Chapter 9 problems.

1. We use Eq. 9.5 to solve for \((x_3, y_3)\).

(a) The \(x\) coordinate of the system’s center of mass is:

\[
x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00 \text{ kg})(-1.20 \text{ m}) + (4.00 \text{ kg})(0.600 \text{ m}) + (3.00 \text{ kg})x_3}{2.00 \text{ kg} + 4.00 \text{ kg} + 3.00 \text{ kg}}
\]

\[= -0.500 \text{ m}.
\]

Solving the equation yields \(x_3 = -1.50 \text{ m}\).

(b) The \(y\) coordinate of the system’s center of mass is:

\[
y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00 \text{ kg})(0.500 \text{ m}) + (4.00 \text{ kg})(-0.750 \text{ m}) + (3.00 \text{ kg})y_3}{2.00 \text{ kg} + 4.00 \text{ kg} + 3.00 \text{ kg}}
\]

\[= -0.700 \text{ m}.
\]

Solving the equation yields \(y_3 = -1.43 \text{ m}\).

5. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We’ll refer to the large 35 cm \(\times\) 10 cm piece (shown to the left of the \(y\) axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at \((x_1, y_1) = (-5.0 \text{ cm}, -2.5 \text{ cm})\). The top 20 cm \(\times\) 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its center of mass is at \((x_2, y_2) = (10 \text{ cm}, 12.5 \text{ cm})\). The bottom 10 cm \(\times\) 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at \((x_3, y_3) = (5 \text{ cm}, -15 \text{ cm})\).

(a) The \(x\) coordinate of the center of mass for the plate is

\[
x_{\text{com}} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm}.
\]

(b) The \(y\) coordinate of the center of mass for the plate is

\[
y_{\text{com}} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm}.
\]
10. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for \( x_{\text{com}} \) and Eq. 9-17 for \( v_{\text{com}} \). At \( t = 3.0 \) s, the location of the automobile (of mass \( m_1 \)) is

\[
x_1 = \frac{1}{2} at^2 = \frac{1}{2} (4.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 18 \text{ m},
\]

while that of the truck (of mass \( m_2 \)) is \( x_2 = vt = (8.0 \text{ m/s})(3.0 \text{ s}) = 24 \text{ m} \). The speed of the automobile then is \( v_1 = at = \left(4.0 \text{ m/s}^2\right)(3.0 \text{ s}) = 12 \text{ m/s} \), while the speed of the truck remains \( v_2 = 8.0 \text{ m/s} \).

(a) The location of their center of mass is

\[
x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000 \text{ kg})(18 \text{ m}) + (2000 \text{ kg})(24 \text{ m})}{1000 \text{ kg} + 2000 \text{ kg}} = 22 \text{ m}.
\]

(b) The speed of the center of mass is

\[
v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000 \text{ kg})(12 \text{ m/s}) + (2000 \text{ kg})(8.0 \text{ m/s})}{1000 \text{ kg} + 2000 \text{ kg}} = 9.3 \text{ m/s}.
\]
14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically aligned. Because of that alignment, \( v_{2y} = v_{1y} = 10.0 \text{ m/s} \). Because the initial value of \( v_2 \) is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

\[
v_{2p} = \sqrt{v_2^2 - v_{2y}^2} = \sqrt{300} \text{ m/s}
\]

for the initial value of the \( y \) component of particle 2's velocity. Equation 2-16 (or conservation of energy) readily yields \( y_{\text{max}} = \frac{300}{19.6} = 15.3 \text{ m} \). Thus, we obtain

\[
H_{\text{max}} = m_2 y_{\text{max}} / m_{\text{total}} = \frac{(3.00 \text{ g})(15.3 \text{ m})}{(8.00 \text{ g})} = 5.74 \text{ m}.
\]

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply \( (10.0 \text{ m/s}) \hat{i} \) (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

\[
a_{\text{com}} = m_2 g / m_{\text{total}} = \frac{(3.00 \text{ g})(9.8 \text{ m/s}^2)}{(8.00 \text{ g})} = 3.68 \text{ m/s}^2.
\]

Thus, \( a_{\text{com}} = (-3.68 \text{ m/s}^2) \hat{j} \).
39. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let \( m_s \) be the mass of the stone and \( v_s \) be its velocity after it is kicked; let \( m_m \) be the mass of the man and \( v_m \) be his velocity after he kicks the stone. Then

\[ m_s v_s + m_m v_m = 0 \Rightarrow v_m = -\frac{m_s v_s}{m_m}. \]

We take the axis to be positive in the direction of motion of the stone. Then

\[ v_m = -\left(\frac{0.068 \text{ kg}}{91 \text{ kg}}\right)(4.0 \text{ m/s}) = -3.0 \times 10^{-3} \text{ m/s}, \]

or \( |v_m| = 3.0 \times 10^{-3} \text{ m/s} \). The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

42. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is \( m \); its initial velocity is \( \bar{v}_0 = \nu \hat{i} \); the mass of the less massive piece is \( m_1 \); its velocity is \( \bar{v}_1 = 0 \); and, the mass of the more massive piece is \( m_2 \). We note that the conditions \( m_2 = 3m_1 \) (specified in the problem) and \( m_1 + m_2 = m \) generally assumed in classical physics (before Einstein) lead us to conclude

\[ m_1 = \frac{1}{4} m \quad \text{and} \quad m_2 = \frac{3}{4} m. \]

Conservation of linear momentum requires

\[ m \bar{v}_0 = m_1 \bar{v}_1 + m_2 \bar{v}_2 \Rightarrow m \bar{v}_i = 0 + \frac{3}{4} m \bar{v}_2 \]

which leads to \( \bar{v}_2 = \frac{4}{3} \nu \hat{i} \). The increase in the system’s kinetic energy is therefore

\[ \Delta K = \frac{1}{2} m_1 \nu_1^2 + \frac{1}{2} m_2 \nu_2^2 - \frac{1}{2} m v_0^2 = 0 + \frac{1}{2} \left(\frac{3}{4} m\right) \left(\frac{4}{3} \nu\right)^2 - \frac{1}{2} m v^2 = \frac{1}{6} m v^2. \]
44. We can think of the sliding-until-stopping as an example of kinetic energy converting into thermal energy (see Eq. 8-29 and Eq. 6-2, with \( F_N = mg \)). This leads to \( v^2 = 2 \mu_g d \) being true separately for each piece. Thus we can set up a ratio:

\[
\left( \frac{v_k}{v_r} \right)^2 = \frac{2 \mu_k gd_k}{2 \mu_r gd_r} = \frac{12}{25}.
\]

But (by the conservation of momentum) the ratio of speeds must be inversely proportional to the ratio of masses (since the initial momentum before the explosion was zero). Consequently,

\[
\left( \frac{m_k}{m_L} \right)^2 = \frac{12}{25} \Rightarrow m_k = \frac{2}{5} \sqrt{3} m_L = 1.39 \text{ kg}.
\]

Therefore, the total mass is \( m_k + m_L \approx 3.4 \text{ kg} \).

51. In solving this problem, our \( +x \) direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

\[(0.0035 \text{ kg})v = (1.8035 \text{ kg})(1.4 \text{ m/s}) \Rightarrow v = 721 \text{ m/s}.
\]

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed \( v \) found in part (a)).

\[(0.0035 \text{ kg})v_0 = (1.20 \text{ kg})(0.630 \text{ m/s}) + (0.00350 \text{ kg})(721 \text{ m/s})
\]

which yields \( v_0 = 937 \text{ m/s} \).
53. With an initial speed of \( v_i \), the initial kinetic energy of the car is \( K_i = \frac{1}{2} m_c v_i^2 \). After a totally inelastic collision with a moose of mass \( m_m \), by momentum conservation, the speed of the combined system is
\[
 m_c v_i = (m_c + m_m) v_f \quad \Rightarrow \quad v_f = \frac{m_c v_i}{m_c + m_m},
\]
with final kinetic energy
\[
 K_f = \frac{1}{2} (m_c + m_m) v_f^2 = \frac{1}{2} (m_c + m_m) \left( \frac{m_c v_i}{m_c + m_m} \right)^2 = \frac{1}{2} \frac{m^2_c}{m_c + m_m} v_i^2.
\]
(a) The percentage loss of kinetic energy due to collision is
\[
 \frac{\Delta K}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{m_c}{m_c + m_m} = \frac{m_m}{m_c + m_m} = \frac{500 \text{ kg}}{1000 \text{ kg} + 500 \text{ kg}} = \frac{1}{3} = 33.3\%.
\]
(b) If the collision were with a camel of mass \( m_{\text{camel}} = 300 \text{ kg} \), then the percentage loss of kinetic energy would be
\[
 \frac{\Delta K}{K_i} = \frac{m_{\text{camel}}}{m_c + m_{\text{camel}}} = \frac{300 \text{ kg}}{1000 \text{ kg} + 300 \text{ kg}} = \frac{3}{13} = 23\%.
\]
(c) As the animal mass decreases, the percentage loss of kinetic energy also decreases.

56. (a) Let \( m_A \) be the mass of the block on the left, \( v_{Ai} \) be its initial velocity, and \( v_{Af} \) be its final velocity. Let \( m_B \) be the mass of the block on the right, \( v_{Bi} \) be its initial velocity, and \( v_{Bf} \) be its final velocity. The momentum of the two-block system is conserved, so
\[
 m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}
\]
and
\[ v_f = \frac{m_A v_{A_i} + m_B v_{B_i} - m_B v_{B_f}}{m_A} = \frac{(1.6 \text{ kg})(5.5 \text{ m/s}) + (2.4 \text{ kg})(2.5 \text{ m/s}) - (2.4 \text{ kg})(4.9 \text{ m/s})}{1.6 \text{ kg}} \]

= 1.9 \text{ m/s.}

(b) The block continues going to the right after the collision.

(c) To see whether the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

\[ K_i = \frac{1}{2} m_A v_{A_i}^2 + \frac{1}{2} m_B v_{B_i}^2 = \frac{1}{2} (1.6 \text{ kg})(5.5 \text{ m/s})^2 + \frac{1}{2} (2.4 \text{ kg})(2.5 \text{ m/s})^2 = 31.7 \text{ J.} \]

The total kinetic energy after is

\[ K_f = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} (1.6 \text{ kg})(1.9 \text{ m/s})^2 + \frac{1}{2} (2.4 \text{ kg})(4.9 \text{ m/s})^2 = 31.7 \text{ J.} \]

Since \( K_i = K_f \) the collision is found to be elastic.
61. Let $m_1$ be the mass of the cart that is originally moving, $v_{i1}$ be its velocity before the collision, and $v_{1f}$ be its velocity after the collision. Let $m_2$ be the mass of the cart that is originally at rest and $v_{2f}$ be its velocity after the collision. Conservation of linear momentum gives $m_1v_{i1} = m_1v_{1f} + m_2v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2}m_1v_{i1}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2.$$

Solving for $v_{1f}$ and $v_{2f}$, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{i1}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{i1}.$$

The speed of the center of mass is $v_{com} = \frac{m_1v_{1f} + m_2v_{2f}}{m_1 + m_2}$.

(a) With $m_1 = 0.34 \text{ kg}$, $v_{i1} = 1.2 \text{ m/s}$ and $v_{1f} = 0.66 \text{ m/s}$, we obtain

$$m_2 = \frac{v_{i1} - v_{1f}}{v_{i1} + v_{1f}} m_1 = \frac{1.2 \text{ m/s} - 0.66 \text{ m/s}}{1.2 \text{ m/s} + 0.66 \text{ m/s}} (0.34 \text{ kg}) = 0.0987 \text{ kg} \approx 0.099 \text{ kg}.$$

(b) The velocity of the second cart is:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{i1} = \frac{2(0.34 \text{ kg})}{0.34 \text{ kg} + 0.099 \text{ kg}} (1.2 \text{ m/s}) = 1.9 \text{ m/s}.$$

(c) From the above, we find the speed of the center of mass to be

$$v_{com} = \frac{m_1v_{1f} + m_2v_{2f}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(1.2 \text{ m/s}) + 0}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$

Note: In solving for $v_{com}$, values for the initial velocities were used. Since the system is isolated with no external force acting on it, $v_{com}$ remains the same after the collision, so the same result is obtained if values for the final velocities are used. That is,

$$v_{com} = \frac{m_1v_{1f} + m_2v_{2f}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(0.66 \text{ m/s}) + (0.099 \text{ kg})(1.9 \text{ m/s})}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$
71. We apply the conservation of linear momentum to the x and y axes respectively,

\[ m_1 v_{1y} = m_1 v_{1y} \cos \theta_1 + m_2 v_{2y} \cos \theta_2 \]
\[ 0 = m_1 v_{1y} \sin \theta_1 - m_2 v_{2y} \sin \theta_2. \]

We are given \( v_{2y} = 1.20 \times 10^5 \text{ m/s} \), \( \theta_1 = 64.0^\circ \) and \( \theta_2 = 51.0^\circ \). Thus, we are left with two unknowns and two equations, which can be readily solved.

(a) We solve for the final alpha particle speed using the y-momentum equation:

\[ v_{1y} = \frac{m_1 v_{1y} \sin \theta_1}{m_2 \sin \theta_2} = \frac{(16.0) (1.20 \times 10^5) \sin (51.0^\circ)}{(4.00) \sin (64.0^\circ)} = 4.15 \times 10^5 \text{ m/s}. \]

(b) Plugging our result from part (a) into the x-momentum equation produces the initial alpha particle speed:

\[ v_0 = \frac{m_1 v_{1y} \cos \theta_1 + m_2 v_{2y} \cos \theta_2}{m_1} \]
\[ = \frac{(4.00) (4.15 \times 10^5) \cos (64.0^\circ) + (16.0) (1.2 \times 10^5) \cos (51.0^\circ)}{4.00} \]
\[ = 4.84 \times 10^5 \text{ m/s}. \]
Chapter 9, Problem X30.

\[ v_{\text{initial}} = V_i = 340 \text{ (m/s)} \]

\[ v_{\text{final}} = V_f = 340 + 4.92 \text{ (m/s)} = 344.92 \text{ (m/s)} \]

\[ v_{\text{relative}} = V_r = 840 \text{ (m/s)} \]

According to the equation (9.88),

\[ V_f - V_i = V_r \ln \left( \frac{M_{\text{initial}}}{M_{\text{final}}} \right) \]

Let \( M_{\text{final}} = M_{\text{initial}} (1 - \Delta) \), so we need to find \( \Delta \).

\[ \frac{V_f - V_i}{V_r} = \ln \left( \frac{M_{\text{initial}}}{M_{\text{final}} (1 - \Delta)} \right) = \ln \left( \frac{1}{1 - \Delta} \right) \]

\[ \ln \left( \frac{1}{1 - \Delta} \right) = \frac{V_f - V_i}{V_r} \Rightarrow \frac{1}{1 - \Delta} = \exp \left( \frac{V_f - V_i}{V_r} \right) \]

\[ 1 - \Delta = \exp \left( - \frac{V_f - V_i}{V_r} \right) \Rightarrow \Delta = 1 - \exp \left( - \frac{V_f - V_i}{V_r} \right) \]

\[ \Delta = 1 - \exp \left( - \frac{(344.92 - 340)}{840} \right) = 1 - \exp \left( - \frac{4.92}{840} \right) \approx 0.00584023238. \]