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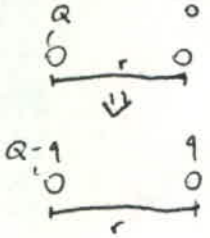
PHYS 2B FALL 2012

Easy:

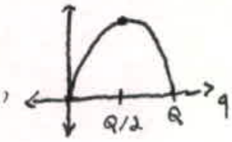
(1)



$$F = \frac{1}{4\pi\epsilon_0}\frac{(Q-q)\cdot q}{r^2}$$

r isn't changing, so what q makes the maximum of $(Q-q)\cdot q$?

$q = Q/2$,



So then $q/Q = 1/2$

(3) $q_1 = 26.0\ \mu C$, $q_2 = -47.0\ \mu C$, $|F| = 5.70\ N$

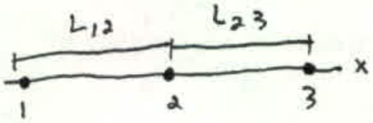
$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$$

$$5.70\,N = \frac{(8.99\times10^9\ N\cdot m^2/C^2)\,|26.0\ \mu C|\cdot|-47.0\ \mu C|}{r^2}$$

$$r = \sqrt{\frac{(8.99\times10^9\ N\cdot m^2/C^2)(26.0\times10^{-6}\ C)(47.0\times10^{-6}\ C)}{5.70\ N}}$$

$$r = 1.39\ m$$

(7)



1 and 2 fixed, 3 free. $\vec{F}_{\substack{net \\ on\,3}} = 0$

$L = L_{23} = L_{12}$, what is q_1/q_2?

$$0 = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{(L_{12}+L_{23})^2}\hat{x} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{(L_{23})^2}\hat{x}$$

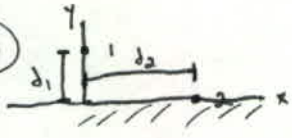
$$0 = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{(2L)^2} + \frac{q_2}{L^2}\right)\hat{x}$$

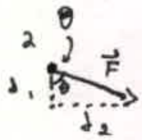
$$\frac{q_1}{(2L)^2} + \frac{q_2}{L^2} = 0 \Rightarrow \frac{q_1}{4L^2} = -\frac{q_2}{L^2} \Rightarrow \frac{q_1}{q_2} = -4$$

(36) a) positron b) electron

$1 = 0 + x \Rightarrow x = 1$ $0 = 1 + x \Rightarrow x = -1$

(39)





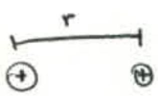
$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} \cdot \sin\theta \qquad \sin\theta = \frac{d_2}{r}$$

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2| \cdot d_2}{r^3} = \frac{(8.99 \times 10^9\ N \cdot m^2/C^2)(4 \cdot 1.60 \times 10^{-19}\ C)(6 \cdot 1.60 \times 10^{-19}\ C)(6.00 \times 10^{-3}\ m)}{\left(\left((2.00 \times 10^{-3}\ m)^2 + (6.00 \times 10^{-3}\ m)^2\right)^{1/2}\right)^3}$$

$$= 1.31 \times 10^{-22}\ N$$

(44)



$$F_E = F_G$$

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = m_p g$$

$$r = \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{m_p g}\right)^{1/2} = \sqrt{\frac{(8.99 \times 10^9\ N \cdot m^2/C^2)(1.60 \times 10^{-19}\ C)^2}{(1.67 \times 10^{-27}\ kg)(9.81\ m/s^2)}}$$

$$r = 1.19 \times 10^{-1}\ m = 0.119\ m$$

(49)

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

$$= \frac{(8.99 \times 10^9\ N \cdot m^2/C^2)\left(\frac{1.60}{3} \times 10^{-19}\ C\right)^2}{(2.6 \times 10^{-15}\ m)^2}$$

$$F = 3.8\ N$$

(59) 75.0 kg of electrons.

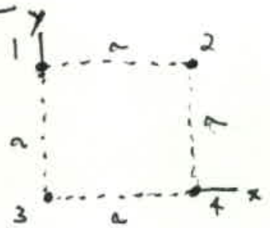
$$N_e = \frac{75.0\ \text{kg}}{9.11\times10^{-31}\ \text{kg}}$$

$$Q = N_e \cdot e = \frac{75.0\ \text{kg}}{9.11\times10^{-31}\ \text{kg}} \cdot 1.60\times10^{-19}\ C$$

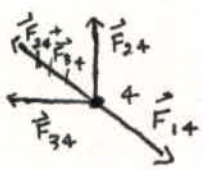
$$= 1.32\times10^{13}\ C$$

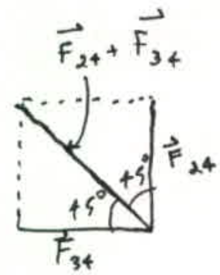
Medium:

(10)



$q_1 = q_4 = Q,$
$q_2 = q_3 = q$





a) $$\vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34} = 0$$

$$\vec{F}_{24} + \vec{F}_{34} = -\vec{F}_{14}$$

so $$|\vec{F}_{24} + \vec{F}_{34}| = |\vec{F}_{14}|$$

45-45-90

$$|\vec{F}_{24} + \vec{F}_{34}| = \sqrt{2}\cdot F_{34}$$
$$= \sqrt{2}\cdot F_{24}$$

$$\sqrt{2}\, F_{24} = F_{14}$$

(-) because we know q is the opposite sign of Q.

$$\frac{Q}{q} < 0$$

$$\sqrt{2}\cdot\frac{1}{4\pi\epsilon_0}\frac{(-q)Q}{a^2} = \frac{1}{4\pi\epsilon_0}\frac{(+Q^2)}{(a^2+a^2)}$$

$$-\sqrt{2}\cdot\frac{qQ}{a^2} = \frac{Q^2}{2a^2}$$

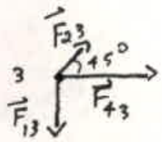
$$\frac{Q}{q} = -2\sqrt{2}$$

b) Besides the trivial $q=0$, No. This is because to have cancellation on 2 particles as seen in (a), the condition $q_1=q_4=Q$ and $q_2=q_3=q$ must be kept to achieve Force vectors at 45° angles. Because $\frac{Q}{q} = -2\sqrt{2}$ was the only solution with this condition, and the forces on particles 2 and 3 $\neq 0$ in this case, there is no q that will yield all 0's for net forces.

(11) Refer to Figure in 10.

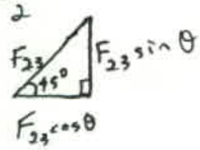
$q_1 = -q_2 = 100\ nC$

$q_3 = -q_4 = 200\ nC$, $a = 5.0\ cm$



$\theta = 45°$

$\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$



b) $F_y = -F_{13} + F_{23} \cdot \frac{\sqrt{2}}{2}$

a) $F_x = F_{43} + F_{23} \cdot \frac{\sqrt{2}}{2}$

$$F_y = \frac{-(8.99\times10^9 \frac{N\cdot m^2}{C^2})(100\times10^{-9}\ C)(200\times10^{-9}\ C)}{(5\times10^{-2}\ m)^2} + \frac{\sqrt{2}}{2}\frac{(8.99\times10^9 \frac{N\cdot m^2}{C^2})(100\times10^{-9}\ C)(200\times10^{-9}\ C)}{2((5\times10^{-2})^2\ m^2)}$$

$$F_y = -7.192\times10^{-2}\ N + \frac{\sqrt{2}}{4}(7.192\times10^{-2}\ N)$$

$$F_y = -4.65\times10^{-2}\ N = -0.0465\ N$$

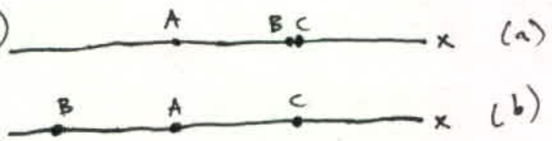
$F_{43} = 2\cdot F_{13}$ due to charge.

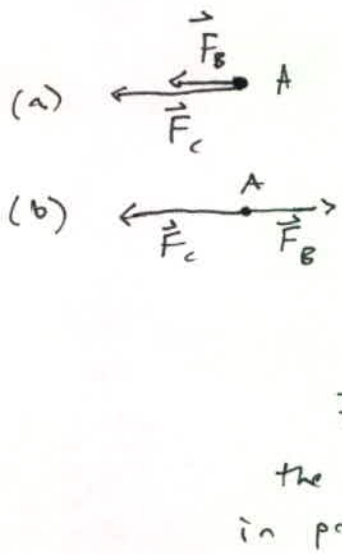
$$F_x = 2\cdot F_{13} + F_{23}\cdot\frac{\sqrt{2}}{2}$$

$$F_x = 2(7.192\times10^{-2}\ N) + \frac{\sqrt{2}}{4}(7.192\times10^{-2}\ N)$$

$$F_x = 0.169\ N$$

(18)





$\vec{F}_C$ has to be to negative x because it dominates $\vec{F}_B$.

The net force in magnitude decreases from (a) to (b) meaning $\vec{F}_B$ had to be along the same direction as $\vec{F}_C$ in part (a).

$$F^{(a)} = -F_B - F_c = -2.014\times10^{-23}\ N$$

$$F^{(b)} = +F_B - F_c = -2.877\times10^{-24}\ N$$

Add 2 equations,

$$-2F_c = -2.3017\times10^{-23}\ N$$

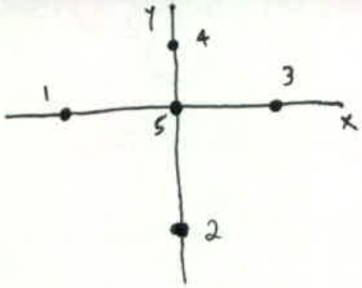
$$F_c = 1.15085\times10^{-23}\ N$$

$$F_B = -2.877\times10^{-24}\ N + F_c$$

$$F_B = 8.6315\times10^{-24}\ N$$

$$\frac{F_c}{F_B} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q_A q_c}{r^2}}{\frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{r^2}} = \frac{q_c}{q_B} \text{ so } \frac{q_c}{q_B} = \frac{1.15085\times10^{-23}\ N}{8.6315\times10^{-24}\ N} = 1.333$$

(29)



$q_1 = q_2 = q_3 = q_4 = -e$

$q_5 = +e$

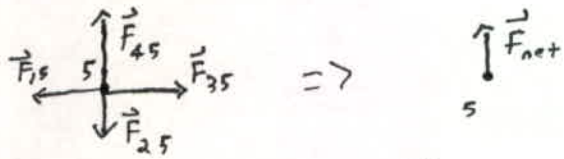
$y_2 = -10.0 \text{ cm}$ fixed

$y_4 = 5.00 \text{ cm}$

$x_1 = -10.0 \text{ cm}$, $x_3 = 10.0 \text{ cm}$ initially, not fixed.

Particle 5 fixed.

Initially,



a) Need to move particle 1 so that the direction of $\vec{F}_{net}$ is rotated $30°$ counterclockwise.

$$F_y = F_{45} - F_{25}$$

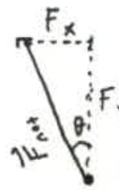
$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cdot e^2}{(5.00\times10^{-2}\text{ m})^2} - \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cdot e^2}{(10.0\times10^{-2}\text{ m})^2}$$

$$F_y = \left(2.697\times10^{12}\ \frac{\text{N}}{\text{C}^2}\right)\cdot e^2$$

$F_x = F_{15} - F_{35}$, so that $\vec{F}_x$ goes towards $-x$.

$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cdot e^2}{x_1^2} - \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cdot e^2}{(10.0\times10^{-2}\text{ m})^2}$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot e^2\left[\frac{1}{x_1^2} - \left(1.00\times10^2\ \frac{1}{\text{m}^2}\right)\right]$$



$\theta = 30°$

$\tan\theta = \dfrac{F_x}{F_y}$

$\tan 30° = \dfrac{1}{\sqrt{3}}$

$$\tan 30° = \frac{\left[\frac{1}{x_1^2} - 100.\ \frac{1}{\text{m}^2}\right]}{(3.00\times10^2\ 1/\text{m}^2)}$$

$$\frac{1}{x_1^2} = (300.\ \text{m}^{-2})\tan 30° + 100.\ \text{m}^{-2}$$

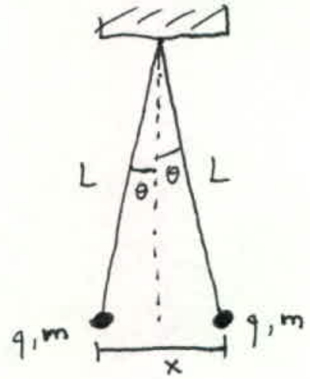
$$x_1 = \left(300.\cdot\frac{1}{\sqrt{3}} + 100.\right)^{-1/2}\text{ m}$$

$x_1 = 0.0605 \text{ m} = 6.05 \text{ cm}$ to the left of the origin.

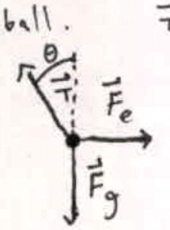
so $x_1 = -6.05 \text{ cm}$

b) to balance F_x so that $F_x = 0$, we need $x_3 = 6.05 \text{ cm}$ so that $\vec{F}_{net}$ is back to straight up.

(42)



FBD of right ball. $\vec{T}$ is tension from the thread.



$\tan\theta \approx \sin\theta$

Since the balls are in equilibrium, $\vec{F}_{net} = 0$

$$F_x = 0 = F_e - T_x \quad , \quad F_y = 0 = T_y - F_g$$

$$T_x = F_e \qquad\qquad T_y = F_g$$

a)

$\sin\theta = \dfrac{\frac{1}{2}x}{L}$

$$\tan\theta = \frac{T_x}{T_y}$$

$$\sin\theta = \frac{T_x}{T_y} = \frac{F_e}{F_g}$$

$$\frac{\frac{1}{2}x}{L} = \frac{F_e}{F_g}$$

$$x = 2L \cdot \frac{F_e}{F_g} = \frac{2L\left(\frac{1}{4\pi\epsilon_0}\frac{q^2}{x^2}\right)}{(mg)}$$

Bring x's over $\rightarrow$ $$x^3 = \frac{q^2 L}{2\pi\epsilon_0 mg} \Rightarrow x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg}\right)^{1/3}$$

b) $L = 120$ cm, $m = 10$ g, $x = 5.0$ cm, what is $|q|$.

First solve for q using previous equation.

$$x^3 = \frac{q^2 L}{2\pi\epsilon_0 mg}$$

$$|q| = \left(\frac{2\pi\epsilon_0 mg\,x^3}{L}\right)^{1/2} = \left(\frac{(0.01\ \text{kg})(9.81\ \text{m/s}^2)(0.05\ \text{m})^3}{2\cdot(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.2\ \text{m})}\right)^{1/2}$$

$$|q| = 2.38\times10^{-8}\ \text{C} = 2\times10^{-8}\ \text{C}$$

↳ only 1 sig fig.

(54) We know from earlier exercises that splitting the charge in half will yield the maximum force, 3.0 mm separation.

so 6.0 μC is split into 2 3.0 μC charges.

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2} = \frac{\left(8.99\times10^9\ \frac{N\cdot m^2}{C^2}\right)(3.0\times10^{-6}\ C)^2}{(3.0\times10^{-3}\ m)^2}$$

$$F_E = 9.0\times10^3\ N$$

(55) a) $\alpha = \frac{1}{2}$ from earlier exercises.

b) This is the only part of F_E that changes with α.

c) $F_E \sim (Q-\alpha Q)(\alpha Q) = Q^2\alpha(1-\alpha)$

$$\frac{1}{2} = \frac{F_E'}{F_E(\alpha=\frac{1}{2})} = \frac{\alpha(1-\alpha)}{\frac{1}{2}\left(\frac{1}{2}\right)} = \frac{\alpha-\alpha^2}{\frac{1}{4}}$$

$$\frac{1}{8} = \alpha-\alpha^2$$

$$\alpha^2-\alpha+\frac{1}{8} = 0$$

$$\alpha = \frac{1\pm\sqrt{1-\frac{1}{2}}}{2} = \frac{1\pm\frac{1}{\sqrt{2}}}{2} = \frac{1}{2}\pm\frac{\sqrt{2}}{4}$$

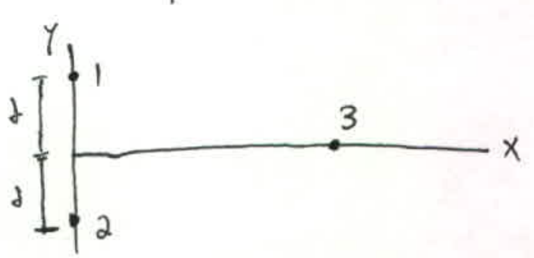
For exact results, $\alpha = 0.854$

$\alpha = 0.146$

Hard:

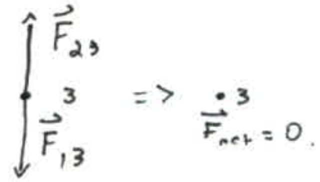
(23) $q_1 = q_2 = 3.20\times10^{-19}\ C$ $\quad d = 17.0$ cm, $q_3 = 6.40\times10^{-19}\ C$

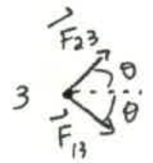
moved gradually from $x=0$ to $x=5.0$ m



a) F_E on particle 3 is minimum, maximum (b)?

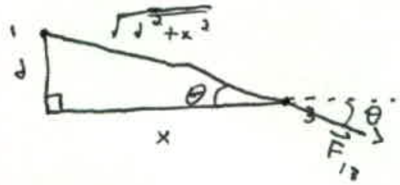
a) $x = 0$ is where F_E is a minimum.



b)  $\Rightarrow$ $\xrightarrow{\vec{F}_{13}+\vec{F}_{23}}$ only an x-component.

since the x component of $\vec{F}_{13}$ and $\vec{F}_{23}$ are equal,
we can say $\vec{F}_{net}$ has an x-component of $2(\vec{F}_{13})_x$

$$F_{net} = 2 \cdot F_{13} \cos\theta$$



$$F_{net} = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{(d^2+x^2)} \cdot \frac{x}{\sqrt{d^2+x^2}}$$

$$\cos\theta = \frac{x}{\sqrt{d^2+x^2}}$$

$$F_{net} = \frac{q_1 q_3}{2\pi\epsilon_0} \cdot \frac{x}{(d^2+x^2)^{3/2}}$$

We want to find the maximum of F_{net} with respect to x.
Sounds like a derivative is needed.

$$\frac{dF_{net}}{dx} = 0 = \frac{q_1 q_3}{2\pi\epsilon_0}\left[\frac{1}{(d^2+x^2)^{3/2}} - \frac{\frac{3}{2}(2x)(x)}{(d^2+x^2)^{5/2}}\right]$$

$$0 = \frac{d^2+x^2-3x^2}{(d^2+x^2)^{5/2}}$$

$$d^2 - 2x^2 = 0$$

$$x^2 = \frac{d^2}{2}$$

$x = +\frac{d}{\sqrt{2}}$ since we are in $(+x)$ of xy-plane.

$x = 12.0$ cm, this is in the range of $x=0$ to $x=5.0$ m.

c) minimum value $F_{net} = 0$

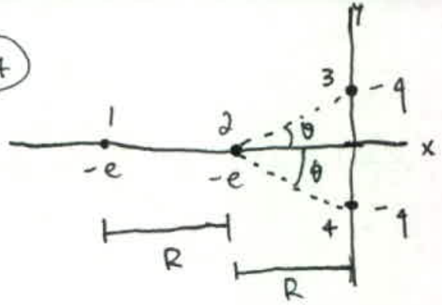
d) maximum F_{net} →

$$F_{net} = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3 \cdot x}{(d^2+x^2)^{3/2}} = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3 \left(\frac{d}{\sqrt{2}}\right)}{\left(3d^2/2\right)^{3/2}} = \frac{2 \cdot 2}{3\sqrt{3}} \cdot \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{d^2}$$

$$F_{net} = \frac{2 \cdot 2}{3\sqrt{3}} \frac{(8.99 \times 10^9\ N \cdot m^2/C^2)(3.20 \times 10^{-19}\ C)(6.40 \times 10^{-19}\ C)}{(17.0 \times 10^{-2}\ m)^2}$$

$$F_{net} = 4.90 \times 10^{-26}\ N$$

(34)

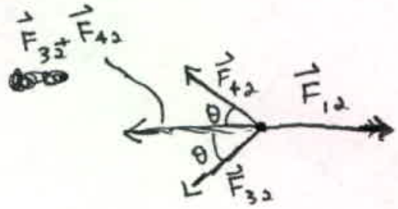


$q > 0$ to keep particle 2 in place.

1, 3, and 4 are fixed.

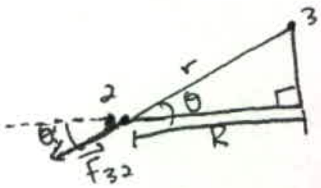
$q \leq 5e$ physically possible.

a) smallest, b) 2nd smallest, c) third smallest values of θ for which electron 2 is held in place.



$$\vec{F}_{12} + \vec{F}_{42} + \vec{F}_{32} = 0$$

$$\vec{F}_{32} + \vec{F}_{42} = -2 F_{32} \cos\theta \cdot \hat{i}$$



$$F_{12} - 2 F_{32} \cos\theta = 0$$

$$F_{12} = 2 F_{32} \cos\theta$$

$$\cos\theta = \frac{R}{r}$$

$$r = \frac{R}{\cos\theta} \longrightarrow$$

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2} = 2 \left(\frac{1}{4\pi\epsilon_0} \frac{eq}{(R/\cos\theta)^2} \right) \cos\theta$$

$$\frac{e^2}{R^2} = 2\cos\theta \cdot \frac{eq}{R^2} \cdot \cos^2\theta$$

$$\frac{e}{q} = 2\cos^3\theta$$

$$\cos^3\theta = \frac{e}{2q}$$

$$\cos\theta = \left(\frac{e}{2q}\right)^{1/3}$$

$$\theta = \cos^{-1}\left(\left(\frac{e}{2q}\right)^{1/3} \right)$$

To get the smallest value of θ, we want our $\cos\theta = \left(\frac{e}{2q}\right)^{1/3}$ to be as close to 1 as possible. (as $\cos^{-1}(1) = 0$) - smallest possible θ.

That means q should be as small as possible to make $\left(\frac{e}{2q}\right)^{1/3}$ as large as possible.

Our range of q's is $q = \{e, 2e, 3e, 4e, 5e\}$ due to quantization

Then we have (a) $q = e$, (b) $q = 2e$, and (c) $q = 3e$ to plug in.

(a) $\theta = \cos^{-1}\left(\left(\frac{1}{2}\right)^{1/3} \right) = 37.5°$

(b) $\theta = \cos^{-1}\left(\left(\frac{1}{4}\right)^{1/3} \right) = 51.0°$

(c) $\theta = \cos^{-1}\left(\left(\frac{1}{6}\right)^{1/3} \right) = 56.6°$