0 i}{2R}$. Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right) = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(5.78\times10^{-3}\ \mathrm{A}\right)}{2\left(0.0189\ \mathrm{m}\right)}\left(1+\frac{1}{\pi}\right) = 2.53\times10^{-7}\,\mathrm{T}.$$

$\vec{B}_C$ points out of the page, or in the +z direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\mathrm{T})\hat{\mathrm{k}}$

(b) Now, $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(5.78\times10^{-3}\ \mathrm{A}\right)}{2\left(0.0189\ \mathrm{m}\right)}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\mathrm{T}.$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\mathrm{T}(\cos 17.66°\,\hat{\mathrm{i}} + \sin 17.66°\,\hat{\mathrm{k}}) = (1.92\times10^{-7}\,\mathrm{T})\hat{\mathrm{i}} + (6.12\times10^{-8}\,\mathrm{T})\hat{\mathrm{k}}.$$

21. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the *y* axis (passing through *P*), consisting of two equal magnetic field *y*-components. Using Eq. 29-17,

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where i = 4.00 A, r = $r = \sqrt{d_2^2 + d_1^2/4} = 5.00\ \mathrm{m}$, and

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{4.00\ \mathrm{m}}{6.00\ \mathrm{m}/2}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°.$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(4.00\ \mathrm{A})}{\pi(5.00\ \mathrm{m})}\sin 53.1° = 2.56\times10^{-7}\ \mathrm{T}.$$

22. The fact that $B_y = 0$ at x = 10 cm implies the currents are in opposite directions. Thus,

$$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

using Eq. 29-4 and the fact that $i_1 = 4i_2$. To get the maximum, we take the derivative with respect to x and set equal to zero. This leads to $3x^2 - 2Lx - L^2 = 0$, which factors and becomes $(3x + L)(x - L) = 0$, which has the physically acceptable solution: $x = L$. This produces the maximum B_y: $\mu_0 i_2/2\pi L$. To proceed further, we must determine L. Examination of the datum at $x = 10$ cm in Fig. 29-49(b) leads (using our expression above for B_y and setting that to zero) to $L = 30$ cm.

(a) The maximum value of B_y occurs at $x = L = 30$ cm.

(b) With $i_2 = 0.003$ A we find $\mu_0 i_2/2\pi L = 2.0$ nT.

(c) and (d) Figure 29-49(b) shows that as we get <u>very</u> close to wire 2 (where its field strongly dominates over that of the more distant wire 1) B_y points along the $-y$ direction. The right-hand rule leads us to conclude that wire 2's current is consequently is *into the page*. We previously observed that the currents were in opposite directions, so wire 1's current is *out of the page*.

23. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge q is $\vec{B} = (\mu_0 i/2\pi d)\,\hat{k}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 iq}{2\pi d}\left(\vec{v} \times \hat{k}\right).$$

In this situation, $\vec{v} = v\left(-\hat{j}\right)$ (where v is the speed and is a positive value), and $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left(\left(-\hat{j}\right)\times\hat{k}\right) = -\frac{\mu_0 iqv}{2\pi d}\,\hat{i} = -\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.350\mathrm{A})(1.60\times10^{-19}\mathrm{C})(200\mathrm{m/s})}{2\pi(0.0289\ \mathrm{m})}\,\hat{i}$$
$$= (-7.75\times10^{-23}\ \mathrm{N})\hat{i}.$$

24. Initially, we have $B_{\mathrm{net},y} = 0$ and $B_{\mathrm{net},x} = B_2 + B_4 = 2(\mu_0 i/2\pi d)$ using Eq. 29-4, where $d = 0.15$ m. To obtain the 30º condition described in the problem, we must have

$$B_{\mathrm{net},y} = B_{\mathrm{net},x}\tan(30°) \quad \Rightarrow \quad B_1' - B_3 = 2\left(\frac{\mu_0 i}{2\pi d}\right)\tan(30°)$$

where $B_3 = \mu_0 i/2\pi d$ and $B_1' = \mu_0 i/2\pi d'$. Since $\tan(30º) = 1/\sqrt{3}$, this leads to

(a) We now make the assumption that wire #2 must be at −π/2 rad (−90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the θ_1 = 90º datum in Fig. 29-57(b)), where there is a *maximum* in $B_{\text{net }x}$ (equal to +6 μT), we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(4.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 4.0\text{ A}.$$

(c) The fact that Fig. 29-57(b) increases as θ_1 progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net }y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-57(c)).

(d) Referring now to Fig. 29-57(b) we note that there is no *x*-component of magnetic field from wire 1 when θ_1 = 0, so that plot tells us that B_{2x} = +2.0 μT. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 2.0\text{ A}.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

31. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *P* do not contribute to the field at that point. We use the result of Problem 29-21 to evaluate the contributions to the field at *P*, noting that the nearest wire segments (each of length *a*) produce magnetism into the page at *P* and the further wire segments (each of length 2*a*) produce magnetism pointing out of the page at *P*. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} = \frac{\sqrt{2}\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(13\text{ A})}{8\pi(0.047\text{ m})}$$
$$= 1.96\times10^{-5}\text{T} \approx 2.0\times10^{-5}\text{T}.$$

(b) The direction of the field is into the page.

32. Initially we have

$$B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$$

using Eq. 29-9. In the final situation we use Pythagorean theorem and write cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle θ_2 (shown in Fig. 29-61) then its components are

$$B_{2x} = B_2 \sin\theta_2, \quad B_{2y} = -B_2 \cos\theta_2 .$$

The magnitude-squared of their net field is then (by Pythagoras' theorem) the sum of the square of their net x-component and the square of their net y-component:

$$B^2 = (B_2 \sin\theta_2)^2 + (B_1 - B_2 \cos\theta_2)^2 = B_1^2 + B_2^2 - 2B_1 B_2 \cos\theta_2 .$$

(since $\sin^2\theta + \cos^2\theta = 1$), which we could also have gotten directly by using the law of cosines. We have

$$B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}.$$

With the requirement that the net field have magnitude B = 80 nT, we find

$$\theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1 B_2}\right) = \cos^{-1}(-1/4) = 104°,$$

where the positive value has been chosen.

35. Equation 29-13 gives the magnitude of the force between the wires, and finding the x-component of it amounts to multiplying that magnitude by $\cos\phi = \frac{d_2}{\sqrt{d_1^2 + d_2^2}}$. Therefore, the x-component of the force per unit length is

$$\frac{F_x}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2 + d_2^2)} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\times10^{-3}\ \text{A})(6.80\times10^{-3}\ \text{A})(0.050\ \text{m})}{2\pi[(0.0240\ \text{m})^2 + (0.050\ \text{m})^2]}$$
$$= 8.84\times10^{-11}\ \text{N/m}.$$

36. We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d} + \frac{1}{2d} + \frac{1}{3d} + \frac{1}{4d}\right)\hat{\text{j}} = \frac{25\mu_0 i^2 l}{24\pi d}\,\hat{\text{j}} = \frac{25\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\text{A}\right)^2(10.0\text{m})}{24\pi\left(8.00\times10^{-2}\,\text{m}\right)}\,\hat{\text{j}}$$
$$= (4.69\times10^{-4}\,\text{N})\,\hat{\text{j}}.$$

(b) Similarly, for wire 2, we have

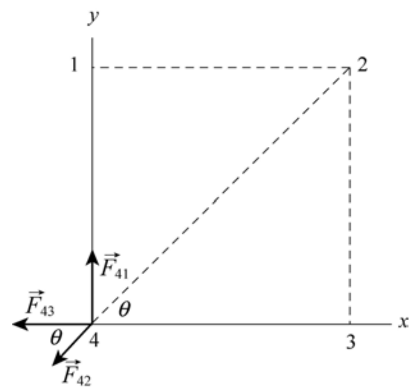
$$\vec{F}_2 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\mathrm{j}} = \frac{5\mu_0 i^2 l}{12\pi d}\,\hat{\mathrm{j}} = (1.88\times10^{-4}\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(c) $F_3 = 0$ (because of symmetry).

(d) $\vec{F}_4 = -\vec{F}_2 = (-1.88\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$, and

(e) $\vec{F}_5 = -\vec{F}_1 = -(4.69\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$.

37. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.



The components of $\vec{F}_4$ are given by

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

and

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$

Thus,

$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\,\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(7.50\,\mathrm{A}\right)^2}{4\pi\left(0.135\,\mathrm{m}\right)}$$

$$= 1.32\times10^{-4}\,\mathrm{N/m}$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°.$$

In unit-vector notation, we have

$$\vec{F}_1 = (1.32\times10^{-4}\,\mathrm{N/m})[\cos162°\,\hat{\mathrm{i}} + \sin162°\,\hat{\mathrm{j}}] = (-1.25\times10^{-4}\,\mathrm{N/m})\hat{\mathrm{i}} + (4.17\times10^{-5}\,\mathrm{N/m})\hat{\mathrm{j}}$$

38. (a) The fact that the curve in Fig. 29-64(b) passes through zero implies that the currents in wires 1 and 3 exert forces in opposite directions on wire 2. Thus, current i_1 points *out of the page*. When wire 3 is a great distance from wire 2, the only field that affects wire 2 is that caused by the current in wire 1; in this case the force is negative according to Fig. 29-64(b). This means wire 2 is attracted to wire 1, which implies (by the discussion in Section 29-2) that wire 2's current is in the same direction as wire 1's

current: *out of the page*. With wire 3 infinitely far away, the force per unit length is given (in magnitude) as 6.27×10^{-7} N/m. We set this equal to $F_{12} = \mu_0 i_1 i_2 / 2\pi d$. When wire 3 is at x = 0.04 m the curve passes through the zero point previously mentioned, so the force between 2 and 3 must equal F_{12} there. This allows us to solve for the distance between wire 1 and wire 2:

$$d = (0.04 \text{ m})(0.750 \text{ A})/(0.250 \text{ A}) = 0.12 \text{ m}.$$

Then we solve 6.27×10^{-7} N/m= $\mu_0 i_1 i_2 /2\pi d$ and obtain i_2 = 0.50 A.

(b) The direction of i_2 is out of the page.

39. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting d = 0.500 m, we find the net force (per meter length) using Eq. 29-13, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.00 \times 10^{-7}\text{ N/m}$.

40. Using Eq. 29-13, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing toward wire 3, which is at the lower right). Only the forces (or their components) along the diagonal direction contribute. With θ = 45°, we find the force per unit meter on wire 1 to be

$$F_1 = |\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}| = 2F_{12}\cos\theta + F_{13} = 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} = \frac{3}{2\sqrt{2}\pi}\left(\frac{\mu_0 i^2}{a}\right)$$

$$= \frac{3}{2\sqrt{2}\pi}\frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(15.0\text{A}\right)^2}{\left(8.50 \times 10^{-2}\text{m}\right)} = 1.12 \times 10^{-3}\text{ N/m.}$$

The direction of $\vec{F}_1$ is along $\hat{r} = (\hat{\text{i}} - \hat{\text{j}})/\sqrt{2}$. In unit-vector notation, we have

$$\vec{F}_1 = \frac{(1.12 \times 10^{-3}\text{ N/m})}{\sqrt{2}}(\hat{\text{i}} - \hat{\text{j}}) = (7.94 \times 10^{-4}\text{ N/m})\hat{\text{i}} + (-7.94 \times 10^{-4}\text{ N/m})\hat{\text{j}}$$

41. The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with i_1 = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp \text{ sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y} dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$\begin{aligned} F &= \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)} \\ &= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(30.0\,\text{A}\right)\left(20.0\,\text{A}\right)\left(8.00\,\text{cm}\right)\left(300\times10^{-2}\,\text{m}\right)}{2\pi\left(1.00\,\text{cm} + 8.00\,\text{cm}\right)} = 3.20\times10^{-3}\,\text{N}, \end{aligned}$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\text{N})\hat{\text{j}}$ in unit-vector notation.

42. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(15\,\text{A/m}^2\right)\left(6\right)\left(0.20\,\text{m}\right)^2 = 4.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

43. We use Eq. 29-20 $B = \mu_0 i r/2\pi a^2$ for the B-field inside the wire ($r < a$) and Eq. 29-17 $B = \mu_0 i/2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\,\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\,\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\,\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

44. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\,\text{A} + 3.0\,\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-2.0\,\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\,\text{A} - 5.0\,\text{A} - 3.0\,\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-13.0\,\text{A}\right) = -1.6\times10^{-5}\,\text{T}\cdot\text{m}.$$

45. (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B}\cdot d\vec{s} = -\mu_0 i = -(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(2.0\,\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

46. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(7i - 6i + 3i + i\right) = 5\mu_0 i = 5\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(4.50\times10^{-3}\,\text{A}\right) = 2.83\times10^{-8}\,\text{T}\cdot\text{m}.$$

47. For $r \le a$,

$$B\left(r\right) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J\left(r\right)2\pi r\,dr = \frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r\,dr = \frac{\mu_0 J_0 r^2}{3a}.$$

(a) At $r = 0,\ B = 0$.

(b) At $r = a/2$, we have

$$B\left(r\right) = \frac{\mu_0 J_0 r^2}{3a} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m}/2)^2}{3(3.1\times10^{-3}\,\text{m})} = 1.0\times10^{-7}\,\text{T}.$$

(c) At $r = a$,

$$B\left(r = a\right) = \frac{\mu_0 J_0 a}{3} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m})}{3} = 4.0\times10^{-7}\,\text{T}.$$

48. (a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi\left(3R\right)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\text{A})/8 = 3.00\times10^{-3}\,\text{A}$.

(b) The direction is into the page.

49. (a) We use Eq. 29-24. The inner radius is r = 15.0 cm, so the field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(0.800\,\text{A}\right)\left(500\right)}{2\pi\left(0.150\,\text{m}\right)} = 5.33\times10^{-4}\,\text{T}.$$

(b) The outer radius is r = 20.0 cm. The field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(0.800\,\text{A}\right)\left(500\right)}{2\pi\left(0.200\text{m}\right)} = 4.00\times10^{-4}\,\text{T}.$$

50. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 3.60 A, $\ell = 0.950$ m, and N = 1200. This yields B = 0.00571 T.

51. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 0.30 A, $\ell = 0.25$ m, and N = 200. This yields $B = 3.0\times10^{-4}$ T .

52. We find N, the number of turns of the solenoid, from the magnetic field $B = \mu_0 in = \mu_o iN/\ell$: $N = B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN=\frac{2\pi rB\ell}{\mu_0 i}=\frac{2\pi\left(2.60\times10^{-2}\text{ m}\right)\left(23.0\times10^{-3}\text{ T}\right)\left(1.30\text{ m}\right)}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(18.0\text{ A}\right)}=108\text{ m.}$$

53. The orbital radius for the electron is

$$r=\frac{mv}{eB}=\frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$i=\frac{mv}{e\mu_0 nr}=\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(0.0460\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(100/0.0100\text{m}\right)\left(2.30\times10^{-2}\text{m}\right)}$$
$$=0.272\text{A}.$$

54. As the problem states near the end, some idealizations are being made here to keep the calculation straightforward (but are slightly unrealistic). For circular motion (with speed, $v_\perp$, which represents the magnitude of the component of the velocity perpendicular to the magnetic field [the field is shown in Fig. 29-19]), the period is (see Eq. 28-17)

$$T = 2\pi r/v_\perp = 2\pi m/eB.$$

Now, the time to travel the length of the solenoid is $t=L/v_{\parallel}$ where $v_{\parallel}$ is the component of the velocity in the direction of the field (along the coil axis) and is equal to $v\cos\theta$ where θ = 30º. Using Eq. 29-23 (B = $\mu_0 in$) with n = N/L, we find the number of revolutions made is $t/T = 1.6\times10^6$.

55. (a) We denote the $\vec{B}$ fields at point P on the axis due to the solenoid and the wire as $\vec{B}_s$ and $\vec{B}_w$, respectively. Since $\vec{B}_s$ is along the axis of the solenoid and $\vec{B}_w$ is perpendicular to it, $\vec{B}_s\perp\vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis we then must have B_s = B_w. Thus,

$$B_s=\mu_0 i_s n=B_w=\frac{\mu_0 i_w}{2\pi d}\ ,$$

which gives the separation d to point P on the axis:

$$d=\frac{i_w}{2\pi i_s n}=\frac{6.00\text{ A}}{2\pi\left(20.0\times10^{-3}\text{ A}\right)\left(10\text{ turns/cm}\right)}=4.77\text{ cm}.$$

(b) The magnetic field strength is

$$B=\sqrt{2}B_s=\sqrt{2}\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\text{ A}\right)\left(10\text{ turns/0.0100 m}\right)=3.55\times10^{-5}\text{T}.$$

56. We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 i R^2 N}{2\left[R^2 + (R/2)^2\right]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R} = \frac{8\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(200)\left(0.0122\,\mathrm{A}\right)}{5\sqrt{5}\left(0.25\,\mathrm{m}\right)} = 8.78\times10^{-6}\ \mathrm{T}.$$

$\vec{B}_P$ is in the positive x direction.

57. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\,\mathrm{A\cdot m^2}\ .$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\ .$$

We solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(2.36\,\mathrm{A\cdot m^2}\right)}{2\pi\left(5.0\times10^{-6}\ \mathrm{T}\right)}\right)^{1/3} = 46\,\mathrm{cm}\ .$$

58. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0\,.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50.$$

59. The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = (200)(0.30\,\mathrm{A})\pi(0.050\,\mathrm{m})^2 = 0.47\,\mathrm{A\cdot m^2}\ .$$

60. Using Eq. 29-26, we find that the net y-component field is

$$B_y = \frac{\mu_0 i_1 R^2}{2\pi(R^2+z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2\pi(R^2+z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ (see Fig. 29-73(a)) and $z_2^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-73(b) corresponding to $B_y = 0$ would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1. We obtain $i_1 = (45/16\pi)$ A ≈ 0.90 A.

(b) With loop 2 at $y = 0.06$ m (see Fig. 29-73(b)) we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2+L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2+y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A ≈ 2.7 A.

61. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(15\,\text{A}\right)}{2\left(0.12\,\text{m}\right)} = 7.9\times10^{-5}\,\text{T}.$$

(b) The torque has magnitude equal to

$$\begin{aligned}\tau &= |\vec{\mu}_2 \times \vec{B}_1| = \mu_2 B_1 \sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1\\ &= \pi\left(50\right)\left(1.3\text{A}\right)\left(0.82\times10^{-2}\,\text{m}\right)^2\left(7.9\times10^{-5}\,\text{T}\right)\\ &= 1.1\times10^{-6}\,\text{N}\cdot\text{m}.\end{aligned}$$

62. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$\begin{aligned}B &= \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(0.0562\,\text{A}\right)}{4}\left(\frac{1}{0.0572\,\text{m}}+\frac{1}{0.0936\,\text{m}}\right)\\ &= 4.97\times10^{-7}\,\text{T}.\end{aligned}$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-6(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2 + b^2) = \frac{\pi(0.0562\text{A})}{2}[(0.0572\text{m})^2 + (0.0936\text{m})^2] = 1.06\times10^{-3}\,\text{A}\cdot\text{m}^2.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

63. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\begin{aligned}\vec{\mu} &= \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = \left(ia^2\right)\left(\hat{\text{j}} - \hat{\text{i}} + \hat{\text{i}}\right) = ia^2\hat{\text{j}} \\ &= \left(6.0\,\text{A}\right)\left(0.10\,\text{m}\right)^2\hat{\text{j}} = (6.0\times10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\text{j}}\,.\end{aligned}$$

(b) Since both points are far from the cube we can use the dipole approximation. For $(x, y, z) = (0, 5.0\text{ m}, 0)$,

$$\vec{B}(0,\,5.0\text{ m},\,0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\text{ T}\cdot\text{m/A})(6.0\times10^{-2}\text{m}^2\cdot\text{A})\,\hat{\text{j}}}{2\pi(5.0\text{ m})^3} = (9.6\times10^{-11}\text{ T})\,\hat{\text{j}}\,.$$

64. (a) The radial segments do not contribute to $\vec{B}_P$, and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}} - \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}$$

where $i = 0.200$ A. This yields $\vec{B} = -2.75\times10^{-8}\,\hat{\text{k}}$ T, or $|\vec{B}| = 2.75\times10^{-8}$ T.

(b) The direction is $-\hat{\text{k}}$, or into the page.

65. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r\,,$$

we find that $r = 0.00128$ m gives the desired field value.

66. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B = 0.050$ m wire B. Using Eq. 29-4, we have

68. We take the current (i = 50 A) to flow in the $+x$ direction, and the electron to be at a point P, which is r = 0.050 m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 29-4 with Eq. 28-2, we obtain

$$\vec{F}_e = \left(-e\mu_0 i/2\pi r\right)\left(\vec{v} \times \hat{\mathrm{k}}\right).$$

(a) The electron is moving down: $\vec{v} = -v\hat{\mathrm{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\mathrm{i}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{i}}\,,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N .

(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{\mathrm{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\mathrm{j}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{j}}\,,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N .

(c) Now, $\vec{v} = \pm v\hat{\mathrm{k}}$ so $\vec{F}_e \propto \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0.$

69. (a) By the right-hand rule, the magnetic field $\vec{B}_1$ (evaluated at a) produced by wire 1 (the wire at bottom left) is at ϕ = 150° (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at ϕ = 210°. By symmetry $\left(\vec{B}_1 = \vec{B}_2\right)$ we observe that only the x-components survive, yielding

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \left(2\,\frac{\mu_0 i}{2\pi\ell}\,\cos 150°\right)\hat{\mathrm{i}} = (-3.46\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$$

where i = 10 A, ℓ = 0.10 m, and Eq. 29-4 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\mathrm{k}}$ direction) of value

$$i_b = B\frac{2\pi r}{\mu_0} = \left(3.46\times10^{-5}\ \mathrm{T}\right)\frac{2\pi(0.087\ \mathrm{m})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 15\,\mathrm{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and Eq. 29-4 has again been used.

(b) As stated above, to cancel this, wire b must carry current into the page (that is, the $-z$ direction).

to the sheet and only has a horizontal component. That is, the field at P must be purely horizontal, as drawn in Fig. 29-83.

(b) The path used in evaluating $\oint \vec{B}\cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P' respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown next. Ampere's law yields

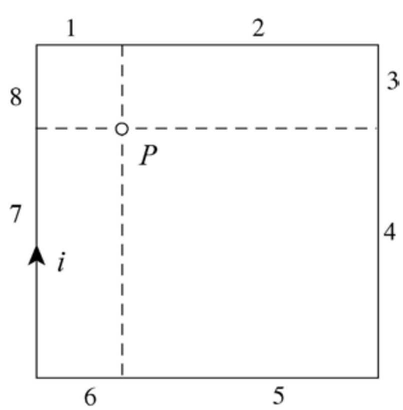
$$2B\Delta x = \mu_0 \lambda \Delta x \;\;\Rightarrow\;\; B = \frac{1}{2}\mu_0 \lambda.$$

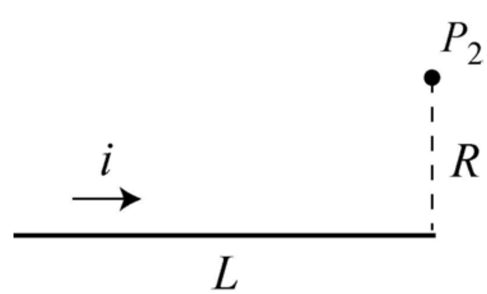
82. Equation 29-17 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)} = 1.25\times10^{-6}\ \text{T},$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement. Using the right-hand rule, we find both horizontal components point in the $+x$ direction. Thus, in unit-vector notation, we have $\vec{B} = (1.25\times10^{-6}\ \text{T})\hat{\text{i}}$.

83. The two small wire segments, each of length $a/4$, shown in Fig. 29-85 nearest to point P, are labeled 1 and 8 in the figure (below left). Let $-\hat{\text{k}}$ be a unit vector pointing into the page.





We use the result of Problem 29-17: namely, the magnetic field at P_2 (shown in Fig. 29-43 and upper right) is