**Problem 1**  

Classically, \[ M = m_1 + m_2 \] . Momentum conservation gives: 

\[ m_1 u - m_2 u = M u' \Rightarrow u' = \frac{m_1 - m_2}{m_1 + m_2} u \]  

(a)

(b) Relativistically:

Momentum conservation: \[ \gamma m_1 u - \gamma m_2 u = \gamma ' M u' \]

Energy conservation: \[ \gamma m_1 c^2 + \gamma m_2 c^2 = \gamma ' M c^2 \Rightarrow \gamma ' M = \gamma (m_1 + m_2) \Rightarrow \]

Substituting in momentum conservation eq. \[ \gamma (m_1 - m_2) u = \gamma ' M u' = \gamma (m_1 + m_2) u' \]

\[ u' = \frac{m_1 - m_2}{m_1 + m_2} u \]  

(b)  

It is same as value found classically.

(c) From energy equation,

\[ M = \frac{\gamma}{\gamma'} (m_1 + m_2) \]

\[ \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} , \quad \gamma' = \frac{1}{\sqrt{1 - u'^2/c^2}} \]

\[ \gamma > 1 \Rightarrow \frac{\gamma}{\gamma'} > 1 \Rightarrow M > m_1 + m_2 \]

because in an inelastic collision, kinetic energy gets transformed into mass.
Problem 2

(a) \( \lambda_0 = \frac{hc}{4.96 \text{meV}} \) = \( \lambda_0 = \frac{12,400 \times 11,600 \text{Å}}{4.96 \times 5000} \)
\[ \lambda_0 = 5800 \text{Å} \]

(b) if \( T = 5000 \text{°K}, \ T' = 10,000 \text{°K} = 2T \), we have:
\[
\frac{hc}{\lambda_0 \text{meV} T} = 4.96, \quad \frac{hc}{\lambda_0 \text{meV} T'} = \frac{4.96}{2} = 2.48
\]

Power is proportional to \( \frac{1}{e^{\frac{hc}{\lambda_0 \text{meV} T}} - 1} \) so
\[
\frac{\text{Power}(T')}{\text{Power}(T)} = \frac{e^{4.96} - 1}{e^{2.48} - 1} = \frac{141.59}{10.94} = 12.94
\]

So power increases by factor 12.94

(c) Total power is proportional to \( T^4 \) (Stefan law)

\( T \) increases by factor of 2 \( \Rightarrow \) total power increases by factor 16

(d) At very large \( T \)
\[
\frac{1}{e^{\frac{hc}{\lambda_0 \text{meV} T}} - 1} \approx \frac{1}{1 + \frac{hc}{\lambda_0 \text{meV}}} = \frac{\lambda_0 \text{meV} T}{hc} \quad \text{(using } e^x \approx 1 + x)\]

So power is proportional to \( T \) \( \Rightarrow \) power increases by factor of 2 at very large \( T \)irs >> do
Problem 3

The ions are in their ground state, so \( n = 1 \) for the electron.

\[
E_1 = -E_0 z^2 \quad \text{with } E_0 = 13.6 \text{ eV}.
\]

To ionize, photons have to have energy

\[
hf = \frac{hc}{\lambda} = E_0 z^2 \Rightarrow \lambda = \frac{hc}{E_0 z^2} = \frac{12,400 \text{ eV} \cdot \text{Å}^2}{13.6 \text{ eV} \cdot z^2}
\]

\[
= \frac{911.76 \text{ Å}}{z^2}.
\]

So to ionize atoms with \( z = 1, 2, 3, 4, \ldots \)

requires \( \lambda = 911 \text{ Å}, \ 228 \text{ Å}, \ 101 \text{ Å}, \ 57 \text{ Å}, \ldots \)

Here, radiation in range \( 100 \text{ Å} \) to \( 150 \text{ Å} \) \Rightarrow \boxed{Z = 3} (a)

The highest-energy photon has energy

\[
hf = \frac{hc}{\lambda} \quad \text{with } \lambda = 100 \text{ Å} \Rightarrow
\]

\boxed{hf = 124 \text{ eV}}. The incident energy is \( E_0 z^2 = 13.6 \text{ eV} \times 9 = 122.4 \text{ eV} \)

\Rightarrow \text{ maximum kinetic energy of electron } = 124 \text{ eV} - 122.4 \text{ eV} = \boxed{1.6 \text{ eV}} (b)

(c) Wavelength required for transition from \( n = 1 \) to \( m > 1 \):

\[
\frac{hc}{\lambda} = z^2 E_0 \left(1 - \frac{1}{m^2}\right) \Rightarrow \lambda = \frac{hc}{z^2 E_0} \left(1 - \frac{1}{m^2}\right) = \frac{101.307 \text{ Å}}{1 - \frac{1}{m^2}}
\]

\[
\delta \lambda_1 = \frac{101.307 \text{ Å}}{3} = \frac{135.08 \text{ Å}}{4}
\]

\[
\delta \lambda_2 = \frac{101.307 \text{ Å}}{8} = \frac{113.97 \text{ Å}}{9}
\]

\[
\delta \lambda_3 = \frac{101.307 \text{ Å}}{15} = 108.06 \text{ Å}
\]

\[
\delta \lambda_m < \delta \lambda_4 \quad \text{for } m > 4
\]
Problem 4

\[ E_1 = \frac{\hbar^2 \pi^2}{2me L^2} = \frac{3.81 \text{eV} \cdot A^2 \cdot \pi^2}{25 \text{Å}^2} = 1.50 \text{eV} \]

(b) Inside the barrier, the electron wavefunction is

\[ \psi_n e^{-\alpha x} = e^{-x/\delta} \]

with

\[ \alpha = \left( \frac{2me}{\hbar^2} (U-E) \right)^{1/2} = \left( \frac{1}{3.81 \text{eV} \cdot A^2} (5 \text{eV} - 1.5 \text{eV}) \right)^{1/2} = 3.57 \text{Å}^{-1} \]

\[ \delta = 1/\alpha = 0.28 \text{ Å} \]

So the effective width of the well is \( 5 \text{Å} + \delta = 5.28 \text{Å} \).

So a better estimate for the ground state energy is

\[ E_1' = \frac{\hbar^2 \pi^2}{2me (L+\delta)^2} = 1.50 \text{eV} \cdot \frac{L^2}{(L+\delta)^2} = 1.50 \text{eV} \left( \frac{5}{5.28} \right)^2 = 1.35 \text{eV} \]

\[ E_1' = 1.35 \text{eV} \]

(c)

\[ T = e^{-2\alpha L} = e^{-2 \times 3.57 \text{Å}^{-1} \times 1 \text{Å}} = e^{-7.14} = 7.9 \times 10^{-4} \]

(d) Momentum \( \mathbf{p} = me \mathbf{v} = \hbar \mathbf{k} = \frac{\hbar \pi}{L} \)

\[ \frac{\mathbf{p}}{\mathbf{C}} = \frac{\hbar \pi}{meC^2 L} = \frac{1973 \text{eV} \cdot \text{Å}}{0.511 \times 10^6 \text{eV} \cdot \text{Å}^2} = 0.0024 \]

\[ U = 7.3 \times 10^{15} \text{Å} \]

(e) The electron travels 2L = 10 Å every time it hits the right well.

So in 1 s it hits \( \frac{U}{2L} \) times

\[ \text{times} = \frac{7.3 \times 10^{15}}{2} \]

(f) If every time it hits barrier the probability of tunneling out is \( T = 7.9 \times 10^{-4} \)

the probability it will tunnel out in a 1 s time interval is \( \approx 1 \)
Problem 5

(a) \[ E_{n_1, n_2} = \frac{k^2 \pi^2}{2me} \left( \frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} \right) = \frac{k^2 \pi^2}{2me L_1^2} \left( n_1^2 + n_2^2 \frac{L_1^2}{L_2^2} \right) = 1.50 \text{eV} \left( n_1^2 + n_2^2 \times 2.778 \right) \]
\[ E_{11} = 5.67 \text{eV}, \quad E_{21} = 10.17 \text{eV}, \quad E_{12} = 18.17 \text{eV}, \quad E_{31} = 17.66 \text{eV} \]

So the 3 lowest levels are:
\[ n_1, n_2 = 1, 1 ; \quad E_{11} = 5.76 \text{eV} \]
\[ n_1, n_2 = 2, 1 ; \quad E_{21} = 10.17 \text{eV} \]
\[ n_1, n_2 = 3, 1 ; \quad E_{31} = 17.66 \text{eV} \]

(b) To have \( \Psi(x, y = L_2/2) = 0 \) we need \( n_2 = 2 \), so lower level is 12.
\[ (n_1, n_2) = (1, 2) ; \quad E_{12} = 18.17 \text{eV} \] (Wave function)
\[ \Psi(x, y) = \sqrt{\frac{2}{L_1}} \sqrt{\frac{2}{L_2}} \sin \frac{\pi x}{L_1} \sin \frac{2\pi y}{L_2} \]

(c) For 5 electrons:
\[ 12 \quad \uparrow \quad 12 \quad \uparrow \]
\[ 31 \quad \uparrow \downarrow \quad 31 \quad \uparrow \downarrow \]
\[ 21 \quad \uparrow \downarrow \quad \text{absorb photon} \quad 21 \quad \uparrow \downarrow \]
\[ 11 \quad \uparrow \downarrow \quad 11 \quad \uparrow \downarrow \]

\[ E = 2E_{11} + 2E_{21} + E_{31} = 49.34 \text{eV} \]

The lowest excitation is electron in B1 \( \rightarrow \) 12.

\[ \Delta E = E_{12} - E_{31} = 18.17 \text{eV} - 17.66 \text{eV} = 0.51 \text{eV} = \frac{hc}{\lambda} \]
\[ = \frac{\lambda = \frac{hc}{\Delta E} = 24,314 \AA} {\lambda} \]
\[ \Psi(r, \theta, \phi) = C r^2 e^{-r/3a_0} \sin^2 \theta \cos \phi \]

(a) Since exponential part \( e^{-r/na_0} \Rightarrow n = 3 \)
Since \( R(r) \) has no zeroes, \( l = n-1 \Rightarrow l = 2 \)
Since a azimuthal part \( e^{im \phi} \Rightarrow m \equiv 1 \)

(b) \( P(r) = r^2 R^2(r) = C^2 \Gamma^6 e^{-2r/3a_0} \)
\[ P'(r) \times 6 \Gamma^5 - \frac{2}{3a_0} \Gamma^6 = 0 \Rightarrow \Gamma = \frac{6.3a_0}{2} \]
In the Bohr atom, \( \Gamma = n^2a_0 \), \( n = 3 \Rightarrow \Gamma_3 = 9a_0 \) same

(c) Numerically:
\[ L = \int_0^\infty dr \ P(r) = C^2 \int_0^\infty dr \ r^6 e^{-2r/3a_0} \]
\[ <r> = \int_0^\infty dr \ r P(r) = C^2 \int_0^\infty dr \ r^7 e^{-2r/3a_0} \]
Using \[ \int_0^\infty dr \ r^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}} \]
\[ <r> = \frac{7!}{\left(\frac{2}{3a_0}\right)^7} \cdot \frac{(\frac{2}{3a_0})^7}{6!} = \frac{7}{2} \frac{2}{3a_0} = \frac{21}{2} a_0 = 10.5a_0 \]

(d) \[ \left( \frac{1}{r} \right) = \int_0^\infty dr \ \frac{1}{r} P(r) = \frac{5!}{\left(\frac{2}{3a_0}\right)^6} \cdot \frac{(\frac{2}{3a_0})^7}{6!} = \frac{1}{6} \frac{2}{3a_0} = \frac{1}{9a_0} \]
In the Bohr atom, \[ \frac{1}{\Gamma n} = \frac{1}{n^2a_0} = \frac{1}{9a_0} \] same
Problem 7

\[ m_e = 1 \]

\[ n = 2, l = 1 \]

\[ n = 1 \]

In magnetic field: \( U = -\mathbf{\mu} \cdot \mathbf{B} = -\mu_B B \)

without spin, \( \mu_z = -\mu_B m_e \rightarrow U = +\mu_B B m_e \)

So: \( E_2 \rightarrow E_2 + \mu_B B m_e \)

\[ \mu_B B m_e = 5.79 \times 10^{-5} \text{eV} \]

\[ \Delta m_e = 8.69 \times 10^{-4} \text{meV} \]

\[ m_e = \frac{8.69 \times 10^{-4} \text{meV}}{1.6 \times 10^{-19} \text{eV}} \]

\[ h\Delta E = E_2 - E_1 = 13.6 \text{eV} \times \frac{3}{4} = 10.2 \text{eV} \]

(b) With spin: \( \mu_z = -\frac{e}{2m_e} (m_e + 2m_s) = -\mu_B (m_e + 2m_s) \)

\[ U = -\mu_z B = \mu_B B (m_e + 2m_s) = \mu_B B \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \]

\( m_e = 1, m_s = \frac{1}{2} \)

\( m_e = 0, m_s = \frac{1}{2} \)

\( m_e = 1, m_s = -\frac{1}{2} \)

\( m_e = 0, m_s = -\frac{1}{2} \)

\( m_e = -1, m_s = -\frac{1}{2} \)

(c) The difference in energy between the \( m_e = 1 \) and \( m_e = 0 \) states is \( 8.69 \times 10^{-4} \text{eV} \)

At low temperatures, most electrons will be in lowest-energy state \( m_e = -1 \).

Since the relative probability is given by the Boltzmann factor

\[ e^{-(E(m_e=0) - E(m_e=-1))/k_B T} \]

this becomes small when

\[ \frac{\hbar}{k_B} \begin{pmatrix} 1 \end{pmatrix} \]
**Problem 8**

\[ \Psi(x) = C \cos(hx), \quad [p] = \frac{\hbar}{i} \frac{d}{dx} \]

(a) \[ \Omega \] is a sharp observable if \([\Omega] \Psi = \eta \Psi, \] with \( \eta \) a number.

\[ [p] \Psi = \frac{\hbar}{i} \frac{d}{dx} C \cos(hx) = -\frac{\hbar}{i} C \sin(hx) \neq \eta C \cos(hx) \]

\[ \Downarrow \quad p \text{ is not a sharp observable} \]

(b) \[ [p^2] = \frac{\hbar}{i} \frac{d}{dx} \frac{\hbar}{i} \frac{d}{dx} = -\hbar \frac{d^2}{dx^2} \]

\[ [p^2] \Psi = -\hbar^2 \frac{d^2}{dx^2} C \cos(hx) = -\hbar^2 \sin(hx) = \eta \Psi \]

with \( \eta = \hbar^2 \) a number. So \( p^2 \) is a sharp observable.

(c) Since \( p^2 \) is sharp, there is no uncertainty,

\[ \Delta (p^2) = 0. \]

\[ \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \]

\[ \langle p \rangle = 0 \text{ because wave function is even: } \int_{-a}^{a} dx \cos(hx) \overset{\circ}{\frac{d}{dx}} \cos(hx) = 0 \]

\[ \langle p^2 \rangle = \hbar^2 \int_{-a}^{a} dx \mid \Psi(x) \mid^2 = \hbar^2 \quad \text{since } \Psi \text{ is normalized} \]

\[ \Downarrow \quad \Delta p = \hbar \]

...