Chapter 27

1. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0$. We solve for i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2R$.

(b) For R_1, $P_1 = i^2R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0\ \text{W}$,

(c) and for R_2, $P_2 = i^2R_2 = (0.50\ \text{A})^2\ (8.0\ \Omega) = 2.0\ \text{W}$.

If i is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0\ \text{W}$

(e) and for ε_2, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0\ \text{W}$.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

2. The current in the circuit is

$$i = (150\ \text{V} - 50\ \text{V})/(3.0\ \Omega + 2.0\ \Omega) = 20\ \text{A}.$$

So from $V_Q + 150\ \text{V} - (2.0\ \Omega)i = V_P$, we get $V_Q = 100\ \text{V} + (2.0\ \Omega)(20\ \text{A}) - 150\ \text{V} = -10\ \text{V}$.

3. (a) The potential difference is $V = \varepsilon + ir = 12\ \text{V} + (50\ \text{A})(0.040\ \Omega) = 14\ \text{V}$.

(b) $P = i^2r = (50\ \text{A})^2(0.040\ \Omega) = 1.0\times10^2\ \text{W}$.

(c) $P' = iV$ = (50 A)(12 V) = 6.0×10^2 W.

(d) In this case $V = \varepsilon - ir$ = 12 V – (50 A)(0.040 Ω) = 10 V.

(e) $P_r = i^2 r$ =(50 A)^2(0.040 Ω) = 1.0×10^2 W.

4. (a) The loop rule leads to a voltage-drop across resistor 3 equal to 5.0 V (since the total drop along the upper branch must be 12 V). The current there is consequently i = (5.0 V)/(200 Ω) = 25 mA. Then the resistance of resistor 1 must be (2.0 V)/i = 80 Ω.

(b) Resistor 2 has the same voltage-drop as resistor 3; its resistance is 200 Ω.

5. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time Δt = 6.0 min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0\text{ A})(6.0\text{ V})\,(6.0\text{ min})\,(60\text{ s/min}) = 1.1\times 10^4\text{ J}.$$

We note the conversion of time from minutes to seconds.

6. (a) The cost is (100 W · 8.0 h/2.0 W · h) ($0.80) = $\$3.2 \times 10^2$.

(b) The cost is (100 W · 8.0 h/10^3 W · h) ($0.06) = $0.048 = 4.8 cents.

7. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\text{ V})^2(2.0\,\text{min})\,(60\,\text{s}/\text{min})}{1.0\,\Omega + 5.0\,\Omega} = 80\text{ J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\text{ V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)\,(2.0\text{ min})\,(60\text{ s}/\text{min}) = 67\text{ J}.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

8. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

(b) The simultaneous solution also gives $r_2 = 0.30\ \Omega$.

15. Let the emf be V. Then $V = iR = i'(R + R')$, where $i = 5.0$ A, $i' = 4.0$ A, and $R' = 2.0\ \Omega$. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\ \text{A})(2.0\ \Omega)}{5.0\ \text{A} - 4.0\ \text{A}} = 8.0\ \Omega.$$

16. (a) Let the emf of the solar cell be ε and the output voltage be V. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10\ \text{V} = \varepsilon - (0.10\ \text{V}/500\ \Omega)r$$
$$0.15\ \text{V} = \varepsilon - (0.15\ \text{V}/1000\ \Omega)r.$$

We solve for ε and r.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\,\text{V}}{(1000\,\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

17. To be as general as possible, we refer to the individual emfs as ε_1 and ε_2 and wait until the latter steps to equate them ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\varepsilon_1 + \varepsilon_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

(a) The current in the circuit is

$$i = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R},$$

and the requirement of zero terminal voltage leads to $\varepsilon_1 = ir_1$, or

$$R = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{\varepsilon_1} = \frac{(12.0\ \text{V})(0.016\,\Omega) - (12.0\ \text{V})(0.012\,\Omega)}{12.0\ \text{V}} = 0.0040\ \Omega.$$

Note that $R = r_1 - r_2$ when we set $\varepsilon_1 = \varepsilon_2$.

(b) As mentioned above, this occurs in battery 1.

18. The currents i_1, i_2 and i_3 are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3)-\varepsilon_2 R_3}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\,\text{V})(10\,\Omega+5.0\,\Omega)-(1.0\,\text{V})(5.0\ \Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\ \Omega)} = 0.275\ \text{A},$$

$$i_2 = \frac{\varepsilon_1 R_3-\varepsilon_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\ \text{V})(5.0\,\Omega)-(1.0\ \text{V})(10\,\Omega+5.0\ \Omega)}{(10\,\Omega)(10\ \Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\ \Omega)} = 0.025\ \text{A},$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}\ .$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\text{ A})\,(10\ \Omega) = +0.25\text{ V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -\,(-\,0.250\text{ A})\,(5.0\ \Omega) - 1.0\text{ V} = +0.25\text{ V}.$$

19. (a) Since $R_{\text{eq}} < R$, the two resistors (R = 12.0 Ω and R_x) must be connected in parallel:

$$R_{\text{eq}} = 3.00\,\Omega = \frac{R_x R}{R+R_x} = \frac{R_x\left(12.0\,\Omega\right)}{12.0\,\Omega + R_x}.$$

We solve for R_x: $R_x = R_{\text{eq}}R/(R - R_{\text{eq}})$ = (3.00 Ω)(12.0 Ω)/(12.0 Ω – 3.00 Ω) = 4.00 Ω.

(b) As stated above, the resistors must be connected in parallel.

20. Let the resistances of the two resistors be R_1 and R_2, with $R_1 < R_2$. From the statements of the problem, we have

$$R_1R_2/(R_1 + R_2) = 3.0\ \Omega \text{ and } R_1 + R_2 = 16\ \Omega.$$

So R_1 and R_2 must be 4.0 Ω and 12 Ω, respectively.

(a) The smaller resistance is R_1 = 4.0 Ω.

(b) The larger resistance is R_2 = 12 Ω.

21. The potential difference across each resistor is V = 25.0 V. Since the resistors are identical, the current in each one is $i = V/R$ = (25.0 V)/(18.0 Ω) = 1.39 A. The total current through the battery is then i_{total} = 4(1.39 A) = 5.56 A. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus

$$i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\text{ V})/(18.0\ \Omega) = 5.56\text{ A}.$$

22. (a) R_{eq} (FH) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) R_{eq} (FG) = (5.00 Ω) $R/(R$ + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} (FG) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

23. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0\,.$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0\text{ V}}{100\,\Omega} = 0.050\text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0\,.$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0\text{ V} - 5.0\text{ V} - 4.0\text{ V}}{50\,\Omega} = -0.060\text{ A}\,,$$

or $|\,i_2\,| = 0.060$ A. The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0 \text{ V} + 5.0 \text{ V} = 9.0 \text{ V}.$$

24. We note that two resistors in parallel, R_1 and R_2, are equivalent to

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \; R_{12} = \frac{R_1 R_2}{R_1 + R_2}.$$

This situation consists of a parallel pair that are then in series with a single $R_3 = 2.50\ \Omega$ resistor. Thus, the situation has an equivalent resistance of

$$R_{\text{eq}} = R_3 + R_{12} = 2.50\Omega + \frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega + 4.00\Omega} = 4.50\Omega.$$

25. Let r be the resistance of each of the narrow wires. Since they are in parallel the resistance R of the composite is given by

$$\frac{1}{R} = \frac{9}{r},$$

or $R = r/9$. Now $r = 4\rho\ell/\pi d^2$ and $R = 4\rho\ell/\pi D^2$, where ρ is the resistivity of copper. Note that $A = \pi d^2/4$ was used for the cross-sectional area of a single wire, and a similar expression was used for the cross-sectional area of the thick wire. Since the single thick wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2} \;\Rightarrow\; D = 3d.$$

26. The part of R_0 connected in parallel with R is given by $R_1 = R_0 x/L$, where $L = 10$ cm. The voltage difference across R is then $V_R = \varepsilon R'/R_{\text{eq}}$, where $R' = RR_1/(R + R_1)$ and

$$R_{\text{eq}} = R_0(1 - x/L) + R'.$$

Thus,

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L) + RR_1/(R+R_1)}\right)^2 = \frac{100R(\varepsilon x/R_0)^2}{\left(100R/R_0 + 10x - x^2\right)^2},$$

where x is measured in cm.

27. Since the potential differences across the two paths are the same, $V_1 = V_2$ (V_1 for the left path, and V_2 for the right path), we have $i_1 R_1 = i_2 R_2$, where $i = i_1 + i_2 = 5000$ A. With $R = \rho L/A$ (see Eq. 26-16), the above equation can be rewritten as

$$i_1 d = i_2 h \quad \Rightarrow \quad i_2 = i_1 (d/h) .$$

With $d/h = 0.400$, we get $i_1 = 3571\,\text{A}$ and $i_2 = 1429\,\text{A}$. Thus, the current through the person is $i_1 = 3571\,\text{A}$, or approximately $3.6\,\text{kA}$.

28. Line 1 has slope R_1 = 6.0 kΩ. Line 2 has slope R_2 = 4.0 kΩ. Line 3 has slope R_3 = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with R_3 gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4\ \text{k}\Omega + 2.0\ \text{k}\Omega = 4.4\ \text{k}\Omega .$$

The current through the battery is therefore $i = \varepsilon / R_{123} = (6\ \text{V})/(4.4\ \text{k}\Omega)$ and the voltage drop across R_3 is (6 V)(2 kΩ)/(4.4 kΩ) = 2.73 V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2. Then Ohm's law gives the current through R_2: (6 V – 2.73 V)/(4 kΩ) = 0.82 mA .

29. (a) The parallel set of three identical R_2 = 18 Ω resistors reduce to R= 6.0 Ω, which is now in series with the R_1 = 6.0 Ω resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R$ = 12 Ω. Thus, the current through R' is (12V)/R' = 1.0 A, which is the current through R. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, i_1 = 0.333 A.

(b) The direction of i_1 is clearly rightward.

(c) We use Eq. 26-27: $P = i^2R'$ = (1.0 A)^2(12 Ω) = 12 W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

30. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$10.0\ \text{V} - i_1R_1 - (i_1 + i_2)\, R_3 = 0$$

$$5.00\ \text{V} - i_2R_2 - (i_1 + i_2)\, R_3 = 0.$$

(a) Solving, we find i_2 = 0, and

(b) $i_3 = i_1 + i_2$ = 1.25 A (downward, as was assumed in writing the equations as we did).

31. (a) We reduce the parallel pair of identical 2.0 Ω resistors (on the right side) to R' = 1.0 Ω, and we reduce the series pair of identical 2.0 Ω resistors (on the upper left side) to R'' = 4.0 Ω. With R denoting the 2.0 Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series, which are equivalent to

$$R + R' + R'' = 7.0\ \Omega$$

across which the voltage is 7.0 V (by the loop rule, this is 12 V – 5.0 V), implying that the current is 1.0 A (clockwise). Thus, the voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is 12 – 1 = 11 V. Noting the orientation of the battery, we conclude $V_1 = -11$ V.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is 5.0 + 4.0 = 9.0 V. Noting the orientation of the battery, we conclude V_2 = –9.0 V. This can be verified by considering the voltage across R and the value we obtained for V_1.

32. (a) For typing convenience, we denote the emf of battery 2 as V_2 and the emf of battery 1 as V_1. The loop rule (examining the left-hand loop) gives $V_2 + i_1 R_1 - V_1 = 0$. Since V_1 is held constant while V_2 and i_1 vary, we see that this expression (for large enough V_2) will result in a negative value for i_1, so the downward sloping line (the line that is dashed in Fig. 27-43(b)) must represent i_1. It appears to be zero when V_2 = 6 V. With $i_1 = 0$, our loop rule gives $V_1 = V_2$, which implies that V_1 = 6.0 V.

(b) At V_2 = 2 V (in the graph) it appears that i_1 = 0.2 A. Now our loop rule equation (with the conclusion about V_1 found in part (a)) gives R_1 = 20 Ω.

(c) Looking at the point where the upward-sloping i_2 line crosses the axis (at V_2 = 4 V), we note that i_1 = 0.1 A there and that the loop rule around the right-hand loop should give

$$V_1 - i_1 R_1 = i_1 R_2$$

when i_1 = 0.1 A and i_2 = 0. This leads directly to R_2 = 40 Ω.

33. First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40\text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8\text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4$ = 16.8 V/(16.0 Ω) = 1.05 A.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.05\text{ A} + 1.40\text{ A} = 2.45\text{ A},$$

so its voltage is V_2 = (2.00 Ω)(2.45 A) = 4.90 V.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4$ = 21.7 V (implying that the current through it is $i_3 = V_3$/(2.00 Ω) = 10.85 A).

The junction rule now gives the current in R_1 as $i_1 = i_2 + i_3$ = 2.45 A + 10.85 A = 13.3 A, implying that the voltage across it is V_1 = (13.3 A)(2.00 Ω) = 26.6 V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6 \text{ V} + 21.7 \text{ V} = 48.3 \text{ V}.$$

34. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in R_3, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A, which implies that the voltage drop across R_3 is 8.0 V. The loop rule then tells us that the voltage drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

35. (a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2 R_2 - i_1 R_1 = 0$$
$$\varepsilon - 2 i_1 R_1 - (i_1 - i_2) R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A, $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore, $V_A - V_B = i_1 R_1 = 5.25$ V.

(b) It follows also that $V_B - V_C = i_3 R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1 R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2 R_2 = 6.75$ V.

36. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$
$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0.$$

Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) The direction is downward. See the results in part (a).

Clearly the only physically interesting solution to this is $n = 8$. Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

43. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{eq}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR}.$$

Since the problem requires $R_{eq} = 10\ \Omega = R$, we must select $n = m$. Next we make use of Eq. 27-16. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{total} = n^2P$, where $P = 1.0\text{ W}$ is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{total} \ge 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

44. (a) Resistors R_2, R_3, and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega) + (50.0\Omega)(75.0\Omega) + (50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\text{ V}/(118.8\ \Omega) = 5.05 \times 10^{-2}\text{ A}$.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0\text{V} - (5.05 \times 10^{-2}\text{ A})(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}\text{ A}$.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\text{ A})(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}\text{ A}$.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\text{ A} - 2(1.90 \times 10^{-2}\text{ A}) = 1.25 \times 10^{-2}\text{ A}$.

45. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\text{ V} - 2.0\text{ V}}{4(1.0\ \Omega) + 2.0\ \Omega} = 0.33\text{ A}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\text{ A})(2.0\ \Omega) + 4.0\text{ V} = 3.3\text{ V}$.

46. (a) When $R_3 = 0$ all the current passes through R_1 and R_3 and avoids R_2 altogether. Since that value of the current (through the battery) is 0.006 A (see Fig. 27-55(b)) for $R_3 = 0$ then (using Ohm's law)

$$R_1 = (12\text{ V})/(0.006\text{ A}) = 2.0\times10^3\ \Omega.$$

(b) When $R_3 = \infty$ all the current passes through R_1 and R_2 and avoids R_3 altogether. Since that value of the current (through the battery) is 0.002 A (stated in problem) for $R_3 = \infty$ then (using Ohm's law)

$$R_2 = (12\text{ V})/(0.002\text{ A}) - R_1 = 4.0\times10^3\ \Omega.$$

47. (a) The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}.$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}$$

and

53. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop to obtain

$$\varepsilon - iR_2 - i_1R_1 - ir = 0.$$

We apply the loop rule to the right-hand loop to obtain

$$i_1R_1 - (i - i_1)R_V = 0.$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} i_1.$$

We substitute this into the first equation to obtain

$$\varepsilon - \frac{(R_2 + r)(R_1 + R_V)}{R_V} i_1 + R_1 i_1 = 0.$$

This has the solution

$$i_1 = \frac{\varepsilon R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}.$$

The reading on the voltmeter is

$$i_1R_1 = \frac{\varepsilon R_V R_1}{(R_2 + r)\,(R_1 + R_V) + R_1R_V} = \frac{(3.0\text{V})\,(5.0\times10^3\,\Omega)\,(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0\times10^3\,\Omega) + (250\,\Omega)(5.0\times10^3\,\Omega)}$$
$$= 1.12\ \text{V}.$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1R_1 = \frac{\varepsilon R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\ \Omega + 300\ \Omega + 100\,\Omega} = 1.15\,\text{V}.$$

The fractional error is (1.12 – 1.15)/(1.15) = –0.030, or –3.0%.

54. (a) $\varepsilon = V + ir$ = 12 V + (10.0 A) (0.0500 Ω) = 12.5 V.

(b) Now $\varepsilon = V' + (i_{\text{motor}} + 8.00\ \text{A})r$, where

$$V' = i'_A R_{\text{light}} = (8.00\ \text{A})\ (12.0\ \text{V}/10\ \text{A}) = 9.60\ \text{V}.$$

Therefore,

$$i_{\text{motor}} = \frac{\varepsilon - V'}{r} - 8.00\,\text{A} = \frac{12.5\,\text{V} - 9.60\,\text{V}}{0.0500\,\Omega} - 8.00\,\text{A} = 50.0\,\text{A}.$$

55. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\frac{R_1}{R_s}i_1 \quad \Rightarrow \quad R_x = \frac{R_2 R_s}{R_1}.$$

56. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V} \quad \Rightarrow R' = \frac{RR_V}{R + R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}} = R_A + R_0 + R' = R_A + R_0 + \dfrac{RR_V}{R + R_V}$.

(a) The ammeter reading is

$$i' = \frac{\varepsilon}{R_{\text{eq}}} = \frac{\varepsilon}{R_A + R_0 + R_V\, R/(R + R_V)} = \frac{12.0\,\text{V}}{3.00\,\Omega + 100\,\Omega + (300\,\Omega)\,(85.0\,\Omega)/(300\,\Omega + 85.0\,\Omega)}$$
$$= 7.09 \times 10^{-2}\,\text{A}.$$

(b) The voltmeter reading is

$$V = \varepsilon - i'(R_A + R_0) = 12.0\text{ V} - (0.0709\text{ A})(103.00\ \Omega) = 4.70\text{ V}.$$

(c) The apparent resistance is $R' = V/i' = 4.70\text{ V}/(7.09 \times 10^{-2}\text{ A}) = 66.3\ \Omega$.

(d) If R_V is increased, the difference between R and R' decreases. In fact, $R' \to R$ as $R_V \to \infty$.

57. Here we denote the battery emf as V. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{\text{cap}}$, or

$$Ve^{-t/RC} = V(1 - e^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC \ln 2$, or $t = 0.208$ ms.

58. (a) $\tau = RC = (1.40 \times 10^6\ \Omega)(1.80 \times 10^{-6}\text{ F}) = 2.52$ s.

(b) $q_o = \varepsilon C$ = (12.0 V)(1.80 μ F) = 21.6 μC.

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\text{s})\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\,\text{s}.$$

59. During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{\text{eq}} = C\varepsilon$. We require $q = 0.99q_{\text{eq}} = 0.99C\varepsilon$, so

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

60. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln (q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus,

$$t_{1/3} = \tau\ln\left(\frac{q_0}{2q_0/3}\right) = \tau\ln\left(\frac{3}{2}\right) = 0.41\tau \quad \Rightarrow \quad \frac{t_{1/3}}{\tau} = 0.41.$$

(b) $t_{2/3} = \tau\ln\left(\dfrac{q_0}{q_0/3}\right) = \tau\ln 3 = 1.1\tau \quad \Rightarrow \quad \dfrac{t_{2/3}}{\tau} = 1.1.$

61. (a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At t = 1.30 μs we have $V(t)$ = 5.00 V, so 5.00 V = (12.0 V)$(1 - e^{-1.30\,\mu\text{s}/RC})$, which gives

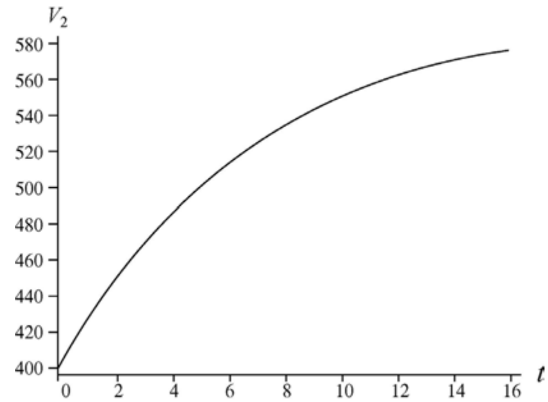
$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R$ = (2.41 μs)/(15.0 kΩ) = 161 pF.

62. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \varepsilon\left(1 - e^{-t/RC}\right)$. We solve for R:

$$R = \frac{t}{C\ln\left[\varepsilon/(\varepsilon - V_L)\right]} = \frac{0.500\,\text{s}}{(0.150\times10^{-6}\,\text{F})\ln\left[95.0\,\text{V}/(95.0\,\text{V} - 72.0\,\text{V})\right]} = 2.35\times10^{6}\,\Omega$$

where we used t = 0.500 s given (implicitly) in the problem.



64. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0 e^{-t/\tau}$, this means $V = V_0 e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\text{s}}{\ln[(1.00\,\text{V})/(100\,\text{V})]} = 2.17\,\text{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0\text{ s})/(2.17\text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = (100\,\text{V})e^{-7.83} = 3.96\times10^{-2}\text{ V}.$$

65. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2\frac{\varepsilon}{R_1+R_2} = (15.0\text{ k}\Omega)\left(\frac{20.0\,\text{V}}{10.0\text{ k}\Omega+15.0\text{ k}\Omega}\right) = 12.0\,\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11×10^{-4} A.

66. We apply Eq. 27-39 to each capacitor, demand their initial charges are in a ratio of 3:2 as described in the problem, and solve for the time. With

$$\tau_1 = R_1C_1 = (20.0\,\Omega)(5.00\times10^{-6}\text{ F}) = 1.00\times10^{-4}\text{ s}$$
$$\tau_2 = R_2C_2 = (10.0\,\Omega)(8.00\times10^{-6}\text{ F}) = 8.00\times10^{-5}\text{ s},$$

$$i\varepsilon = \left(9.55\times10^{-7}\text{ A}\right)\left(4.00\text{ V}\right) = 3.82\times10^{-6}\text{ W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)\,(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the R = 4.0 Ω resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to 4.0 Ω) and ε denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3\left(\varepsilon - ir\right) - \left(2i\right)R = 0.$$

This yields i = 3.0 A. Consequently, i_R = 6.0 A.

(b) The terminal voltage of each battery is $\varepsilon - ir$ = 8.0 V.

(c) Using Eq. 27-17, we obtain $P = i\varepsilon$ = (3)(20) = 60 W.

(d) Using Eq. 26-27, we have $P = i^2r$ = 36 W.

71. (a) If S_1 is closed, and S_2 and S_3 are open, then $i_a = \varepsilon/2R_1$ = 120 V/40.0 Ω = 3.00 A.

(b) If S_3 is open while S_1 and S_2 remain closed, then

$$R_{\text{eq}} = R_1 + R_1\,(R_1 + R_2)/(2R_1 + R_2) = 20.0\ \Omega + (20.0\ \Omega)\times(30.0\ \Omega)/(50.0\ \Omega) = 32.0\ \Omega,$$

so $i_a = \varepsilon/R_{\text{eq}}$ = 120 V/32.0 Ω = 3.75 A.

(c) If all three switches S_1, S_2, and S_3 are closed, then $R_{\text{eq}} = R_1 + R_1\,R'/(R_1 + R')$ where

$$R' = R_2 + R_1\,(R_1 + R_2)/(2R_1 + R_2) = 22.0\ \Omega,$$

that is,

$$R_{\text{eq}} = 20.0\ \Omega + (20.0\ \Omega)\,(22.0\ \Omega)/(20.0\ \Omega + 22.0\ \Omega) = 30.5\ \Omega,$$

so $i_a = \varepsilon/R_{\text{eq}}$ = 120 V/30.5 Ω = 3.94 A.

72. (a) The four resistors R_1, R_2, R_3, and R_4 on the left reduce to

$$R_{\text{eq}} = R_{12} + R_{34} = \frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4} = 7.0\ \Omega + 3.0\ \Omega = 10\ \Omega.$$

With $\varepsilon = 30$ V across R_{eq} the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{eq} = R_{56} + R_7 = \frac{R_5 R_6}{R_5 + R_6} + R_7 = \frac{(6.0\,\Omega)(2.0\,\Omega)}{6.0\,\Omega + 2.0\,\Omega} + 1.5\,\Omega = 3.0\,\Omega\,.$$

With $\varepsilon = 30$ V across R'_{eq} the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2}\, i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30\text{V} - i_4(1.5\,\Omega) - i_5(2.0\,\Omega) = 0$$

readily yields $i_5 = 7.5$ A.

73. (a) The magnitude of the current density vector is

$$J_A = \frac{i}{A} = \frac{V}{(R_1 + R_2)A} = \frac{4V}{(R_1 + R_2)\pi D^2} = \frac{4(60.0\text{V})}{\pi(0.127\Omega + 0.729\Omega)(2.60\times 10^{-3}\,\text{m})^2}$$
$$= 1.32\times 10^7 \text{ A/m}^2\,.$$

(b) $V_A = V R_1/(R_1 + R_2) = (60.0\text{ V})(0.127\,\Omega)/(0.127\,\Omega + 0.729\,\Omega) = 8.90$ V.

(c) The resistivity of wire A is

$$\rho_A = \frac{R_A A}{L_A} = \frac{\pi R_A D^2}{4L_A} = \frac{\pi(0.127\,\Omega)(2.60\times 10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 1.69\times 10^{-8}\,\Omega\cdot\text{m}\,.$$

So wire A is made of copper.

(d) $J_B = J_A = 1.32\times 10^7 \text{ A/m}^2$.

(e) $V_B = V - V_A = 60.0\text{ V} - 8.9\text{ V} = 51.1$ V.

(f) The resistivity of wire B is $\rho_B = 9.68\times 10^{-8}\,\Omega\cdot\text{m}$, so wire B is made of iron.

74. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current i). As if we were presented with a

maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which — as in any good maze — winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\varepsilon = 40$ V.

(a) The current through R is then $i = \varepsilon/R = 4.0$ A.

(b) The direction is upward in the figure.

75. (a) In the process described in the problem, no charge is gained or lost. Thus, q = constant. Hence,

$$q = C_1V_1 = C_2V_2 \quad \Rightarrow \quad V_2 = V_1\frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3.0\times10^3\,\text{V}.$$

(b) Equation 27-39, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0e^{-t/\tau} \;\;\Rightarrow\;\; t = RC\ln\left(\frac{V_0}{V}\right) = \left(300\times10^9\,\Omega\right)\left(10\times10^{-12}\,\text{F}\right)\ln\left(\frac{3000}{100}\right)$$

which yields t = 10 s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0e^{-t/RC}$ for R with the new values V_0 = 1400 V and t = 0.30 s. Thus,

$$R = \frac{t}{C\ln(V_0/V)} = \frac{0.30\,\text{s}}{\left(10\times10^{-12}\,\text{F}\right)\ln(1400/100)} = 1.1\times10^{10}\,\Omega\,.$$

76. (a) We reduce the parallel pair of resistors (at the bottom of the figure) to a single R' =1.00 Ω resistor and then reduce it with its series 'partner' (at the lower left of the figure) to obtain an equivalence of R'' = 2.00 Ω +1.00Ω =3.00 Ω. It is clear that the current through R'' is the i_1 we are solving for. Now, we employ the loop rule, choose a path that includes R'' and all the batteries (proceeding clockwise). Thus, assuming i_1 goes leftward through R'', we have

$$5.00\text{ V} + 20.0\text{ V} - 10.0\text{ V} - i_1R'' = 0$$

which yields i_1 = 5.00 A.

(b) Since i_1 is positive, our assumption regarding its direction (leftward) was correct.

(c) Since the current through the ε_1 = 20.0 V battery is "forward", battery 1 is supplying energy.

78. The current in the ammeter is given by

$$i_A = \varepsilon/(r + R_1 + R_2 + R_A).$$

The current in R_1 and R_2 without the ammeter is $i = \varepsilon/(r + R_1 + R_2)$. The percent error is then

$$\frac{\Delta i}{i} = \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} = \frac{0.10\,\Omega}{2.0\,\Omega + 5.0\,\Omega + 4.0\,\Omega + 0.10\,\Omega}$$
$$= 0.90\%.$$

79. (a) The charge q on the capacitor as a function of time is $q(t) = (\varepsilon C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\varepsilon/R)e^{-t/RC}$. The energy supplied by the emf is then

$$U = \int_0^\infty \varepsilon i\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC} dt = C\varepsilon^2 = 2U_C$$

where $U_C = \frac{1}{2}C\varepsilon^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2 R dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}C\varepsilon^2.$$

80. In the steady state situation, there is no current going to the capacitors, so the resistors all have the same current. By the loop rule,

$$20.0\text{ V} = (5.00\,\Omega)i + (10.0\,\Omega)i + (15.0\,\Omega)i$$

which yields $i = \frac{2}{3}$ A. Consequently, the voltage across the $R_1 = 5.00\ \Omega$ resistor is (5.00 Ω)(2/3 A) = 10/3 V, and is equal to the voltage V_1 across the $C_1 = 5.00\ \mu$F capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(5.00\times10^{-6}\text{ F})\left(\frac{10}{3}\text{V}\right)^2 = 2.78\times10^{-5}\text{ J}.$$

Similarly, the voltage across the $R_2 = 10.0\ \Omega$ resistor is (10.0 Ω)(2/3 A) = 20/3 V and is equal to the voltage V_2 across the $C_2 = 10.0\ \mu$F capacitor. Hence,

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10.0\times10^{-6}\text{ F})\left(\frac{20}{3}\text{V}\right)^2 = 2.22\times10^{-5}\text{ J}$$

Therefore, the total capacitor energy is $U_1 + U_2 = 2.50 \times 10^{-4}$ J.

81. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\varepsilon R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}.$$

82. From $V_a - \varepsilon_1 = V_c - ir_1 - iR$ and $i = (\varepsilon_1 - \varepsilon_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \varepsilon_1 - i(r_1 + R) = \varepsilon_1 - \left(\frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\text{V} - \left(\frac{4.4\,\text{V} - 2.1\,\text{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\,\text{V}. \end{aligned}$$

83. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$. Thus, $R = \dfrac{t}{C \ln(V_0/V)}$.

(a) Then, for $t_{\min} = 10.0\ \mu$s, $R_{\min} = \dfrac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F})\ln(5.00/0.800)} = 24.8\,\Omega$.

(b) For $t_{\max} = 6.00$ ms,

$$R_{\max} = \left(\frac{6.00\,\text{ms}}{10.0\,\mu\text{s}}\right)(24.8\,\Omega) = 1.49 \times 10^4\ \Omega\ ,$$

where in the last equation we used $\tau = RC$.

84. (a) Since $R_{\text{tank}} = 140\ \Omega, i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0 \times 10^{-2}$ A.

(b) Now, $R_{\text{tank}} = (140\ \Omega + 20\ \Omega)/2 = 80\ \Omega$, so $i = 12$ V/(10 Ω + 80 Ω) = 0.13 A.

(c) When full, $R_{\text{tank}} = 20\ \Omega$ so $i = 12$ V/(10 Ω + 20 Ω) = 0.40 A.

85. The internal resistance of the battery is $r = (12\text{ V} - 11.4\text{ V})/50\text{ A} = 0.012\ \Omega < 0.020\ \Omega$, so the battery is OK. The resistance of the cable is

$$R = 3.0\text{ V}/50\text{ A} = 0.060\ \Omega > 0.040\ \Omega,$$

so the cable is defective.