

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation : $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse : $v \rightarrow -v$

Velocity transformation : $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse : $v \rightarrow -v$

Spacetime interval : $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift : $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum : $\vec{p} = \gamma m \vec{u}$; Energy : $E = \gamma mc^2$; Kinetic energy : $K = (\gamma - 1)mc^2$

Rest energy : $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law : $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution : $P(E) = C e^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons : $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect : $eV_s = K_{max} = hf - \phi$, ϕ = work function; Bragg equation : $n\lambda = 2d \sin \vartheta$

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$

Coulomb force : $F = \frac{kq_1 q_2}{r^2}$; Coulomb energy : $U = \frac{kq_1 q_2}{r}$; Coulomb potential : $V = \frac{kq}{r}$

Force in electric and magnetic fields (Lorentz force) : $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering : $\Delta n = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\phi/2)}$; $ke^2 = 14.4 \text{ eV \AA}$; $\hbar c = 1973 \text{ eV \AA}$

Hydrogen spectrum : $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Bohr atom : $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$; $L = m_e v r = n\hbar$ angular momentum

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)

In a scattering experiment with α -particles incident on a silver (Ag) foil ($Z=49$), it is found that for every 600 particles scattered at 90° there are 100 particles scattered at 180° when the energy of the α -particles is 19 MeV. Instead, when the energy of the α -particles is 18 MeV, for every 600 particles scattered at 90° there are 150 particles scattered at 180° .

- For α -particle energy 17 MeV, how many α -particles would be scattered at 180° for every 600 particles scattered at 90° ? Justify your answer.
- How many α -particles are scattered at 45° for every 600 particles scattered at 90° for α -particle energy 18 MeV?
- What can you deduce about the size of the Ag nucleus from these data?

Problem 2 (10 points)

Radiation with wavelengths in the range 240Å to 5000Å is incident on a gas of hydrogen-like ions with atomic number Z at room temperature. The largest wavelength that is absorbed is 303.8Å.

- Find the value of Z for these ions.
- Find all other wavelengths that are absorbed for incident radiation in this wavelength range by this gas. Explain why there are no more absorption lines.
- After the ions absorb radiation they will emit it. State the total number of emission lines for this situation, which quantum numbers they involve, and find all their wavelengths (in Å).

Problem 3 (10 points)

An electron in a hydrogen-like ion is in a state with kinetic energy 21.25 eV and angular momentum $4\hbar$.

- Find the value of Z for this ion.
- Find the speed of this electron, expressed as v/c .
- Find the radius of the orbit of this electron, in Å.
- If the nucleus of this ion (with the same Z) had positrons instead of protons (mass of the positron = m_e) and no neutral particles, what would the total energy of the system in this state? Give your answer in eV.

Hint: For electrons in hydrogen-like ions, $K=-U/2$ relates kinetic and potential energies.

Justify all your answers to all problems. Write clearly.