Physics 2D — Quiz 1 Solutions — Winter 2012

Problem 1

Frank had the idea at t'_F, Bonnie at t'_B, with $t'_B - t'_F = 10^{-6}$s

(t'_B is larger than t'_F since Bonnie had the idea later)

In the reference frame of the patent office though,

$$t_B = \gamma\left(t'_B + \frac{v}{c^2} x_B\right)$$

$$t_F = \gamma\left(t'_F + \frac{v}{c^2} x_F\right)$$

$$t_B - t_F = \gamma\left(t'_B - t'_F - \frac{v}{c^2}(x_F - x_B)\right) \quad ; \quad \text{since } x_F - x_B = L = 900\text{m},$$

$$t_B - t_F = \gamma\left(t'_B - t'_F - \frac{v}{c^2} L\right)$$

So we can have $t_B - t_F > 0$ or $t_B - t_F < 0$ depending on v

For $t_B - t_F = 0 \Rightarrow \frac{v}{c^2} L = t'_B - t'_F = 10^{-6}\text{s} \Rightarrow$

$$\frac{v}{c} = \frac{10^{-6}\text{s}}{900\text{ m}} \cdot 3\times10^8 \frac{\text{m}}{\text{s}} = \frac{300}{900} = 0.333$$. Hence,

(a) For $\frac{v}{c} < 0.333$, Frank gets the patent ($t_B - t_F > 0$)

(b) For $\frac{v}{c} > 0.333$, Bonnie gets the patent ($t_B - t_F < 0$)

(c) The time it takes light to go from F to B is

$$t = \frac{L}{c} = \frac{900\text{ m}}{3\times10^8\text{ m/s}} = 3\mu\text{s}.$$ Nothing goes faster than light, so it is impossible that B got the idea from F, she got it independently since she got it 1μs after F did.

Problem 2

$$K = (\gamma-1)mc^2 \quad \text{kinetic energy}$$

For electron, $K = 0.511$ MeV. Also $m_e c^2 = 0.511$ MeV. So,

$$K_e = (\gamma_e - 1) m_e c^2 = m_e c^2 \Rightarrow \quad \gamma_e = 2$$

$$\gamma_e = \frac{1}{\sqrt{1 - \frac{u_e^2}{c^2}}} \Rightarrow \frac{1}{\gamma_e^2} = 1 - \frac{u_e^2}{c^2} \Rightarrow \frac{u_e}{c} = 1 - \frac{1}{\gamma_e^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{u_e}{c} = \frac{\sqrt{3}}{2} = 0.866 \quad \Rightarrow \boxed{u_e = 0.866c}$$

For proton, $K = 938.26$ MeV. Also $m_p c^2 = 938.26$ MeV. So,

$\gamma_p = 2$ and $u_p = 0.866c$ also (in opposite direction)

(c) Speed of proton relative to electron:

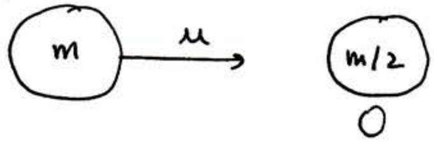
Go to reference frame S' moving with electron. $v = u_e$; and

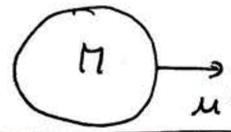
$u_p = -u_e$, so in S'

$$u'_p = \frac{u_p - v}{1 - \frac{u_p v}{c^2}} = \frac{-2u_e}{1 + \frac{u_e^2}{c^2}} = \frac{-2u_e/c}{1 + \frac{u_e^2}{c^2}} \cdot c =$$

$$= -\frac{2 \cdot 0.866}{1 + 0.866^2} \cdot c \boxed{= -0.9897c} \text{ (c)}$$

Problem 3





Classically: $M = \frac{3}{2} m$, from momentum conservation $mu = \frac{3}{2} m u' \Rightarrow u' = \frac{2}{3} u = 0.533c$

Momentum conservation:

$$\gamma m u = \gamma' M u'$$

Energy conservation: $\gamma mc^2 + \frac{m}{2} c^2 = \gamma' M c^2 \Rightarrow$

$\Rightarrow \quad m(\gamma + \frac{1}{2}) = \gamma' M$; divide momentum eq. by this,

$$\frac{\gamma \cancel{m} u}{\cancel{m}(\gamma + \frac{1}{2})} = \frac{\cancel{\gamma'} \cancel{M} u'}{\cancel{\gamma'} \cancel{M}} \Rightarrow u' = \frac{\gamma}{\gamma + \frac{1}{2}} u$$

$u = 0.8c$, $\gamma = \frac{1}{\sqrt{1 - 0.8^2}} = 1.667 \Rightarrow \boxed{u' = 0.615c}$

$\gamma' = \frac{1}{\sqrt{1 - u'^2/c^2}} \Rightarrow \boxed{\gamma' = 1.269}$

From energy eq., $M = \frac{\gamma + 1/2}{\gamma'} m \Rightarrow \boxed{M = 1.707 m}$

(c) $K_i = (\gamma - 1) mc^2 = 0.667 mc^2$

$K_f = (\gamma' - 1) Mc^2 = 0.269 \times 1.707 mc^2 = 0.459 mc^2$

$\Rightarrow$ kinetic energy decreased by $\boxed{K_i - K_f = (0.667 - 0.459) mc^2 = 0.208 mc^2}$

(d) The mass difference is $\Delta m = M - m - \frac{m}{2} = 0.207 m$

and $\boxed{\Delta m c^2 = 0.207 mc^2 = K_i - K_f}$ the extra mass came from decrease in kinetic energy (roundoff error in last decimal)