

Easy:

② $\Delta V = 1.2 \times 10^9 \text{ V}$ potential difference between ground and cloud.

$$\Delta U = q(\Delta V) = (-e)(\Delta V) = -1.2 \times 10^9 \text{ eV}$$

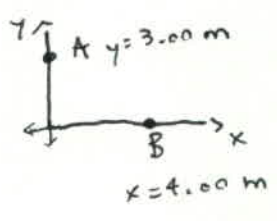
$$|\Delta U| = 1.2 \times 10^9 \text{ eV}$$

④ a) $F = 3.9 \times 10^{-15} \text{ N}$, $q = -e$, $d = 12 \text{ cm}$
2 parallel conducting plates with identical σ change, but opposite

$$E = \frac{F}{q} = \frac{(3.9 \times 10^{-15} \text{ N})}{(-1.6 \times 10^{-19} \text{ C})} = -2.4 \times 10^4 \text{ N/C}$$

b) $V = -E \cdot d = (+2.4 \times 10^4 \text{ N/C})(12 \times 10^{-2} \text{ m}) = 2.9 \times 10^3 \text{ V}$

⑦ $E_y = E_z = 0$, $E_x = (4.00 \text{ N/C})x$.



$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

Need to choose path. Choose L shape to make integral easier.

from $y=3.00 \text{ m}$ to $y=0$ along y axis, then from $x=0$ to $x=4.00 \text{ m}$ along x axis.

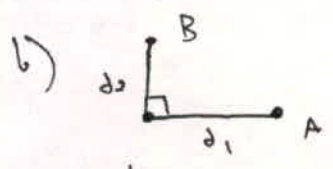
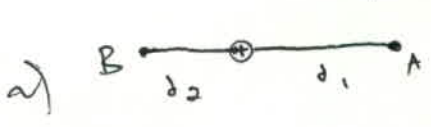
$$\vec{E} \cdot d\vec{y} = 0 \quad \vec{E} \cdot d\vec{x} = E_x dx$$

vertical part is 0.

$$V_B - V_A = - \int_0^4 E_x dx = -(4.00 \text{ N/C}) \int_0^4 x dx = -(4.00 \text{ N/C}) \frac{(4)^2}{2}$$

$$V_B - V_A = -32.0 \text{ V}$$

⑭ $q = 1.0 \mu\text{C}$, $d_1 = 2.0 \text{ m}$, $d_2 = 1.0 \text{ m}$



Same distances for (a) and (b).

$$V_A - V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{d_1} - \frac{1}{4\pi\epsilon_0} \frac{q}{d_2} = \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{d_1} - \frac{1}{d_2} \right)$$

$$V_A - V_B = (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C}) \left(\frac{1}{2} - 1 \right) \text{ m}^{-1} = -4.50 \times 10^3 \text{ V}$$

for both (a) and (b).

dipole moment of NH_3

1.47 D

1 D = 1 debye unit = $3.34 \times 10^{-30} \text{ C}\cdot\text{m}$, $r = 520 \text{ nm}$

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

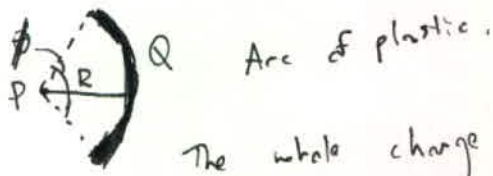
$\theta = 0$ when along the axis.

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.47)(3.34 \times 10^{-30} \text{ C}\cdot\text{m})}{(520 \times 10^{-9} \text{ m})^2}$$

$$V = 1.63 \times 10^{-5} \text{ V}$$

(24) $Q = -25.6 \text{ pC}$, $R = 3.71 \text{ cm}$, $\beta = 120^\circ$, $V = 0$ at ∞ .

Potential at P.



Arc of plastic.

The whole charge Q is at the same distance from P. Therefore I can say

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(-25.6 \times 10^{-12} \text{ C})}{(3.71 \times 10^{-2} \text{ m})}$$

$$V = -6.20 \text{ V}$$

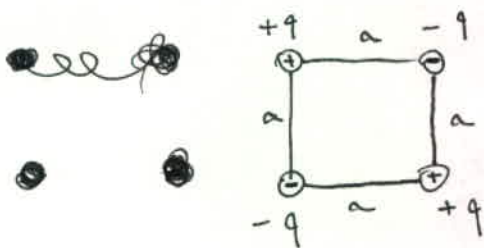
(42) a) 2 electrons separated by 2.00 nm

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(2.00 \times 10^{-9} \text{ m})}$$

$$U = 1.15 \times 10^{-19} \text{ J}$$

b) separation increases means potential energy decreases due to $\frac{1}{r}$.

(43) $q = 2.30 \text{ pC}$, $a = 64.0 \text{ cm}$, initially infinitely far apart and at rest.



$$E_i = 0 \text{ due to } \uparrow$$

$$W = E_f - E_i = E_f$$

No kinetic in E_f so only U .

$$E_f = U.$$

Potential energy works in pairs, so I have to add all of the distinct pairs

$$U = U_{++} + 2U_{+-} + 2U_{-+} + U_{--} = U_{++} + 4U_{+-} + U_{--}$$

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q^2}{a\sqrt{2}} + 4 \frac{(q)(-q)}{a} + \frac{(-q)(-q)}{a\sqrt{2}} \right]$$

$$U = \frac{q^2}{4\pi\epsilon_0 a} \left[\frac{2}{\sqrt{2}} - 4 \right] = \frac{q^2}{4\pi\epsilon_0 a} [\sqrt{2} - 4]$$

$$U = \frac{(2.30 \times 10^{-12} \text{ C})^2 (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{(64.0 \times 10^{-2} \text{ m})} (\sqrt{2} - 4)$$

$$W = E_f = U = -1.92 \times 10^{-13} \text{ J}$$

$$r \gg R_1$$



a) $V_1 = V_2 = V$

b) Note: For those of you in the problem session, I realized the correct method to use to find q . The minimum energy principle is usable but I forgot about the most important part! The following method is much preferred.

Using the fact that $V_1 = V_2 = V$, we will obtain q_1 and q_2 . sphere of charge has same \vec{E} as point charge outside sphere so V is the same also outside the sphere.

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1}$$

$$V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{2R_1}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_2}{2R_1}$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{2R_1}$$

$$q_1 = \frac{q_2}{2} \Rightarrow \frac{q_1}{q_2} = \frac{1}{2}$$

$$q = q_1 + q_2 = \frac{q_2}{2} + q_2 = \frac{3}{2} q_2$$

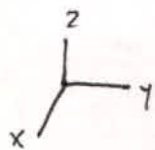
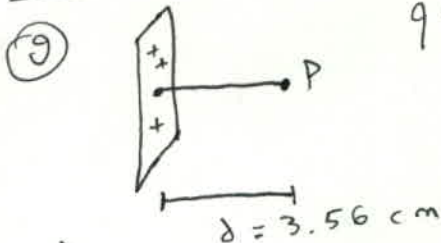
d) $\frac{\sigma_1}{\sigma_2} = \frac{q_1/A_1}{q_2/A_2} = \frac{q_1 A_2}{q_2 A_1} = \frac{q_1 (4\pi(2R_1)^2)}{q_2 (4\pi R_1^2)} = \frac{q_1 \cdot 4}{q_2} = \left(\frac{1}{2}\right)(4) = 2$

c) $q_2 = \frac{2}{3} q$ so (b) $q_1 = \frac{1}{3} q$

Medium:

$\sigma = 5.80 \text{ pC/m}^2$ infinite nonconducting sheet,

$q = 1.60 \times 10^{-19} \text{ C}$



a)

$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{j}$ to the right of the sheet. (+y)

$\vec{F} = q\vec{E}$

$W = \int \vec{F} \cdot d\vec{s}$

choose the path that goes along the y axis in the \hat{j} direction.

$W = q \int \vec{E} \cdot d\vec{s} = q \int_0^d (\frac{\sigma}{2\epsilon_0} \hat{j}) \cdot (dy \hat{j})$

$W = \frac{q\sigma}{2\epsilon_0} \int_0^d dy = \frac{q\sigma}{2\epsilon_0} d$

$W = \frac{(1.60 \times 10^{-19} \text{ C})(5.80 \times 10^{-12} \text{ C/m}^2)(3.56 \times 10^{-2} \text{ m})}{2(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)}$

$W = 1.87 \times 10^{-21} \text{ J}$

b)

$W = -\Delta U = -q\Delta V$

$V_i = 0$ ~ at sheet
 V_f ~ at point P.

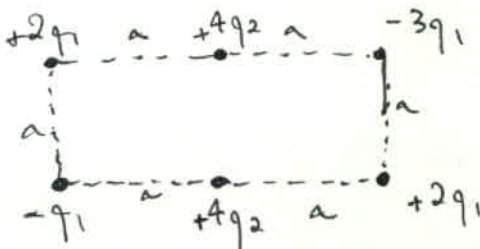
$\Delta V = -\frac{W}{q}$

$V_f - V_i = -\frac{W}{q}$

$V_f = -\frac{W}{q} = -1.17 \times 10^{-2} \text{ V}$

(16)

$a = 39.0 \text{ cm}$, $q_1 = 3.40 \text{ pC}$, $q_2 = 6.00 \text{ pC}$. $V = 0$ at ∞ , what is V at center of rectangle?



The four corners are the same distance so I can add the charges together for the calculation.

$2q_1 - q_1 - 3q_1 + 2q_1 = 0$. So the corner contributions cancel.

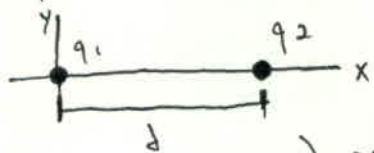
The 2 middle charges are also at the same distance from the center. $(\frac{a}{2})$. add to get $8q_2$.

$$V = \frac{1}{4\pi\epsilon_0} \frac{8q_2}{(a/2)} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{a} \cdot 16 = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2) (6.00 \times 10^{-12} \text{ C}) (16)}{(39.0 \times 10^{-2} \text{ m})}$$

$$V = 2.21 \text{ V}$$

(19)

$$q_1 = 5e, q_2 = -15e, d = 24.0 \text{ cm}, V = 0 \text{ at } \infty.$$



a) positive and b) negative values of x at which $V = 0$ on the x axis.

2 points, 1 in between charges and 1 in $(-x)$.

$$a) V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{x} + \frac{q_2}{d-x} \right] = 0$$

$$\frac{q_1}{x} = \frac{-q_2}{d-x}$$

$$\frac{5e}{x} = \frac{15e}{d-x} \Rightarrow$$

$$15ex = 5ed - 5ex$$

$$x = \frac{d}{4} = 6.00 \text{ cm}$$

$$b) V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{(-x)} + \frac{q_2}{d+x} \right] = 0$$

$$\frac{q_1}{x} = \frac{q_2}{d+x}$$

$$\frac{5e}{x} = \frac{-15e}{d+x}$$

$$5d + 5x = -15x$$

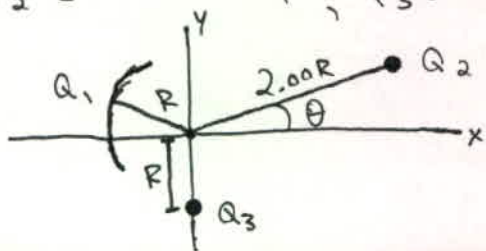
$$20x = -5d$$

$$x = -\frac{d}{4} = -6.00 \text{ cm}$$

(20)

$$Q_1 = +7.21 \text{ pC}$$

$$Q_2 = 4.00 Q_1, Q_3 = -2.00 Q_1, R = 2.00 \text{ m}, \theta = 20.0^\circ$$



V at origin.

Since the origin is the center of curvature for the arc, we can treat it as just a point charge Q_1 a distance R away.

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R} + \frac{Q_3}{R} + \frac{Q_2}{2R} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R} - \frac{2Q_1}{R} + \frac{4Q_1}{2R} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(7.21 \times 10^{-12} \text{ C})}{(2.00 \text{ m})}$$

$$V = 3.24 \times 10^{-2} \text{ V}$$



full circle $\rightarrow R_c = 6.0 \text{ cm}$, $q_c = -3.0 \mu\text{C}$

arc $\rightarrow R_A = 4.0 \text{ cm}$, $q_A = 2.0 \mu\text{C}$ concentric

dipole $\rightarrow p = 1.28 \times 10^{-21} \text{ C}\cdot\text{m}$ perpendicular to radial line.

V at center.

dipole contribution is 0 because $V_D = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$ and $\theta = \frac{\pi}{2}$ so $\cos \frac{\pi}{2} = 0$ then $V_D = 0$.

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_c}{R_c} + \frac{q_A}{R_A} \right] = \frac{1}{4\pi\epsilon_0} \left[\frac{-3 \times 10^{-6} \text{ C}}{6 \times 10^{-2} \text{ m}} + \frac{2 \times 10^{-6} \text{ C}}{4 \times 10^{-2} \text{ m}} \right]$$

$$V = \frac{1}{4\pi\epsilon_0} \left[-5 \times 10^{-5} \frac{\text{C}}{\text{m}} + 5 \times 10^{-5} \frac{\text{C}}{\text{m}} \right] = 0$$

$V = 0$ (turns out this one was easy 😊)

E at

$(3.00 \hat{i} - 2.00 \hat{j} + 4.00 \hat{k}) \text{ m}$ if $V = 2.00xyz^2$, V in volts,
 $(3, -2, 4)$ x, y, z meters.

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

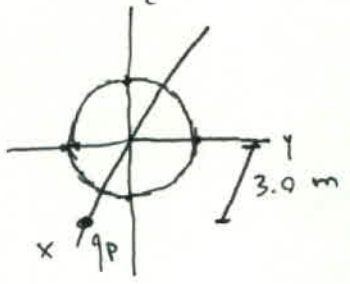
$$E_x = -2yz^2, \quad E_y = -2xz^2, \quad E_z = -4xyz$$

At $(3, -2, 4)$ we have $E_x = (-2)(-2)(4)^2$ $E_y = (-2)(3)(4)^2$ $E_z = (-4)(3)(-2)(4)$

$$E_x = 64 \text{ N/C} \quad E_y = -96 \text{ N/C} \quad E_z = 96 \text{ N/C}$$

$$\text{then } E = \sqrt{E_x^2 + E_y^2 + E_z^2} = 32\sqrt{22} \text{ N/C} = 150. \text{ N/C}$$

46 $q_r = -9.0 \text{ nC}$



thin plastic ring in yz -plane.
 $q_p = -6.0 \text{ pC}$ at $x = 3.0 \text{ m}$
 ring radius $R = 1.5 \text{ m}$

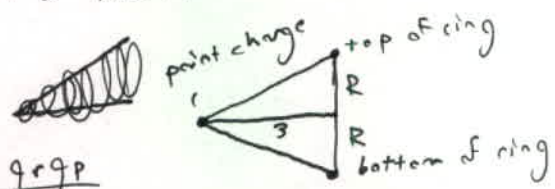
key is external force for sign of ΔU . for W .

$W = U_f - U_i$

when point charge is at origin

$U_f = \frac{q_r q_p}{4\pi\epsilon_0 R}$

$U_i = \frac{q_r q_p}{4\pi\epsilon_0 r}$



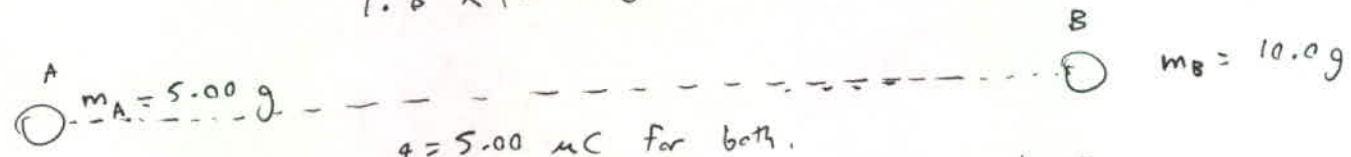
$r = \sqrt{3^2 + R^2}$

$W = U_f - U_i = \frac{q_r q_p}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{\sqrt{3^2 + R^2}} \right)$

$W = (-9.0 \times 10^{-9} \text{ C})(-6.0 \times 10^{-12} \text{ C})(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2) \cdot \left(\frac{1}{1.5} - \frac{1}{\sqrt{11.25}} \right) \text{ m}^{-1}$

$W = 1.8 \times 10^{-10} \text{ J}$
 $1.8 \times 10^{-10} \text{ J}$

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$q = 5.00 \text{ } \mu\text{C}$ for both.
 $d = 1.00 \text{ m}$ length of string \rightarrow nonconducting
 $d \gg R$.

a) $U = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})^2}{1.00 \text{ m}} = 0.225 \text{ J}$

b) $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2} = \frac{U}{d} = \frac{U}{1 \text{ m}} = 0.225 \text{ N}$

$a_A = \frac{F}{m_A}$, $a_B = \frac{F}{m_B} = \frac{F}{10^{-2} \text{ kg}} = 22.5 \text{ m/s}^2$

$a_A = \frac{F}{5 \times 10^{-3} \text{ kg}} = \frac{2F}{10^{-2} \text{ kg}} = 45.0 \text{ m/s}^2$

c) U is split between the 2 metal spheres KE.

$U = KE_A + KE_B$

$U = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

Need another equation to solve for the pair of v_A and v_B .
 Momentum is conserved, so we can use that equation.

$$\sum \vec{p}_i = \sum \vec{p}_F$$

mass A moves to the left, B to the right.
Initially neither move, so $p_i = 0$.

$$0 = \vec{p}_A + \vec{p}_B$$

$$0 = -p_A + p_B$$

$$p_B = p_A$$

$$m_B v_B = m_A v_A$$

$$v_B = \frac{m_A}{m_B} v_A = \frac{1}{2} v_A$$

$$U = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{v_A}{2}\right)^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{8} m_B v_A^2$$

$$v_A^2 = \frac{8U}{4m_A + m_B} = \frac{8U}{3 \times 10^{-2} \text{ kg}}$$

$$|v_A| = \sqrt{\frac{8U}{3 \times 10^{-2} \text{ kg}}} = 7.74 \text{ m/s}$$

$$|v_B| = \frac{|v_A|}{2} = 3.87 \text{ m/s}$$

(67)



$R = 15 \text{ cm}$, $q = 3.0 \times 10^{-8} \text{ C}$, metal sphere.

$$a) \text{ at surface, } E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(3.0 \times 10^{-8} \text{ C})}{(15 \times 10^{-2} \text{ m})^2} = 1.2 \times 10^4 \text{ N/C}$$

b) $V = 0$ at ∞ , what is V at surface?

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = E \cdot R = 1.8 \times 10^3 \text{ V}$$

$$c) V_f - V_R = -500$$

$$V_f = V_R - 500 \quad V_f = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$\frac{1}{4\pi\epsilon_0} \frac{q}{r} = V_R - 500$$

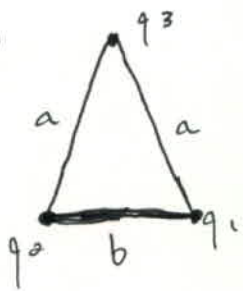
$$r = \frac{q}{4\pi\epsilon_0 (V_R - 500)} = \frac{(3.0 \times 10^{-8} \text{ C})(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{1298 \text{ V}}$$

$r = 0.21 \text{ m}$ is the distance from the center.

distance from surface \rightarrow

$$L = r - R = 5.8 \text{ cm}$$

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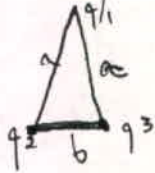
$$q_1 = 10 \mu\text{C}, \quad q_2 = -20 \mu\text{C}, \quad q_3 = 30 \mu\text{C}$$

$$a = 10 \text{ cm}, \quad b = 6.0 \text{ cm}$$

$$W = U_f - U_i$$

a) exchange positions of q_1 and q_3 and b) q_1 and q_2

~~U_f~~



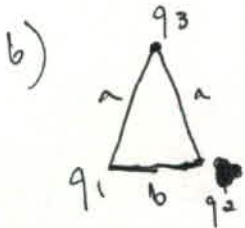
$$U_f = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{a} + \frac{q_1 q_2}{a} + \frac{q_2 q_3}{b} \right]$$

$$U_i = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{a} + \frac{q_2 q_3}{a} + \frac{q_1 q_2}{b} \right]$$

$$W = U_f - U_i = \frac{1}{4\pi\epsilon_0} \left[\frac{q_2 (q_1 - q_3)}{a} + \frac{q_2 (q_3 - q_1)}{b} \right] = \frac{q_2}{4\pi\epsilon_0} \left[\frac{(q_1 - q_3)}{a} - \frac{(q_1 - q_3)}{b} \right]$$

$$W = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) (-20 \times 10^{-6} \text{ C}) (-20 \times 10^{-6} \text{ C}) \left[\frac{1}{.1} - \frac{1}{.06} \right] \text{ m}^{-1}$$

$$W = -24 \text{ J}$$



$$U_f = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{a} + \frac{q_2 q_3}{a} + \frac{q_1 q_2}{b} \right] = U_i$$

Good symmetry here.

$$\text{so } W = U_f - U_i = 0$$

Hard:

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Nonconducting sphere $R = 2.31 \text{ cm}$ $q = 3.50 \text{ fC}$ uniform.
 $V_0 = 0$ at sphere's center.

$$V_0 = V_i = 0, \text{ so}$$

$$V = - \int_{r=0}^r \vec{E} \cdot d\vec{s} = - \int_{r=0}^r E \hat{r} \cdot dr \hat{r} = - \int_{r=0}^r E dr$$

a) $r = 1.45 \text{ cm}$ b) $r = R$

Inside the nonconducting sphere ($r \leq R$), we have

$$E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r$$

so
$$V = - \int_0^r \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r \, dr$$

$$V = \frac{-q}{4\pi\epsilon_0 R^3} \int_0^r r \, dr = \frac{-qr^2/2}{4\pi\epsilon_0 R^3}$$

At $r = 1.45 \text{ cm}$,

$$V = \frac{-(3.59 \times 10^{-15} \text{ C}) (1.45 \times 10^{-2} \text{ m})^2 (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{2 (2.31 \times 10^{-2} \text{ m})^3}$$

$$V = -2.68 \times 10^{-4} \text{ V}$$

b) At $r=R$,

$$V = \frac{-qR^2/2}{4\pi\epsilon_0 R^3} = \frac{-q}{4\pi\epsilon_0 2R}$$

$$V = \frac{-(3.59 \times 10^{-15} \text{ C}) (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{2 (2.31 \times 10^{-2} \text{ m})}$$

$$V = -6.81 \times 10^{-4} \text{ V}$$

32) $\lambda = bx$ nonuniform linear charge distribution, b const.

from $x=0$ to $x=0.20 \text{ m} = x_{\text{max}}$, $b = 20 \text{ nC/m}^2$, $V=0$ at ∞ .

V at a) the origin and b) the point $y = 0.15 \text{ m} = y_{\text{max}}$ on the y axis.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad dq = \lambda dx$$

a)
$$V = \frac{1}{4\pi\epsilon_0} \int_0^{x_{\text{max}}} \frac{\lambda dx}{x} = \frac{1}{4\pi\epsilon_0} \int_0^{x_{\text{max}}} \frac{bx}{x} dx = \frac{b}{4\pi\epsilon_0} \int_0^{x_{\text{max}}} dx$$

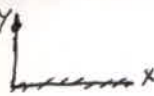
$$V = \frac{bx_{\text{max}}}{4\pi\epsilon_0}$$

$$V = \frac{(20 \times 10^{-9} \text{ C/m}^2) (0.20 \text{ m}) (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{4\pi\epsilon_0}$$

origin

$$V = 36 \text{ V}$$

$$b) \quad r = \sqrt{x^2 + y_{\max}^2}$$



$$V = \frac{1}{4\pi\epsilon_0} \int_0^{x_{\max}} \frac{bx}{\sqrt{x^2 + y_{\max}^2}} dx$$

$$u = x^2 + y_{\max}^2$$

$$du = 2x dx$$

$$x=0 \rightarrow u = y_{\max}^2$$

$$x=x_{\max} \rightarrow u = x_{\max}^2 + y_{\max}^2$$

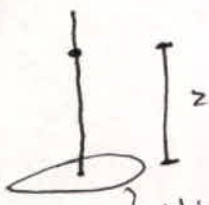
$$V = \frac{b}{4\pi\epsilon_0} \int_{y_{\max}^2}^{x_{\max}^2 + y_{\max}^2} \frac{du/2}{\sqrt{u}}$$

$$V = \frac{b}{4\pi\epsilon_0} \left[\frac{2}{2} \sqrt{u} \right]_{y_{\max}^2}^{x_{\max}^2 + y_{\max}^2} = \frac{b}{4\pi\epsilon_0} \left[\sqrt{x_{\max}^2 + y_{\max}^2} - y_{\max} \right]$$

$$V = (20 \times 10^{-9} \text{ C/m}^2) (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2) \left[\sqrt{.2^2 + .15^2} - .15 \right]$$

$$y = 0.15 \text{ m}$$

$$V = 72 \text{ V}$$



thin ring charge q , radius R .

a) $\textcircled{99}$ show $V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{z^2 + R^2}}$

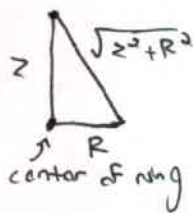
No integral even required.

All points on the ring are an equal distance from the point we wish to calculate potential at. That distance is $\sqrt{z^2 + R^2}$

Then we can immediately write

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{z^2 + R^2}} \text{ since}$$

the whole charge q is at this distance $\sqrt{z^2 + R^2}$ from the point in question.



b) we see from the symmetry that there will only be a z component.

$$\text{(Also } \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0 \text{)}$$

$$E = E_z = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\epsilon_0} \frac{(-\frac{1}{2})(2z)}{(z^2 + R^2)^{3/2}} = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \quad \checkmark$$

checks out with book's version.