Lecture #11:

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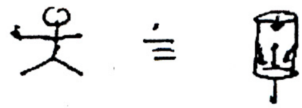
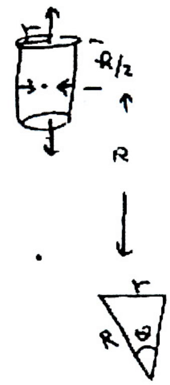
Tidal ripping and pressure at $r = r_g$



In rest frame of astronaut we have:
(a) ripped from head to toe in vertical
(b) squeezed at the side



$$z \quad \Delta F_{\parallel} = (\partial F_g/\partial z)\, R/2 = \frac{GmM}{R^3}\cdot\frac{R}{2}$$

$$x \quad \Delta F_{\perp} = \frac{GmM}{R^2}\cdot\frac{r}{R} = \frac{GMm}{R^3}\cdot r$$

Suppose data is
$M = 1 M_\odot = 2\times10^{33}\,g$
$m = 80\ kg = 8\times10^4\,g$
$R = r_g = 3\ km = 3\times10^5\ cm = 2GM/c^2$
$\mathcal{R} = 2\ m = 2\times10^2\,cm$
$r = 10\ cm$

Pressure: $$P_\perp = \frac{\Delta F_\perp}{A_\perp} = \frac{\Delta F_\perp}{2\pi r \mathcal{R}} = \frac{GMm\cdot r}{R^3}\times\frac{1}{2\pi r\mathcal{R}} = \frac{GMm}{R^3}\times\frac{1}{2\pi\mathcal{R}}$$

$$P_{\parallel}' = \frac{\Delta F_{\parallel}}{A_{\parallel}} = \frac{\Delta F_{\parallel}}{\pi r^2} = \frac{GMm}{R^3}\cdot\frac{\mathcal{R}}{2}\times\frac{1}{\pi r^2} = \frac{GMm}{R^3}\times\frac{1}{2\pi}\times\frac{\mathcal{R}}{r^2}$$

$$P_{\parallel}'/P_\perp = \frac{\mathcal{R}/r^2}{1/\mathcal{R}} = (\mathcal{R}/r)^2 \simeq (20)^2 = 400$$

$$P_{\parallel}' = \frac{6.7\times10^{-8}\times 2\times10^{33}\times(8\times10^4)}{(3\times10^5)^3}\times\frac{2\times10^2}{2\pi\times(10)^2} = 1.3\times10^{14}\ \text{dynes cm}^{-2}$$

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BH 88C

1 atmosphere = 10^6 dynes/cm^2

$P_{\parallel}' \approx 10^8$ atmospheres $\Rightarrow P_{\perp} = 2.5 \times 10^5$ atmospheres

Human body can withstand no more than $P = 10^2$ At.
So astronaut would be finished at Kill radius,
$r = r_K > r_g$

for $\parallel$ $\quad \dfrac{P_{\parallel}(r_K)}{P_{\parallel}(r_g)} = \left(\dfrac{r_g}{r_K}\right)^3 = \dfrac{10^2}{10^8} = \dfrac{1}{10^6}$

$\Rightarrow r_K = 100 \cdot r_g$ or $r_K = 300$ km (180 mi)

Massive Black Holes   Less lethal

Recall $P(R) \propto M/R^3$

$\therefore P(r_g) \propto M/(2GM/c^2)^3 \propto 1/M^2$

Suppose $M = 2\times10^7 M_\odot$ (Mass of BH at center of Andromeda)

$$\frac{P(r_g)_{Galaxy}}{P(r_g)_{star}} = \left(\frac{1}{2\times10^7}\right)^2 = 2.5\times10^{-15}$$

ratio at Schwarzschild radii

$\therefore P(r_g) = 2.5\times10^{-15} \times 1.3\times10^{14} \approx 0.33$ At

no problem

At $r = r_g = 2\times10^7 \cdot 3$ km $= 6\times10^7$ km

But at smaller radius all bets are off.

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Lightlike Geodesics

Go back to Geodesic equations in Schwarzschild metric

t (1) $$\boxed{\frac{dt}{d\lambda} = \frac{E}{\left(1-\frac{2GM/c^2}{r}\right)}}$$ (eq. 1, p. BH-77)

ϕ (2) $$\boxed{\frac{d\phi}{d\lambda} = \frac{L}{r^2}}$$ (eq. 2a, BH-77 $\to K = -\left(\frac{ds}{d\lambda}\right)^2$

r $$\frac{1}{2}\left(\frac{dr}{d\lambda}\right)^2 + \frac{1}{2}\left(1-\frac{2GM/c^2}{r}\right)\left(K+\frac{L^2}{r^2}\right) = \frac{c^2E^2}{2}$$ (eq. 4; BH79

For lightlike geodesics set $K=0$. Rewrite last equation

(3) $$\boxed{\left(\frac{dr}{d\lambda}\right)^2 = c^2E^2 - \frac{L^2}{r^2}\left(1-\frac{2GM/c^2}{r}\right)}$$

Define: $\lambda_{new} \equiv L\cdot\lambda$; $b \equiv L/E$

Eq. (1): since $\frac{1}{\lambda} = \frac{L}{\lambda_{new}}$, eq. (1) becomes:

$$\frac{dt}{d\lambda} = L\frac{dt}{d\lambda_{new}} = \frac{E}{1-\frac{2GM/c^2}{r}} \quad \text{or} \quad \frac{dt}{d\lambda_{new}} = \frac{(E/L)}{1-\frac{2GM/c^2}{r}}$$

or $$\frac{dt}{d\lambda} = \frac{1}{b\left(1-\frac{2GM/c^2}{r}\right)}$$

eq. (2) $$L\frac{d\phi}{d\lambda_{new}} = \frac{L}{r^2} \Rightarrow \frac{d\phi}{d\lambda} = \frac{1}{r^2}$$

Eq. (3) $$L^2\left(\frac{dr}{d\lambda_{new}}\right)^2 = c^2E^2 - \frac{L^2}{r^2}\left(1-\frac{2GM/c^2}{r}\right)$$

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{c^2}{b^2} - \frac{1}{r^2}\left(1-\frac{2GM/c^2}{r}\right)$$

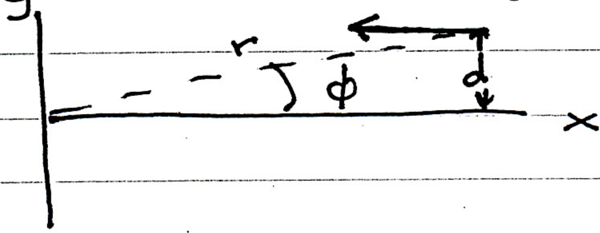
$\lambda = \lambda_{new}$

The lightlike geodesic of the photon depends only on the parameter b not on L nor E separately.

What is b ?.

Consider orbits reaching $r \to \infty$. At such lg. distances from r_g, space is flat and Cartesian coordinates can be used such that

$$x = r\cos\phi \quad ; \quad y = r \cdot \sin\phi$$



Consider light ray moving parallel to x axis at $y = +d$.

$$b = \lim_{r\to\infty} \frac{L}{E} = \lim_{r\to\infty} \frac{r^2 \frac{d\phi}{d\lambda}}{\left(1 - \frac{2GM/c^2}{r}\right)\frac{dt}{d\lambda}} \approx r^2 \frac{d\phi}{dt}$$

As $r \to \infty$, $\quad \tan\phi \simeq \phi = d/r \quad ; \quad dr/dt = -c$

Since $\quad \dfrac{d\phi}{dt} = \dfrac{d\phi}{dr} \times \dfrac{dr}{dt} = -\dfrac{d}{r^2} \times (-c)$

Therefore

$$b \approx \cancel{r^2}\left(\frac{dc}{\cancel{r^2}}\right) = d \cdot c$$

So, b = impact parameter × speed of light

True Impact parameter $\quad d = \dfrac{b}{c}$

Back to radial equation

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{d^2} - V_{phot}(r) \;;\quad V_{phot}(r) = \frac{1}{r^2}\left(1 - \frac{2GM/c^2}{r}\right)$$

Let's rename $b = d \Rightarrow$ $\boxed{\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{b^2} - V_{phot}(r)}$

Find extremum of $V_{phot}(r)$

$$V_{phot}(r) = \frac{1}{r^2} - \frac{2GM/c^2}{r^3} \quad (\text{Note: } V_{phot} = 0 \text{ at } r = 2GM/c^2)$$

$$\frac{\partial V_{phot}}{\partial r} = -\frac{2}{r^3} + \frac{6GM/c^2}{r^4} \Rightarrow r_{extreme} = 3\,GM/c^2$$

To determine whether this is max. or min find

$$\left(\frac{\partial^2 V_{phot}}{\partial r^2}\right)_{r_{extreme}} = \left[\frac{6}{r^4} - \frac{24\,GM/c^2}{r^5}\right]_{r_{extreme}}$$

$$\therefore \left(\frac{\partial^2 V_{phot}}{\partial r^2}\right)_{r_{extreme}} = \frac{6}{(r_{ex})^4}\left[1 - \frac{4(GM/c^2)}{r_{ext}}\right]$$

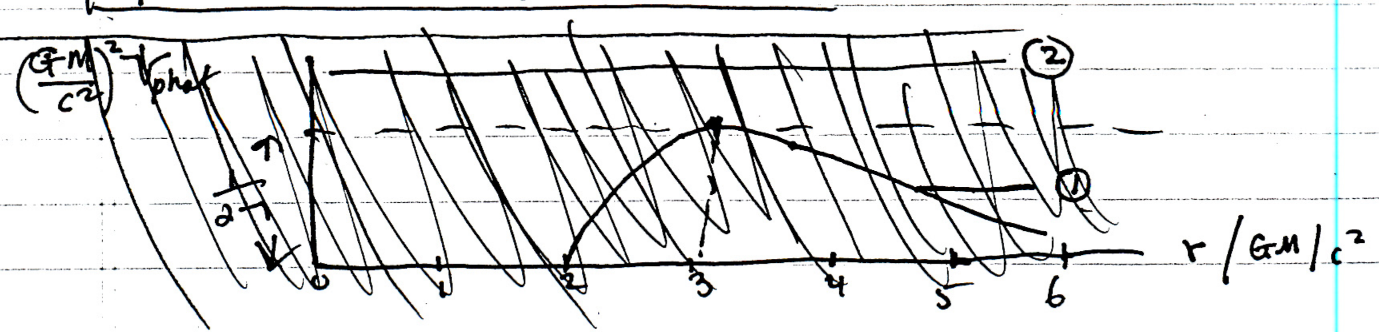
$$\left(\frac{\partial^2 V_{phot}}{\partial r^2}\right)_{r_{extreme}} = \frac{6}{(3GM/c^2)^4}\left[1 - \frac{4(GM/c^2)}{3(GM/c^2)}\right]$$

$$\left(\frac{\partial^2 V_{phot}}{\partial r^2}\right)_{r_{extreme}} = \frac{6}{(3GM/c^2)^4}\left[1 - \frac{4}{3}\right] < 0 \; \therefore \text{ Maximum}$$

Find $V_{phot}(r_{max}) = \frac{1}{r_{max}^2}\left[1 - \frac{2GM/c^2}{r_{ex}}\right]$

$$V_{phot}(r_{max}) = \frac{1}{(3GM/c^2)^2}\left[1 - \frac{2GM/c^2}{3GM/c^2}\right] = \frac{1}{9(GM/c^2)^2} \times \frac{1}{3}$$

$$\boxed{V_{phot}(r_{max}) = \frac{1}{27} \times \frac{1}{(GM/c^2)^2}}$$

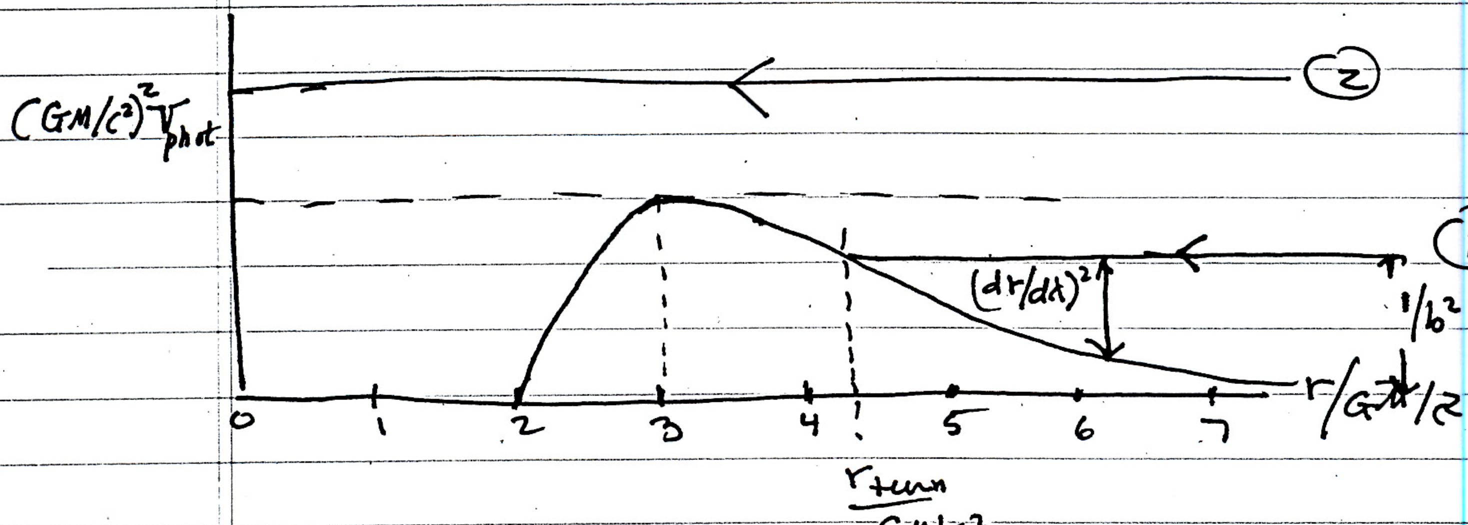


Recall $\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{b^2} - V_{phot}(r)$

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2GM/c^2}{r}\right)$$

Thus we have turning point possibility as with timelike geodesics

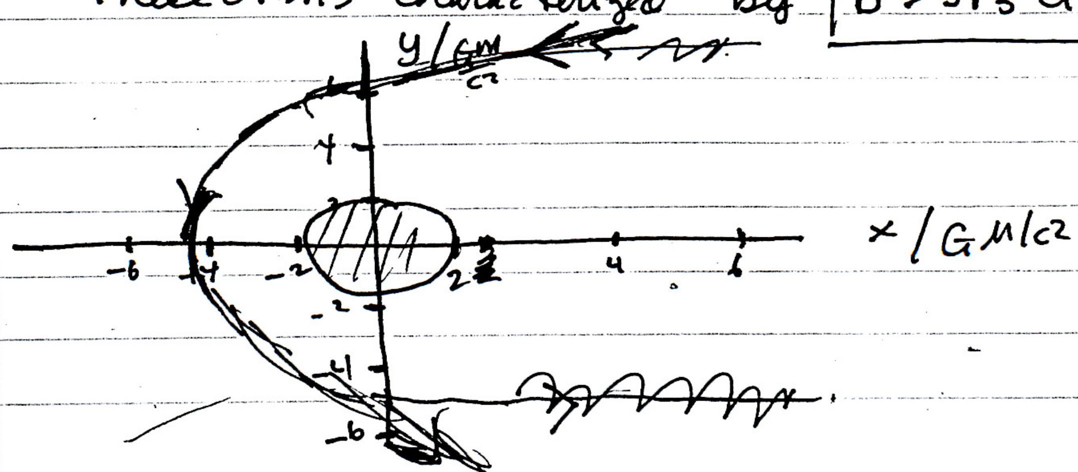
Case ① Suppose $1/b^2 < V_{phot}(r_{max})$



• Turning point at $V_{phot}(r_{turn}) = 1/b^2$

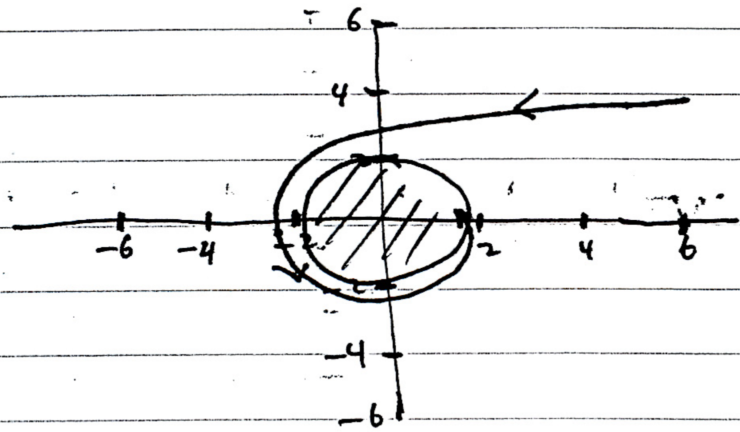
Since $\frac{1}{b^2} < \frac{1}{27} \times \frac{1}{(GM/c^2)^2}$

These orbits characterized by $\boxed{b > 3\sqrt{3}\, GM/c^2}$



Case (2) Suppose $1/b^2 > V_{phot}(r_{max})$

In this case $b < 3\sqrt{3}\, GM/c^2$.
and we have a capture orbit



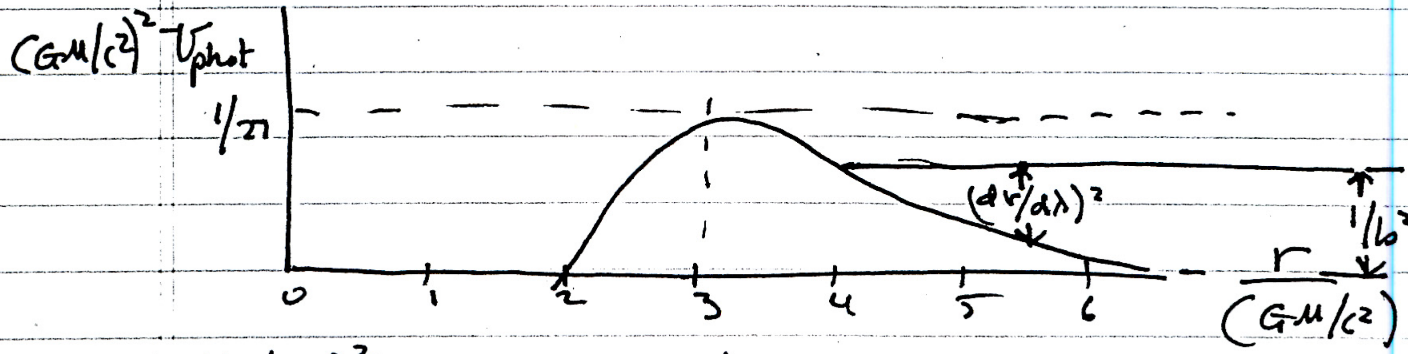
No analogue in Newtonian physics.
Capture can occur in quantum mechanics since particle (e^- -) can lose energy by emitting a photon.
Also in Newtonian physics, presence of a third body allows energy transfer!

Recap: Lightlike geodesics in Schwarzschild metric

Recall: $\left(\frac{dr}{d\lambda}\right)^2 + V_{phot}(r) = \frac{1}{b^2}$

Analogous to Conservation of energy eq.

$$\frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + V_{eff}(r) = E$$



Since $(dr/d\lambda)^2 \geq 0 \Rightarrow 1/b^2 > V_{phot}$

- $(dr/d\lambda)^2 = 1/b^2 - V_{phot}(r)$: distance between $\frac{1}{b^2}$ and V_{phot}
- $V_{max} = \frac{1}{27}(GM/c^2)^{-2}$ at $r = 3GM/c^2$

  since $V_{phot}(r) = \frac{1}{r^2}\left(1 - \frac{2GM/c^2}{r}\right)$

  (1 max contrast with timelike - 1 max + 1 min
- No stable circular orbits unlike timelike
- Critical impact parameter separating capture orbits from scattering orbits given by

  $$\frac{1}{b^2} = V_{max} = \frac{1}{27} \times \frac{1}{(GM/c^2)^2} \Rightarrow$$

  $$b_{crit} = 3\sqrt{3}\, GM/c^2$$

- $\frac{1}{b^2} < \frac{1}{27} \times \frac{1}{(GM/c^2)^2}$ or $b > 3\sqrt{3}\, GM/c^2$

  orbits exhibit turning points

- $\frac{1}{b^2} > \frac{1}{27} \times \frac{1}{(GM/c^2)^2}$ or $b < 3\sqrt{3}\, GM/c^2$

  orbits are capture orbits

Critical Impact Parameter

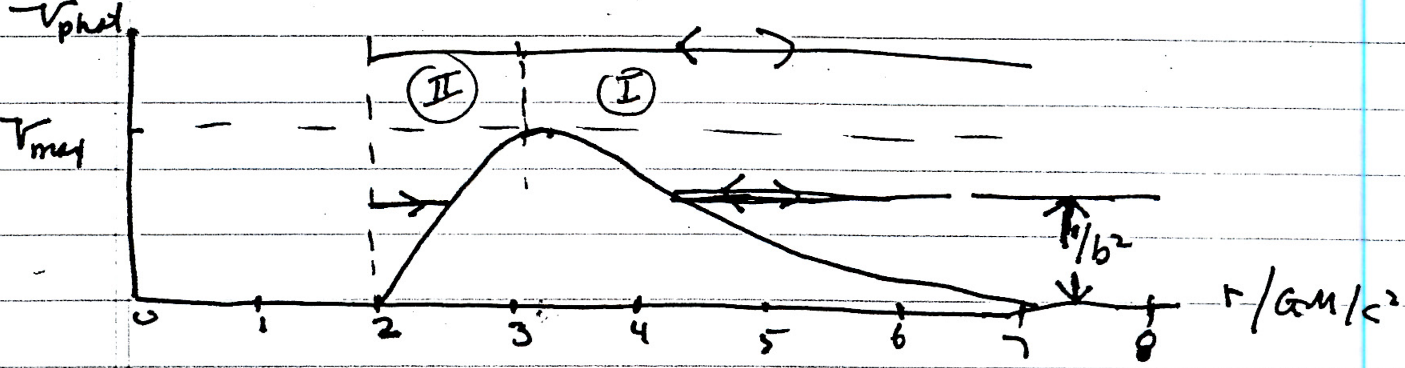
$$b_{crit} = 3\sqrt{3}\, GM/c^2$$

Thus cross-section for photons arriving from $r=\infty$ is: km^2

$$\sigma_{phot} = \pi b_{crit}^2 = \pi \cdot 27\, (GM/c^2)^2 = \pi \times 27 \times (1.5\, km)^2 = 190\,/$$

This is effective area ~~[illegible]~~ of a BH for capturing radiation; i.e. blocking light from background stars

Photons escaping from a Schwarzschild BH region outward motion:



Outgoing:

(I) Emitted at $r \geq 3GM/c^2$

(a) If $V_r > 0$ (moving in positive radial direction) all photons escape

(b) If $V_r < 0$ (moving in negative radial direction) Only photons with $1/b^2 < V_{max} = \frac{1}{27} \times \frac{1}{(GM/c^2)^2}$ escape. Or only photons with

$$\boxed{b > 3\sqrt{3}\, (GM/c^2) \text{ escape}}$$

(c) photons with $V_r < 0$ and $b < 3\sqrt{3}\,(GM/c^2)$ are captured.

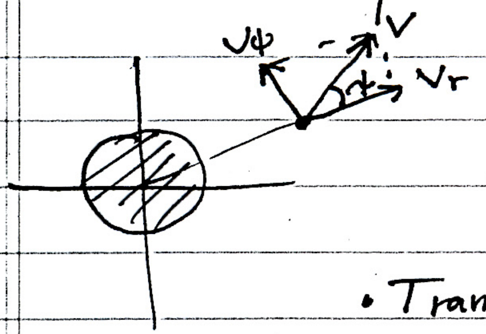
II Emitted at $2GM/c^2 \leq r \leq 3GM/c^2$

(a) If $v_r > 0$: only photons with $1/b^2 > V_{max}$ or $1/b^2 > \frac{1}{27} \cdot \frac{1}{(GM/c^2)^2}$ escape.

Thus only photons with $\boxed{b < 3\sqrt{3}\,(GM/c^2) \text{ escape}}$

(b) If $v_r > 0$ and $1/b^2 < V_{max}$, photons are trapped. Thus ~~only~~ photons with $\boxed{b > 3\sqrt{3}\,(GM/c^2) \text{ are trapped}}$

Compute photon emission from gas near a BH.
What are propagation directions from which photon emitted at r escapes to $r \to \infty$?



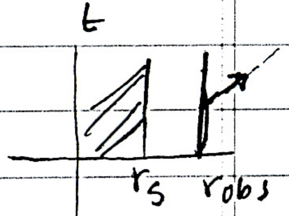
- ψ angle between radial and propagation directions.

- Transverse velocity: $v_\phi = c \sin(\psi)$
- Radial velocity: $v_r = c \cdot \cos(\psi)$

Compute v_ϕ as seen by observer at $r = \text{const}$ (clearly not freely-falling). In this case

$$v_\phi = r \frac{d\phi}{d\tau}$$

(Note $d\tau$ since dt only relevant for observer at $r \to \infty$)



$$v_\phi = r \frac{\frac{d\phi}{dt}}{\frac{d\tau}{dt}} = \frac{r \frac{d\phi}{d\lambda} / dt/d\lambda}{\frac{d\tau}{dt}}$$

- $d\phi / d\lambda = \frac{1}{r^2}$

- $dt / d\lambda = \frac{1}{b(1 - \frac{r_s}{r})}$

- Since $ds^2 = -(1 - \frac{r_s}{r}) c^2 dt^2$ (for $dr = d\theta = d\phi = 0$) and $ds^2 = -c^2 d\tau^2 \Rightarrow d\tau = (1 - \frac{r_s}{r})^{1/2} dt$

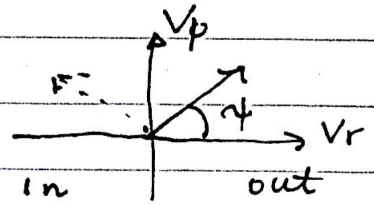
Thus $$v_\phi = \frac{r \times \frac{1}{r^2}}{\frac{1}{b(1 - \frac{r_s}{r})} \times (1 - \frac{r_s}{r})^{1/2}} = \frac{b}{r} \left(1 - \frac{r_s}{r}\right)^{1/2}$$

In physical units

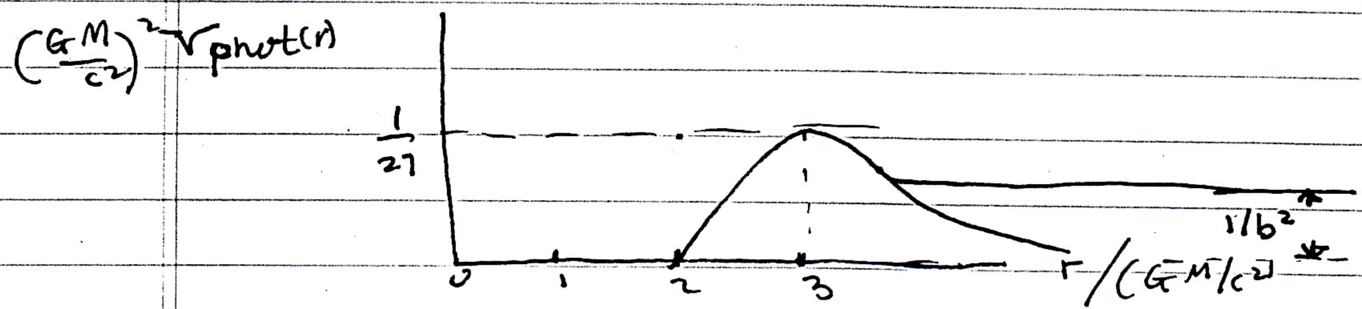
$$\boxed{v_\phi = c \cdot \frac{b}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}}$$

Therefore $\quad \cancel{c} \cdot \sin(\psi) = \cancel{c} \cdot \frac{b}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$

Since $v_r = c \cdot \cos(\psi)$, inward photons with $v_r < 0$ have $\psi > 90°$.



(I) Let $r \gtrsim 3GM/c^2$

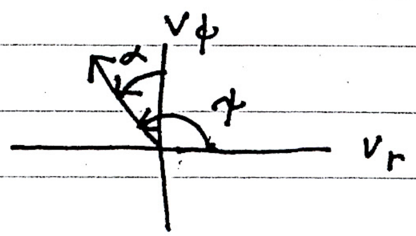


If $v_r < 0$, escape possible only if

$$\frac{1}{b^2} < \frac{1}{27}\left(\frac{1}{(GM/c^2)^2}\right)$$

or only if $b > 3\sqrt{3}\,(GM/c^2)$

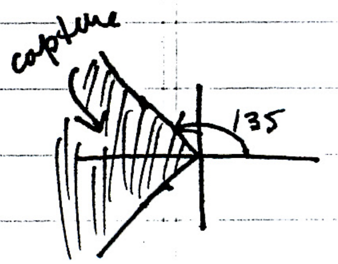
or if $\sin(\psi) > \frac{3\sqrt{3}\,(GM/c^2)}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$



Let $\psi = \frac{\pi}{2} + \alpha$ $\quad \therefore \sin(\psi) = \cos(\alpha)$

Above condition becomes: $\cos(\alpha) > \frac{3\sqrt{3}\,GM/c^2}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$

- $\boxed{\text{Suppose } r = 6 \cdot GM/c^2}$



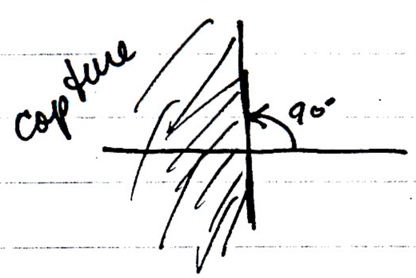
Then $\cos(\alpha) > \frac{3\sqrt{3}}{6}\left(1 - \frac{2}{6}\right)^{1/2} = \frac{\sqrt{3}}{2}\sqrt{\frac{2}{3}} = \frac{1}{\sqrt{2}}$

$\cos(\alpha) > \frac{1}{\sqrt{2}}$ when $\alpha < 45^\circ$ or $\psi < 135^\circ$

- $\boxed{\text{Suppose } r = 3\ GM/c^2}$

$$\cos\alpha > \frac{3\sqrt{3}}{3}\left(1 - \frac{2}{3}\right)^{1/2} = \sqrt{3}\left(\frac{1}{3}\right)^{1/2} = 1$$

$\alpha < 0^\circ$ or $\psi < 90^\circ$



II: what about $r < 3GM/c^2$

- No photons can escape if $v_r < 0$
- If $v_r > 0$, recall that only photons with $1/b^2 > (1/27)(GM/c^2)^{-2}$ will escape implying $b < 3\sqrt{3}(GM/c^2)$.

As a result

$$\sin(\psi) < \frac{3\sqrt{3}(GM/c^2)}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$$

- Suppose photon is emitted infinitesmaly close to Schwarzschild radius; i.e.,

$$r = \frac{2GM}{c^2}(1+\epsilon), \text{ where } \epsilon \ll 1$$

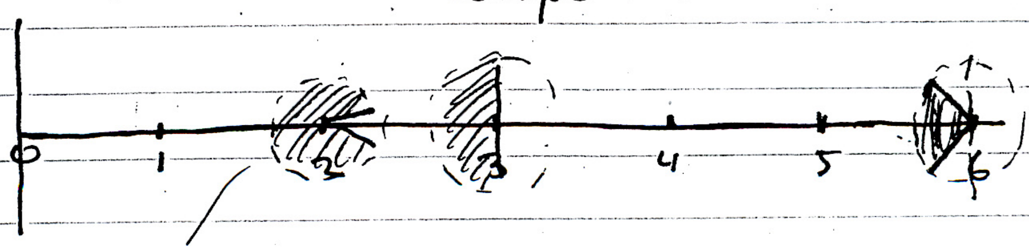
In that case: $\sin(\psi) < \frac{3\sqrt{3}}{2(1+\epsilon)} \times \left(1 - \frac{1}{1+\epsilon}\right)^{1/2}$

or $\sin(\psi) < \frac{3\sqrt{3}}{2}\epsilon^{1/2}$

Therefore

$$\lim_{\epsilon \to 0} \sin(\psi) \to 0$$

Escape Cone



Escape cone closes up as $r \to r_s = 2GM/c^2$

Dark area shows capture directions

Lecture #12:

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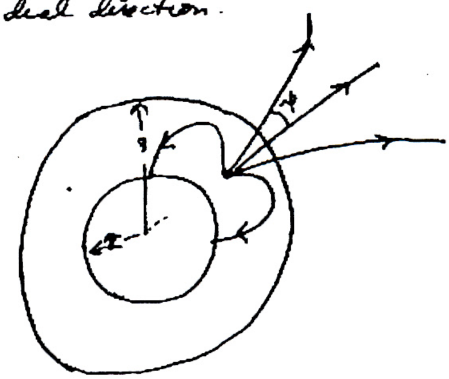
BH96

Last time we found that angle between ray and radial direction must be small for escape

$$\sin\psi = \frac{b}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$$

$$\Rightarrow \sin\psi < \frac{3\sqrt{3}\,(GM/c^2)}{r}\left(1 - \frac{2GM/c^2}{r}\right)^{1/2}$$

Recall since $b \propto l$, this means light needs to have sufficiently small angular momentum to ~~so~~ escape, it moves mainly in radial direction.



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Light Bending and gravitational lenses

Let's derive formula for bending of light in grav. field of a spherical mass M. Its detection was famous for verifying GR. It is also basis for gravitational lensing, which has become a major observational tool in modern astrophysics.

Previously we found

$$\left(\frac{dr}{d\lambda}\right) = \pm\sqrt{\frac{1}{b^2} - V_{phot}}$$

$$\boxed{\left(\frac{dr}{d\lambda}\right) = \pm\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{r_s}{r}\right)}}$$

$$\boxed{\left(\frac{d\phi}{d\lambda}\right) = \frac{1}{r^2}}$$

Get rid of λ: Divide 2^{nd} equation by 1^{st} equation

$$\frac{d\phi}{dr} = \frac{d\phi/d\lambda}{dr/d\lambda} = \frac{1/r^2}{\pm\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{r_s}{r}\right)}}$$

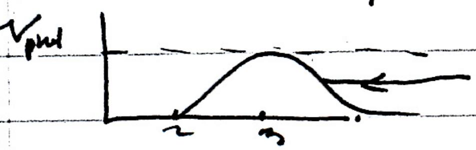
$$\frac{d\phi}{dr} = \frac{\pm 1}{r^2\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{r_s}{r}\right)}}$$

This is equation for angular deviation of light ray, ϕ, as a function of r.

this equation tells us how ϕ changes as photon approaches and then recedes from spherical mass, M.

Consider ~~[illegible]~~ geodesics with $1/b^2 < V_{max}$

or $b > 3\sqrt{3}\, GM/c^2$



To integrate

① Introduce new independent variable

- $u = 1/r$.
- $r = 1/u \Rightarrow dr/du = -1/u^2$

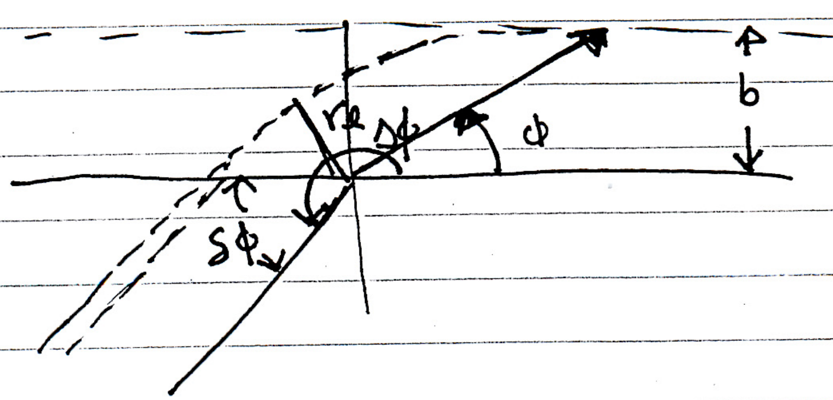
② In that case

$$\frac{d\phi}{du} = \frac{d\phi}{dr} \times \frac{dr}{du} = \frac{d\phi}{dr} \times \left(-\frac{1}{u^2}\right)$$

$$\therefore \quad \frac{d\phi}{du} = \pm \frac{1}{\sqrt{\frac{1}{b^2} - u^2(1 - r_s u)}} \times u^2 \times \left(-\frac{1}{u^2}\right)$$

$$\boxed{\frac{d\phi}{du} = \mp \left(\frac{1}{b^2} - u^2 + r_s u^3\right)^{-1/2}}$$

③ Geometry



$\Delta\phi$ is change in ϕ going from $\phi = 0$
r_e is distance of closest approach
$\delta\phi_{def}$ is deflection angle.

From symmetry ($r = \infty \rightarrow r = r_e$)

$$\frac{\Delta\phi}{2} = \int_0^{u_e} \left(\frac{1}{b^2} - u^2 + r_s u^3\right)^{-1/2} du$$

Need to do numerically to get exact answer.
But in most cases $b > R_{star} \gg r_s$ or
$r \gg r_s$ or $r_s/r \ll 1$

Compare last 2 terms in integrand

$$\frac{r_s u^3}{u^2} = r_s u = \frac{r_s}{r} \ll 1$$

Let $y^2 = u^2 - r_s u^3$ or $y = \sqrt{u^2 - r_s u^3}$

$$\therefore \quad y = u\sqrt{1 - r_s u} \simeq u\left(1 - \frac{r_s u}{2}\right)$$

Since $y = u - \frac{r_s u^2}{2} \Rightarrow \frac{dy}{du} = 1 - r_s u$

$$\text{or} \quad \frac{du}{dy} = (1 - r_s u)^{-1}$$

$$\therefore \frac{du}{dy} \simeq (1 - r_s u)^{-1} \simeq 1 + r_s u = 1 + r_s y$$

Thus

$$\frac{\Delta\phi}{2} = \int_0^{u_e} \left(\frac{1}{b^2} - y^2\right)^{-1/2} \frac{du}{dy} dy = \int_0^{u_e} \frac{(1 + r_s y)\, dy}{\left(\frac{1}{b^2} - y^2\right)^{1/2}}$$

Recall u_e is $1/r_e$ where r_e is turning point. But this is just where $\frac{d\phi}{du} = 0$

or where denominator vanishes; i.e., where $y = 1/b$ or $u_e = 1/b$

$$\therefore \frac{\Delta\phi}{2} = \int_0^{1/b} \frac{(1 + r_s y)\, dy}{\left(\frac{1}{b^2} - y^2\right)^{1/2}}$$

Look up:

$$\frac{\Delta\phi}{2} = \left[\arcsin(by) - r_s \sqrt{\frac{1}{b^2} - y^2} \right]_0^{1/b}$$

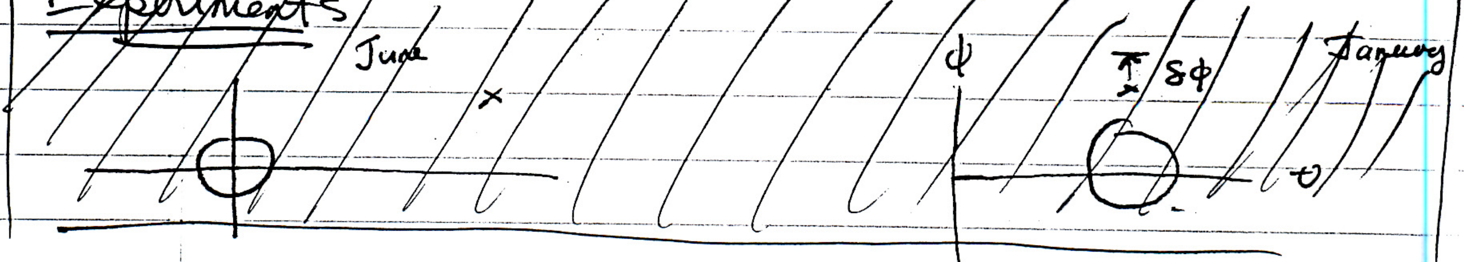
$$\frac{\Delta\phi}{2} = \left[\frac{\pi}{2} - 0 - 0 + \frac{r_s}{b} \right.$$

$$\boxed{\Delta\phi = \pi + \frac{2 r_s}{b}}$$

But from figure: $\Delta\phi - \delta\phi_{def} = \pi$

$$\text{or } \boxed{\delta\phi_{defl} = \frac{2 r_s}{b} = \frac{4 GM}{bc^2}}$$

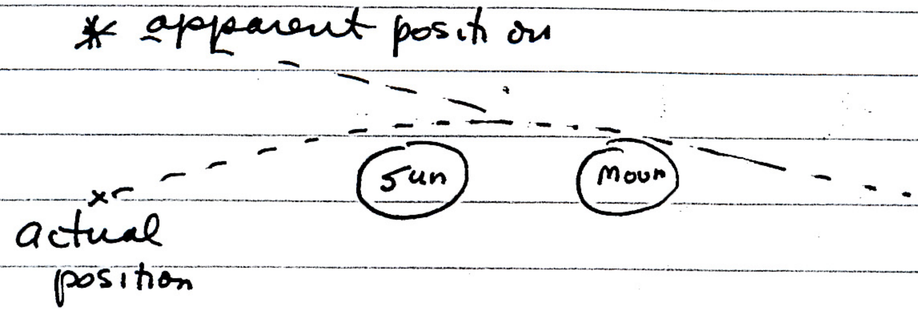
Experiments



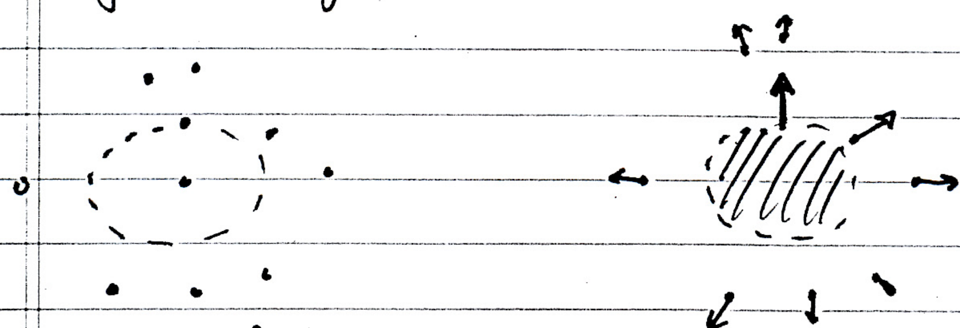
Evaluate: $b = R_{\odot} = 7\times10^{10}\,\text{cm} = 7\times10^{5}\,\text{km}$

$r_s = 3\,\text{km}$

$$\therefore\ \delta\phi_{de} = \frac{2\times3}{7\times10^5} = 8.5\times10^{-6}\ \text{rad or } 1.''75$$



- Optical Experiments: First carr.. by Eddington in 1919



No sun in field

Eclipsing sun (by moon)

Outward radial displacements increasingly by many stars: Note displacement decreases with).

Difficult measurement since fluctuations in earth's atmosphere blur the image of a star by $\Delta\Theta_{se} \simeq 1''$, which is on the order of $\delta\phi_{defl}$. So many stars must be used to improve accuracy of measurement. Today we know that Eddington's earlier claims are questionable. Most recent optical results give $\boxed{\delta\phi_{defl}}$ GR