

## Solutions to Quiz 4, Physics 1b (Levine)

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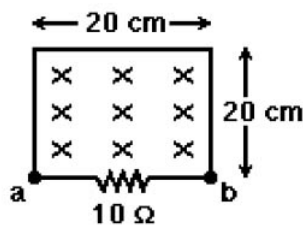
### Answers

**Version A:** 1-d 2-a 3-b 4-d 5-b 6-c 7-c 8-a

**Version B:** 1-a 2-a 3-d 4-b 5-b 6-d 7-c 8-c

**Version C:** 1-d 2-b 3-a 4-b 5-c 6-a 7-c 8-d

### Problem A1/B6/C8



For this problem, we need to use Faraday's Law of Induction, which is:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (1)$$

The magnetic flux  $\Phi_B$  through the loop is equal to the magnetic field in the loop times the area of the loop:  $\Phi_B = BA$ . Since the magnetic field is getting weaker, we know that the magnetic flux is decreasing, so there is some EMF  $\mathcal{E}$  being generated. We'll deal with the direction in just a moment.  $\frac{d\Phi_B}{dt}$  is found using  $\frac{\Delta\Phi_B}{\Delta t}$ , so equation 1 tells us that  $\mathcal{E} = 0.4 \text{ V}$ . EMF is kinda like voltage, so to find the current through the resistor, we use Ohm's law:  $V = IR$ . We find that the current through the resistor is  $\frac{\mathcal{E}}{R} = 40 \text{ mA}$ .

Now, let's figure out the direction of the EMF. According to Lenz's Law, a changing magnetic flux through a loop induces a current in that loop, and that current generates a magnetic field that opposes the change in flux. In this problem, the magnetic field points into the page ( $\otimes$ ), but it's decreasing. The induced current will want to reinforce that diminishing magnetic field. Remember how we find the direction of magnetic field from a current loop: if we curl our fingers in the direction of current, our thumb points in the direction of the magnetic field. In this problem, we can do the reverse: point your thumb into the page, in the direction of the magnetic field generated by the loop, and your fingers will point in the direction of the current: clockwise. Thus our induced 40 mA current flows from b to a (*choice d*).

**Note:** if the magnetic flux were increasing then the induced current would generate a magnetic field to oppose this increase. The created magnetic field would point out of the page ( $\odot$ ), so the current would flow counterclockwise.

## Problem A2/B1/C6

This problem also employs Faraday's law, which I gave you in equation (1). Since this problem deals with a coil of current that is rotating, I'm going to be a bit more mathematically rigorous in how I treat magnetic flux. In the previous problem, I just multiplied the magnetic field by the area of the loop to get the flux. I could do that because the magnetic field was always pointing perpendicular to the plane of the loop (i.e. parallel to  $\vec{A}$ ). In this problem, that is not always true, so we have to say that in general

$$\Phi_B = N(\vec{B} \cdot \vec{A}) = N|B||A|\cos\theta \quad (2)$$

Here  $N$  is the number of turns in the loop, and  $A$  is the area. In our setup  $\theta = \omega t$  because the loop is rotating, so

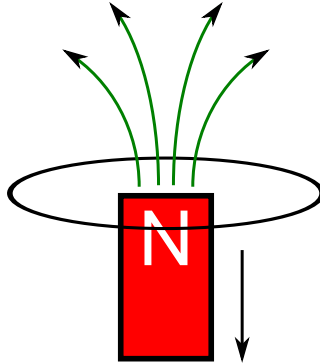
$$\Phi_B = NBA\cos(\omega t)$$

In order to find the EMF through this circuit, let's take the time derivative of magnetic flux:

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} \\ &= -\frac{d}{dt}[NBA\cos(\omega t)] \\ &= -NBA[-\omega\sin(\omega t)] \\ &= NBA\omega\sin(\omega t) \end{aligned}$$

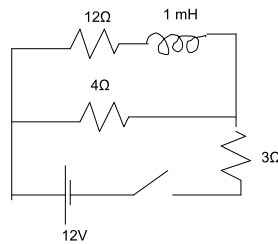
When the magnetic field is parallel to the area vector,  $\theta = \omega t = 0$ . Since  $\sin(0) = 0$ , we know that when the magnetic field is parallel to  $\vec{A}$ , the EMF is zero (*choice a*).

### Problem A3/B5/C4



Magnetic field lines point outward from the north pole of a magnet. As the north pole of our bar magnet drops down past the loop of wire, fewer field lines will pass upward through the loop. Use Lenz's Law like I explained in problem 1: The magnetic flux points up through the loop, and it's decreasing. The loop will have a current induced in it, creating a magnetic field to reinforce the decreasing flux from the bar magnet. If the loop wants to create a magnetic field pointing up (use your thumb), then it must have a current flowing in the direction of your fingers: counterclockwise (*choice b*).

### Problem A4/B3/C1



This problem deals with the dynamics of an R-L circuit, and the way in which an inductor settles down to equilibrium. If a voltage is applied across a resistor and an inductor at time  $t = 0$ , the current through the inductor is given by:

$$I_L(t) = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau}\right) \quad \text{where} \quad \tau = \frac{L}{R} \quad (3)$$

Our switch is closed at time  $t = 0$ .  $e^0 = 1$ , and  $1 - 1 = 0$ , so no current will flow through the inductor immediately after the switch is closed. That means

that all the current generated by the battery will flow through the  $4\ \Omega$  resistor and the  $3\ \Omega$  resistor in series, and  $12\text{ V}/7\ \Omega = 1.71\text{ A}$  (*choice d*).

## Problem A5/B4/C2

We now move from the time-dependent behavior of inductors to their steady-state behavior. We know that the EMF (or voltage) across an inductor is given by:

$$\mathcal{E} = -L \frac{dI}{dt} \quad (4)$$

What this means is that inductors respond to fluctuating currents. After our circuit has been left to sit for a long time, much longer than tau ( $t \gg \tau$ ), all the currents in the circuit will have reached their equilibrium values; that is to say, none of the currents will be fluctuating anymore. If none of the currents are changing in time, then there will be zero EMF across our inductor, and it will behave just like a normal wire. When this happens, we're left with a simple resistor network. We can combine our  $12\ \Omega$  and  $4\ \Omega$  resistors in parallel, then add that in series with our  $3\ \Omega$  resistor to get an equivalent resistance of  $6\ \Omega$ . Dividing, we see that the total current through the circuit is  $2\text{ A}$ . Thus the voltage drop across the two parallel resistors is  $6\text{ V}$ , so the current through the  $4\ \Omega$  resistor is  $1.5\text{ A}$  (*choice b*).

## Problem A6/B8/C7

Inductors are circuit components that store energy in the form of magnetic fields. The total energy stored in an inductor is given by:

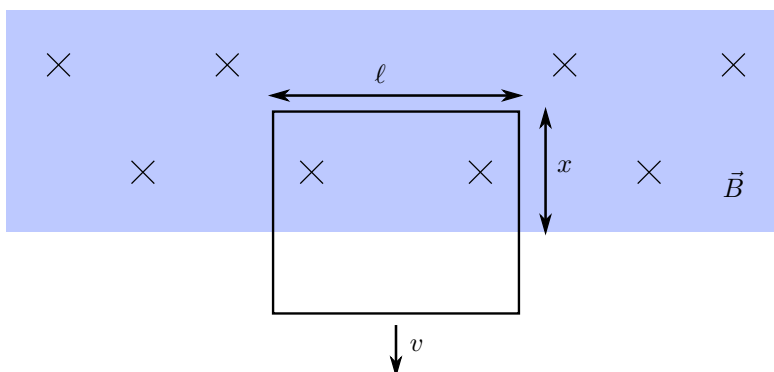
$$U = \frac{1}{2} LI^2 \quad (5)$$

If  $U = 10\text{ J}$  and  $I = 6\text{ A}$ , we can solve to find  $L = 0.555\ \text{H} \approx 0.56\text{ H}$  (*choice c*).

## Problem A7/B7/C5

This is definitely a tricky problem, but it's possible to calculate the terminal velocity of the loop. The first piece of information we can use is the fact that our loop is falling at terminal velocity. Terminal velocity means it's falling at a constant rate, neither accelerating nor decelerating. According to Newton's Laws, this means that the forces on the loop are balanced - the gravitational force pulling it down equals the magnetic force pulling it up. Only the top segment of the loop feels a force upward, because the bottom segment isn't in the magnetic field, and the two side segments feel a magnetic force pushing sideways. So we have:

$$mg = I\ell B$$



In this equation, we know all the values except for the current  $I$ . The current is given by the EMF in the loop, divided by the resistance ( $2\Omega$ ) of the loop:  $I = \frac{\mathcal{E}}{R}$ . Now we have to find  $\mathcal{E}$ .

We can solve for  $\mathcal{E}$  using Faraday's law (1). The magnetic flux through the loop is given by  $\Phi_B = B\ell x$ , where  $x$  is the height of the area of flux (i.e. the height of the region where the loop overlaps the magnetic field). The time derivative of the magnetic flux gives us the EMF:

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \frac{d}{dt}(B\ell x) = B\ell v$$

$v$  is the velocity of the falling loop, and it's what we want to solve for in the end.

We now have two expressions for  $I$ : one is given by Newton's laws above, and the other is given by the EMF. We can set these two expressions equal to each other, eliminating  $I$  and leaving us with an equation we can use to find the velocity:

$$I = \frac{mg}{B\ell} = \frac{B\ell v}{R} \quad \text{so} \quad v = \frac{mgR}{B^2\ell^2}$$

Solving, we find  $v = 5.227$  m/s (*choice c*).

### Problem A8/B2/C3

This question is somewhat different from the usual fare, in that it requires you to have a conceptual understanding of the physics. Lenz's Law basically states that loops don't like it when the magnetic flux through them changes. If the magnetic flux through a loop decreases, a current will be induced in the loop, creating a magnetic field to reinforce the decreasing flux. Conversely, if the flux through a loop increases, a current will be induced, creating a magnetic field that opposes the increasing flux. Or, as the quiz puts it, "inductive effects always act in a sense to oppose the change which caused them" (*choice a*).