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Endpoints of Stellar Evolution
Lecture 7. BH 53

Descriptive Stuff: Let's take a break from heavy-duty mathematics

- **Origin of Black holes**

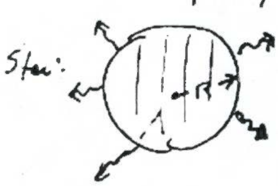
Very soon we will be speaking about orbits around Schwarzschild metric, and we will be considering black holes; i.e., objects in which mass is concentrated within $r = 2GM/c^2$, Schwarzschild radius. Where do they come from??

- **Endpoints of Stellar evolution** (Reach 15.3, 15.4,

Stellar Physics flow chart:

$\left\{ \frac{\partial P}{\partial r} < 0 \right.$ holds up star$\}$

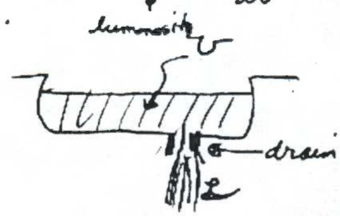
Star:



Internal Energy: $U = M \langle \frac{3}{2} \frac{kT}{m_p} \rangle$

Radiation losses: $\mathcal{L}_{rad} = -\frac{dE}{dt}$

Bathtub analogy:



(Rate of change of Energy)

$$\frac{dE}{dt} = -\mathcal{L}_{rad} < 0$$

Integrate: $\Delta E = -\mathcal{L}\Delta t$: Change in total energy in Δt

But $(\Delta E)_{max} = U$

So $\Delta t = \frac{U}{\mathcal{L}_{rad}}$ (Kelvin-Helmholtz Lifetime)

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Since $M_\odot = 2\times10^{33}$g, $T \simeq 2\times10^7$K

$$U \simeq \frac{2\times10^{33}\times1.5\times1.4\times10^{-16}\times2\times10^7}{1.67\times10^{-24}} \simeq 3.5\times10^{48}\ \text{ergs}$$

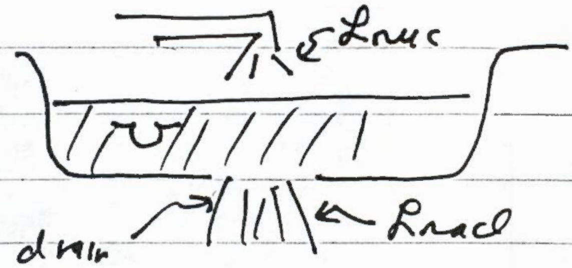
Recap: Energy Balance in the Sun

- **Recall**: Kelvin-Helmholtz time scale

$$t_{KH} \simeq \frac{U_{thermal}}{\mathcal{L}_{rad}} \simeq 3\times10^7 \text{ year}$$

- **Paradox**: But age of sun is much longer. Best bet $t_\odot \simeq 4.6\times10^9$ yr.

- **Solution**: Energy must be supplied at same rate as loss rate $\mathcal{L}_{rad}$: Nuclear Faucet analogy:



Net energy change: $\frac{dE}{dt} = \mathcal{L}_{nuc} - \mathcal{L}_{rad} \simeq 0$

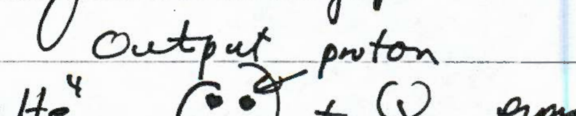
Nuclear Fusion

Bethe & others showed that core of sun hot enough ($T\sim10^7$K) and sufficiently dense ($\rho\simeq 50\,g\,cm^{-3}$) for fusion reactions to supply required energy:

nucleons → A ; Z chg. ; $\leftarrow$ element: ${}_Z^A X$

Input: ${}_1H^1$, ${}_1H^1$, ${}_1H^1$, ${}_1H^1$ $\Longrightarrow$ Output: ${}_2He^4$ + Q energy released

Hydrogen nuclei (protons) ; proton ; neutron



Energy conservation

$$4\gamma m_p c^2 = (\gamma m_{He} c^2) + Q \;:\; \text{But } \gamma \simeq 1 \;(K \ll mc^2)$$

$$Q = 4(m_p c^2) - (m_{He} c^2) = 26\,\text{MeV} \;:\; \epsilon_p = \frac{Q}{4m_p} \simeq 6\times10^{18}\,\frac{erg}{g}$$

Conversion of mass to energy

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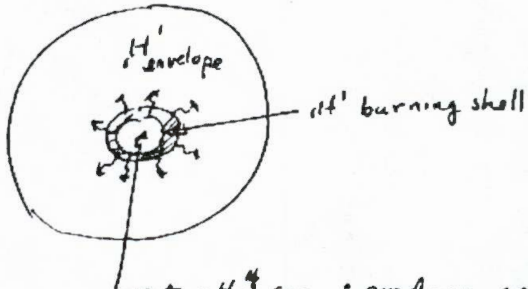
Chemical changes in the core of sun

$_2He^4$ core moves outward with time. At $t \simeq 10^{10}$y, about 10% of total mass of sun will be in $_2He^4$ core. Energy structure looks like this



inert $_2He^4$ core : produces no energy & is isothermal

Coulomb barrier $\propto Z^2$ is 4 times higher for $_2He^4$ reactions: no way at $\sim 10^7$K.

<u>Core contraction & envelope expansion</u>:

When $M_{core}(_2He^4) \geq 0.1 \times M_{TOTAL}$

- core contracts under its heavier weight.
- Excess energy released cannot be radiated away. Rather excess energy does work in pushing out layers of star outward $\Rightarrow$ red giant.

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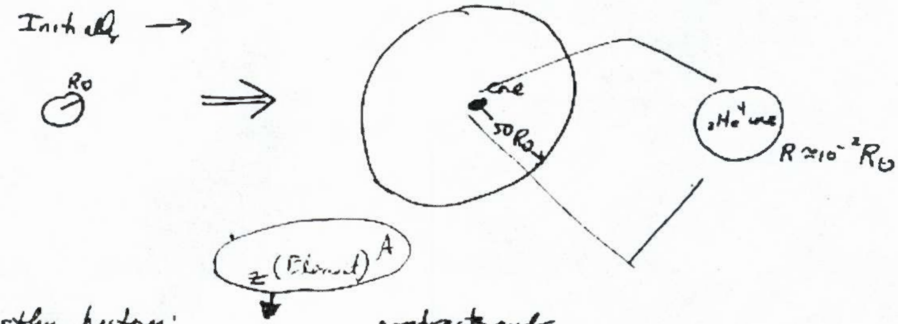

Further history:
Eventually: $_2He^4$ core contracts and heats up and $3\,{}_2He^4 \to {}_6C^{12} + Q$ (If star massive enough) leaving behind very dense $_6C^{12}$ core. Eventually Envelope is ejected and these $_6C^{12}$ or even $_8O^{16}$, $_2He^4$ cores are left over.

Cores & Chandrasekhar limiting mass

These hot cores are white dwarf stars. These are physically distinct from ordinary "main sequence" stars like the sun. They are

- dense $$\rho \simeq \frac{M}{4\pi R_\odot^3/3} \simeq 10^6\ g\,cm^{-3} \simeq 10^6 \rho_\odot$$
- High gravity $$g = \frac{GM}{R^2} \approx 10^4 g_{sun}$$
- Energy Source: Not powered by fusion reactions. Rather T in nuclei is radiated

Degeneracy Pressure

what holds up a white dwarf? Not ordinary pressure: Temperature required:

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$T \simeq 10^9 K$ is not available.

Instead there is a different source of pressure whose source lies at the root of quantum mechanics: the uncertainty principle and Pauli Exclusion principle.

- Pauli Exclusion principle (Fermions; i.e. spin ½ particles) No 2 electrons can have same quantum mechanical state
- Heisenberg Uncertainty principle

  Cannot determine the position and momentum of a particle to an accuracy better than Planck's constant; i.e., $\Delta x \cdot \Delta p_x \geq h$
- Dense gas: (Chandrasekhar)

  The last 2 laws ⇒ that even a gas at $T \simeq 0 K$ has pressure. Why? Because when mean separation between 2 particles is very small, their momenta must differ by more than $\Delta p_x \geq \frac{h}{\Delta x}$. So as Δx gets smaller Δp_x gets larger. The consequent electron motions get very large, and give rise to a "degeneracy" pressure much larger than pressure supplied by thermal motions. And as Chandrasekhar showed the electron degeneracy pressure is sufficient to balance strong gravitational force of a white dwarf.

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Chandrasekhar Mass

- Non-relativistic Pressure

(1) Pressure is force per unit area exerted by gas on real or imaginary wall

(2) Thus it equals: momentum per unit area per unit time transferred to the wall. In time Δt no. of particles incident on wall is given by

$$\Delta N = n \cdot V = n \cdot (\Delta A \cdot v_x \Delta t)$$

So if each particle has momentum p_x, then total increase in momentum is:

momentum, not pressure

$$\Delta P_x = \Delta N \cdot p_x = n \cdot \Delta A \cdot v_x \Delta t \cdot p_x$$

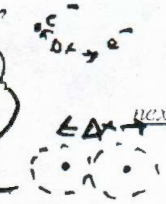
But $\boxed{P_e = \frac{\Delta P_x}{\Delta A \Delta t} = n v_x p_x}$ ← momentum flux

For degenerate e^- gas $P_e = n_e \left(\frac{p_x}{m_e}\right) p_x = \frac{n_e}{m_e} p_x^2$

since non-relativistic $v_x = p_x/m_e$

But momentum $p \simeq \hbar/\Delta x = \hbar n_e^{1/3}$

$$n_e \simeq \frac{1}{\frac{4\pi}{3}\left(\frac{\Delta x}{2}\right)^3} \simeq \frac{2}{\Delta x^3} \Rightarrow \Delta x \sim \frac{1}{n_e^{1/3}}$$



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as a result: $P_e \sim \frac{n_e}{m_e} \hbar^2 n_e^{2/3}$.

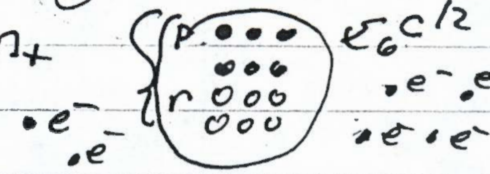
$$P_e \sim \frac{\hbar^2}{m_e} n_e^{5/3}$$

Precise result: $P_e = 0.05 \frac{\hbar^2}{m_e} n_e^{5/3}$

- Compute n_e as function of total mass density

charge Neutrality: Electron negative charge = total positive charge of protons in nuclei.

$n_e = Z n_+$



e.g. ${}_6C^{12}$ nucleus

$\{ m_n \gtrsim m_p \}$

$n_e = \underset{\substack{\uparrow \\ \text{chg. per nucleus}}}{Z}\, n_+ \leftarrow$ density of nuclei

Mass Density $\quad \rho = (A m_p) n_+ + m_e n_e \approx A m_p n_+$

therefore $\quad \boxed{n_e = Z\left(\frac{\rho}{A m_p}\right) = \left(\frac{Z}{A}\frac{\rho}{m_p}\right)}$

Therefore $\quad \boxed{P_e = 0.05 \frac{\hbar^2}{m_e}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{m_p}\right)^{5/3}}$

Since white dwarfs composed of ${}_2He^4$, ${}_6C^{12}$, ${}_8O^{16}$,
$Z/A \simeq 1/2$

"Gravitational" Pressure:

P_e required to balance weight per unit area of overlying matter. Let's call the latter P_c.

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$\therefore P_c \simeq \Sigma' \cdot g$ $\left\{ \begin{array}{l} \Sigma = \text{mass per unit area} \\ g = \text{grav. acceleration} \end{array} \right.$ $\boxed{D\pi}$

Since $\Sigma \sim M/R^2$, $g \simeq \frac{GM}{R^2}$

we have $P_c \sim \frac{GM^2}{R^4}$

More precise calculations give $P_c = 0.77 \frac{GM^2}{R^4}$

Mass-Radius Relationship

So hydrostatic equilibrium satisfied if $P_c = P_e$

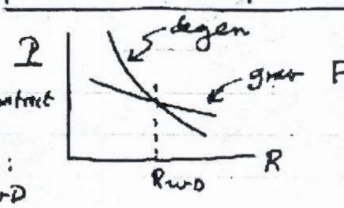
$$\frac{0.77\, GM^2}{R^4} = 0.05 \frac{\hbar^2}{m_e} \left(\frac{Z}{A}\right)^{5/3} \frac{\rho_c^{5/3}}{m_p^{5/3}} \quad \Big\{ \text{central density}$$

Models show: $\rho_c = 6\bar{\rho}$; where average density $\bar{\rho} = M/4\pi R^3/3$

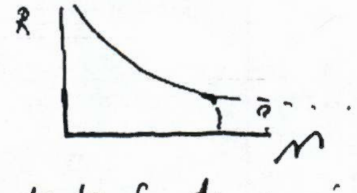
Substituting we get $\boxed{\underbrace{\frac{M^2}{R^4}}_{\text{grav.}} \propto \underbrace{\frac{M^{5/3}}{R^5}}_{\text{degeneracy}}} \Rightarrow R \propto M^{-1/3}$

More precisely $\boxed{R_{WD} = 0.114 \frac{\hbar^2}{G m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} M^{-1/3}}$

What's going on



Stable. $\left\{ \begin{array}{l} \text{(1) If } R > R_{WD};\ P_c > P_e;\ \text{contract back to } R_{WD} \\ \text{(2) If } R < R_{WD};\ P_e > P_c:\ \text{expand up to } R_{WD} \end{array} \right.$



Does relationship $R = R(M)$ extend ~~[illegible]~~ to $M \to \infty$?

or out to finite maximum mass?

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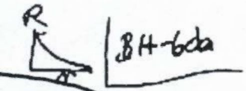
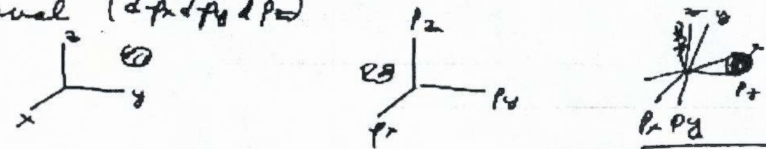


Alternative view of degeneracy:

Phase space density

$= f \underbrace{dp_x dp_y dp_z dx dy dz}_{dV_p}$ — phase space distribution function

Let $dN = f(\underset{\sim}{p}, \underset{\sim}{x})\, d^3p\, d^3x$ be no. of particle's in spatial volume element $(dx\,dy\,dz)$ and momentum "volume" interval $(dp_x dp_y dp_z)$



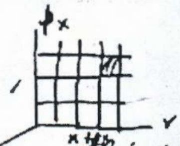
$\boxed{\text{6D phase space}}$

Uncertainty principle:

Says there is a lower bound on the 6D phase-space volume element, dV_p, since uncertainty principle tells me cannot locate particle more accurately than:

$$dp_x dx \geq h;\ dp_y dy \geq h,\ dp_z dz \geq h.$$

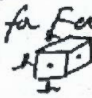
So I divide 6D phase space into "resolution elements" which are boxes with sides: $dp_x dx = h$, $dp_y dy = h$, $dp_z dz = h$



"area" of each box $= h$. Particle somewhere in that box. $(x, x+dx)$, then $dp_x = h/dx$

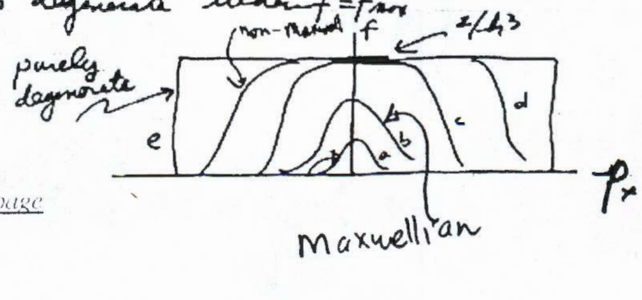
Pauli Exclusion principle: Tells us that in this "minimum" phase-space volume $dV_p = h^3$ I can fit no more than 2 Fermions (opposite spins). Thus the largest phase-space density possible for Fermions:

$$f = \frac{dN}{dV_p} \leq \frac{2}{h^3} = f_{max}$$



$f \propto e^{-E/kT} = e^{-\frac{p^2}{2mkT}}$

Gas degenerate when $f = f_{max}$



(a) Maxwellian

add particles: unfilled states at higher p_x

(e) degenerate distribution

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(BH-66)

Why are e^- degenerate, but ions non-degenerate in white dwarfs?

Non-degenerate

In thermal equilibrium ions and e^- equilibrate and have same energy. Both species follow Maxwellian velocity (momentum) distribution characterized by common temperature, T.

Since energy $E = p^2/2m \Rightarrow p = \sqrt{2mE}$

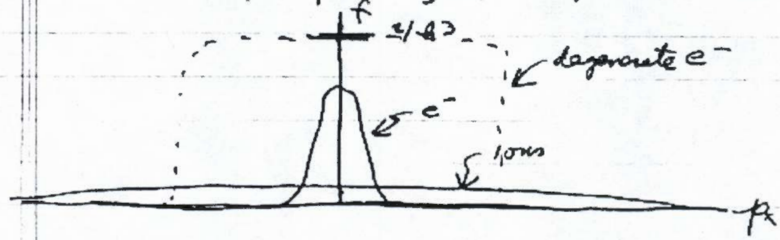
then $\frac{\langle p_I^2\rangle^{1/2}}{\langle p_e^2\rangle^{1/2}} = \sqrt{\frac{m_{Ion}}{m_e}} \simeq \sqrt{A}\sqrt{\frac{m_p}{m_e}} = \sqrt{A}\cdot 43$

≈ 150 for ${}_6C^{12}$ WD $(A=12)$

Phase space volume: $\Delta V_p = V \cdot p^3 \propto m^{3/2}$ (fixed T)

$$\therefore \frac{\Delta V_p(\text{ion})}{\Delta V_p(e^-)} = \left(\frac{A m_p}{m_e}\right)^{3/2} = 3.3\times10^6$$

That means we can place $\sim 3\times10^6$ times more ions in same volume before $f_{ion} \to 2/h^3$



So ions are still an ideal non-degen. gas at WD densities. But even they will become degenerate at sufficiently high densities (few $\times10^6$ higher) 94

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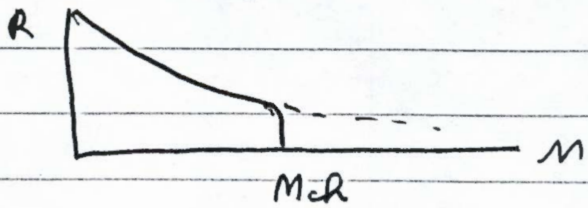
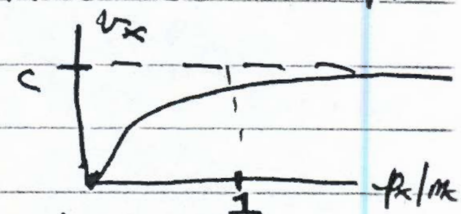
Back to electrons: Analysis so far indicates $R(M) \propto M^{-1/3}$. This indicates $R \to 0$ as $M \to \infty$. But $R \to 0$ at a finite mass, M_{ch}:



Physical reason: Recall, pressure $P = n v_x p_x$.
Since $R \propto M^{-1/3}$, the density, $\rho \propto M/R^3$, goes like: $\boxed{\rho \propto M^2}$. But $p_x \sim \hbar n^{1/3}$ implies $p_x \propto M^{2/3}$. So as $M \uparrow$ so does $p_x \uparrow$.

$v_x \to c$ as M increases

But recall, that v_x does not increase with p_x as $p_x \gg mc$. In that case v_x approaches c asymptotically.



This limits increase of P with ρ when M increases } since $P = n v_x p_x$! R { while p_x increases without limit, flux $n v_x \simeq nc$

In this relativistic limit: $P_e^{rel} = n v_x p_x = n c p_x$

$$P_e^{rel} = n c \hbar n^{1/3} = c \hbar n^{4/3}$$

(Recall in non-relativistic limit $P_e = \frac{n p_x^2}{m_e} = \frac{n \hbar^2 n^{2/3}}{m_e} = \frac{\hbar^2}{m_e} n^{5/3}$)

Precise relativistic result:

$$P_e = 0.12 \hbar c n_e^{4/3} \quad \text{or}$$

$$\boxed{P_e = 0.12 \hbar c \left(\frac{Z}{A} \frac{\rho}{m_p}\right)^{4/3}}$$

Equate this pressure with weight/area of overlying gas: $P_c \propto M^2/R^4$ $\to$ $P_e = P_c$

$$P_e \to \frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4} \; P_c \quad \text{leads to unique mass}$$

$$\boxed{M_{ch} = 0.2 \left(\frac{Z}{A}\right)^2 \left(\frac{\hbar c}{G m_p^2}\right)^{3/2} m_p}$$

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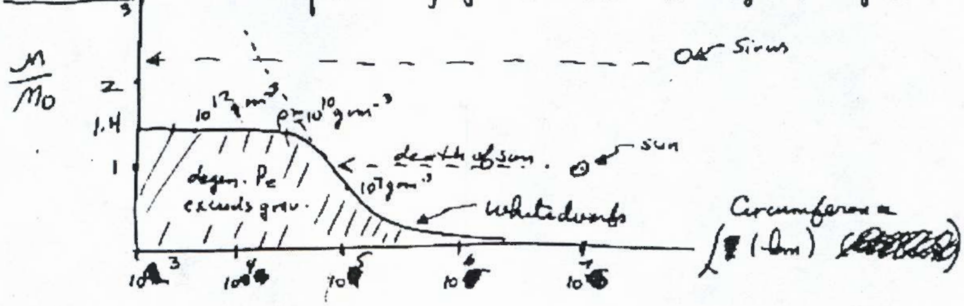
BH-~~GZ~~ GZ

Since $Z/A = 1/2$ in white dwarfs:

$$\boxed{M_{ch} = 1.4 M_\odot}$$

So far, no known white dwarf has $M \geq M_{ch}$

Paradox: Let's plot fig on BH 6⊙ by rotating 90°



Fate of the sun: As it contracts, it eventually settles into white dwarf state

Fate of Sirius: Will implode since it is too massive to become a white dwarf

In fact the idea that stars with $M > 1.4 M_\odot$ would ultimately implode led many ~~astronomy~~ astronomers to reject Chandrasekhar calculation.

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Recap: Endpoints of stellar Evolution

- <u>Main Sequence Phase:</u>
  Most of the lifetime of all stars is spent converting $_1H^1 \to {}_2He^4$. Energy release is sufficient to balance radiative loss rate L_{rad}

- <u>Core-Envelope Structure</u>
  When $_2He^4$ core mass exceeds critical value, $_2He^4$ core contracts & $_1H^1$ envelope expands: envelope becomes Red Giant Star. Eventually envelope is expelled and core ⇒ white dwarf star

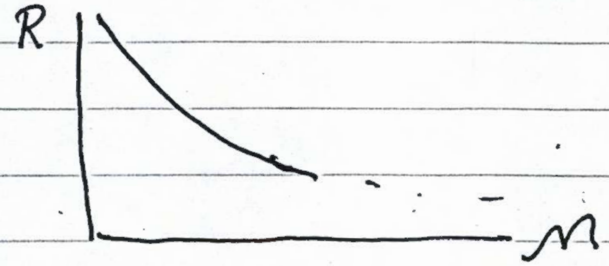
- <u>White Dwarf Stars</u>
  $M_{WD} \sim 1 M_\odot$; $R_{WD} \simeq 10^{-2} R_\odot$
  Held up by degeneracy pressure
  $$P_e = .05\frac{\hbar^2}{m_e}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{m_p}\right)^{5/3} \propto \frac{M^{5/3}}{R^5}$$

This must balance Force per unit area given by $P_c = .77\frac{GM^2}{R^4}$

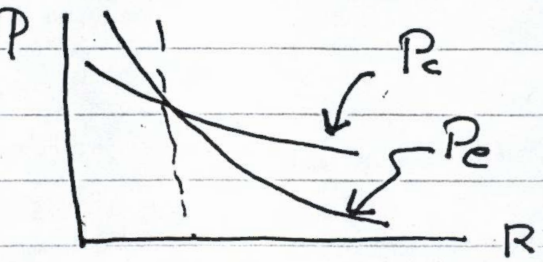
Balance $P_c = P_e$ yields
$$R_{WD} = \frac{0.114\,\hbar^2}{G m_e m_p}\left(\frac{Z}{A}\right)^{5/3} M^{-1/3}$$

:

More massive stars contract to smaller radii



because star can adjust to stable equilibrium



- Chandrasekhar Mass

But above results valid for non-relativistic matter where $v_x = p_x/m_e$. But we saw that

- $\rho \propto M/R^3$ and $R \propto M^{-1/3} \Rightarrow \rho \propto M^2$
  (more mass WD's are more dense)
- Since $p_x \sim \hbar n^{1/3} \Rightarrow p_x \propto (M^2)^{1/3} = M^{2/3}$

Momentum increases arbitrarily thus $v_x \to c$ limit

Since $P_e = n v_x p_x \to n \underset{\uparrow \text{limit}}{c} p_x$

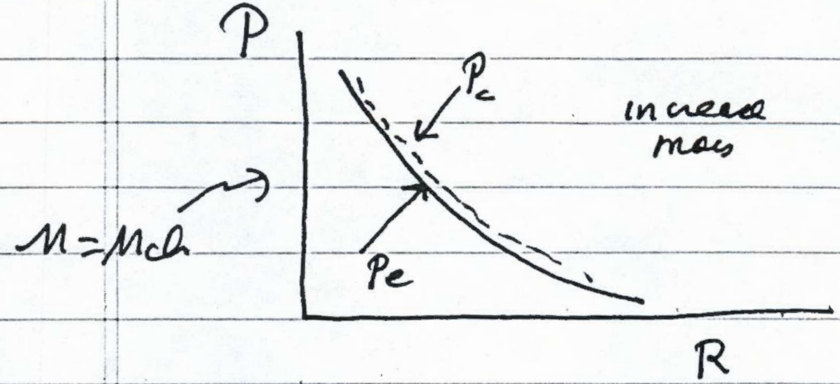
$$P_e = n c \hbar n^{1/3} \propto \rho^{4/3}$$

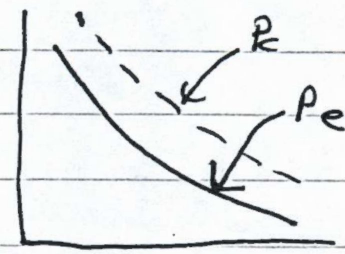
Balance:

$$P_e \propto \frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4}$$

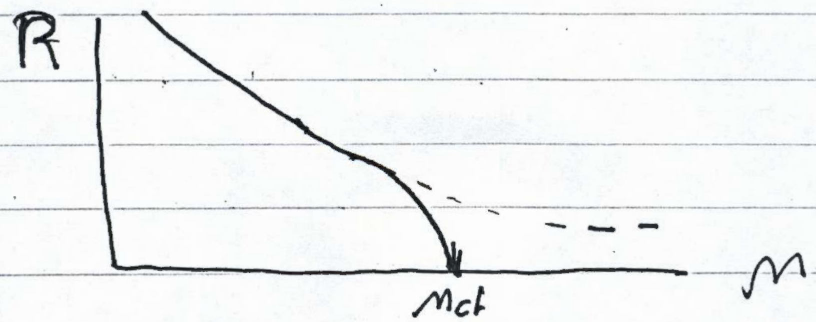
$$\Rightarrow \text{Unique mass } M_{ch} = 0.2\left(\frac{Z}{A}\right)^2\left(\frac{\hbar c}{G m_p^2}\right)^{3/2} m_p = 1.4 M_\odot$$

This is upper limit since equilibrium unstable

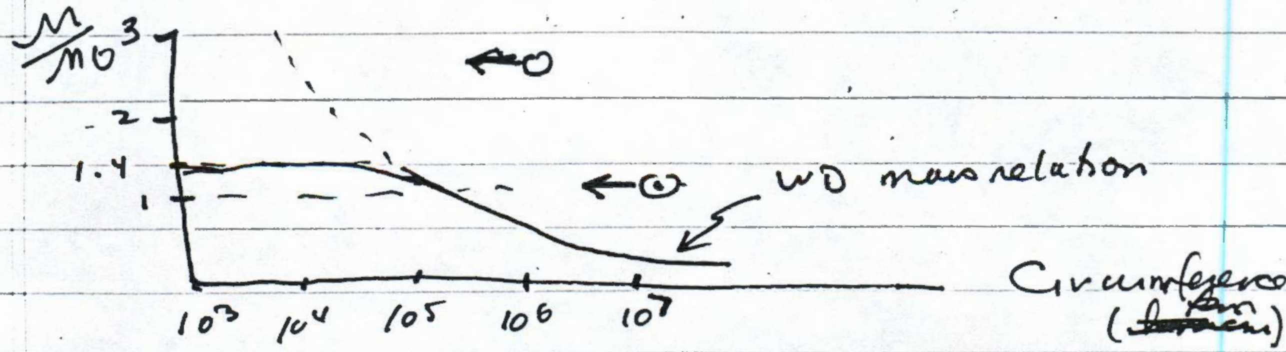




At given R P_c increases moe than P_e as M increases. Stai cannot adjust and collapses



Let's ~~[illegible]~~ rotate this counterclockwise by 90° & flip it



Sun contracts to form WD What about the cores of more massive stars?.

Implode?

Neutron Stars

what happens when $M > M_{ch}$? Star now contracts to densities beyond $\rho_{WD} \simeq 10^6\ g\,cm^{-3}$.
~~[illegible]~~ Would $R \to 0$.

- Electron Degeneracy: Degenerate e^- pressure cannot support star with $M > M_{ch}$
- Neutron Degeneracy: But what about other Fermions (spin 1/2) particles such as n, p? Can they prevent collapse by exerting degenerate pressure.

Massive Stars: Cores reach high central temperatures, higher than WDs, as they evolve.

T: Recall $P_c \sim \frac{GM^2}{R^4}$

Ideal gas Law: $P_{gas} = \frac{R}{\cancel{\mu}} \rho T \simeq \frac{R}{\mu}\left(\frac{M}{R^3}\right) T_c$

Thus hydrostatic balance: $P_c = P_{gas}$

$$\Rightarrow \frac{GM^2}{R^4} \sim \frac{R}{\cancel{\mu}}\left(\frac{M}{R^3}\right) T_c$$

$$\Rightarrow \boxed{T_c \sim \frac{GM}{R \cdot R}}$$

So for these stars, $2He^4$ cores can burn via fusion reaction since kinetic energies sufficiently large to tunnel through

Coulomb barriers. In fact very massive stars can burn $_6C^{12} \to {_8O^{16}} \to {_{14}Si^{28}} \to {_{26}Fe^{56}}$ all the way to $_{26}Fe^{56}$.

Photo-disintegration

Stars cannot burn via fusion reactions to heavier elements (say something later about why this is true). Because of high T ($\sim 10^9 K$), photons are energetic γ-rays that photo-disintegrate complex nuclei. All of nucleosynthesis.
All we have left are free p, n, e^- ($Z=1, A=1$)

Neutronization: All the protons disappear via inverse β decay:

$$e^- + p \to n + \tilde{\nu}_e$$

Energetics:

$$E_{before} = E_{after}$$

$$m_e c^2 + m_p c^2 = m_n c^2 + Q$$

$$\Rightarrow \quad Q = m_e c^2 - (m_n - m_p) c^2$$

$$Q = 0.51 MeV - 1.3 MeV = -0.8 MeV$$

Reaction is endothermic ($Q < 0$) which requires energy input to proceed.

But we made a mistake: while n, p are non-relativistic, electrons are moving at relativistic speeds:

$$\langle \tfrac{1}{2} m v^2 \rangle = \tfrac{3}{2} kT$$

$$\Rightarrow \quad v = \sqrt{\frac{3kT}{m}} \qquad \begin{cases} \text{at } T = 10^9 K \\ v_n \simeq c/43 \\ v_e \sim c \end{cases}$$

So in this case

$$\gamma_e m_e c^2 + m_p c^2 = m_n c^2 + Q$$

$$Q = \gamma m_e c^2 - (m_n - m_p)c^2$$

For exothermic, $Q > 0$, reaction we must have

$$\gamma_e > \frac{m_n - m_p}{m_e} = 2.6$$

Since $\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \Rightarrow \gamma^2 = \frac{1}{1-(v/c)^2} \therefore 1 - \left(\frac{v}{c}\right)^2 = \frac{1}{\gamma^2}$

$$\Rightarrow (v/c)^2 = 1 - \frac{1}{\gamma^2} \Rightarrow \left(\frac{v_e}{c}\right)^2 = \sqrt{1 - \frac{1}{(2.6)^2}}$$

$\frac{v}{c} = .9$ which is comparable to e^- speed at $T \sim 10^9 K$

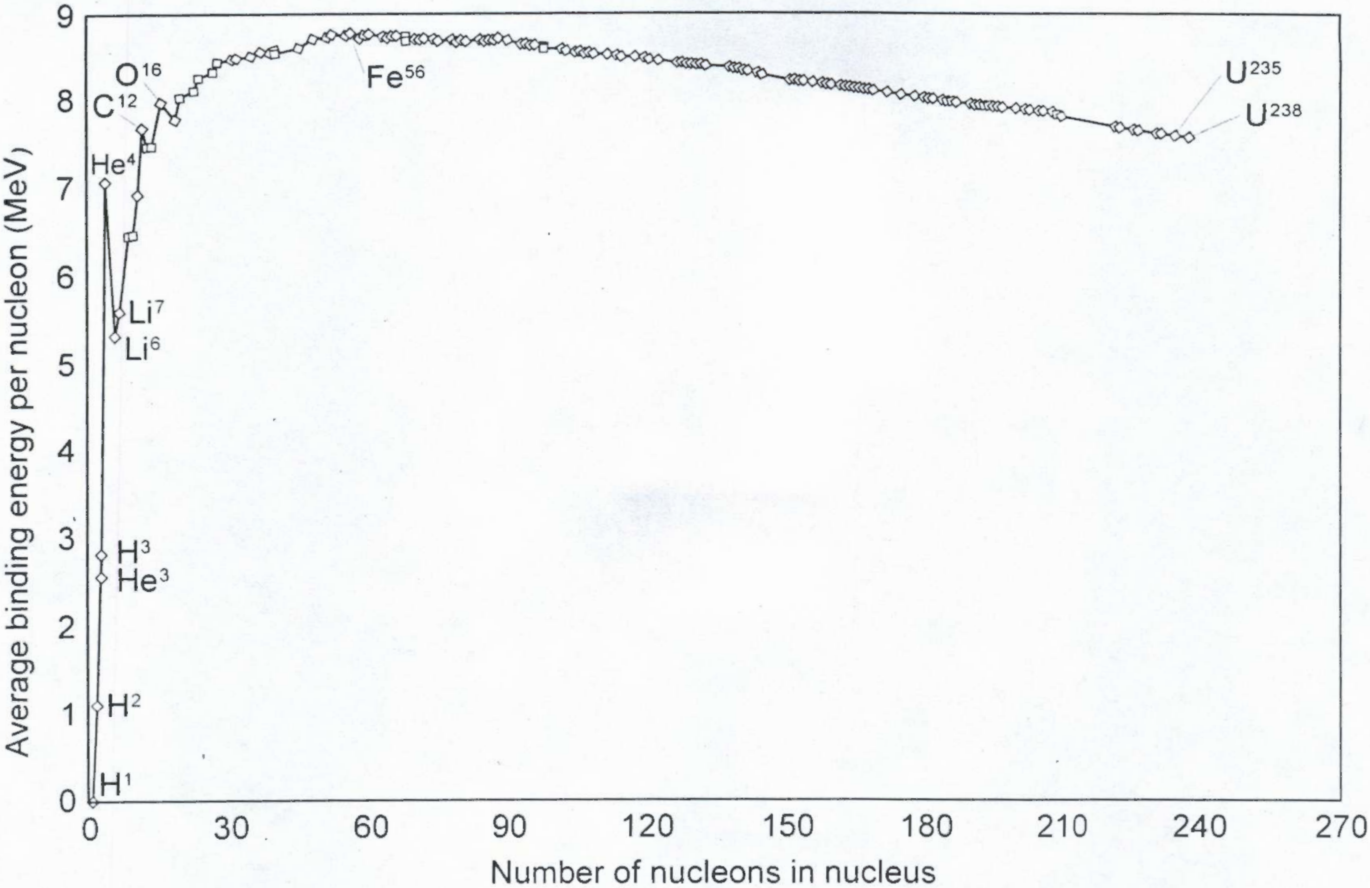
<u>Result</u>: Gravitating sphere of neutrons.

We can use the same physics as before Heisenberg Uncertainty principle, Pauli exclusion principle to find $R = R(M)$ relation for these neutron-star cores

<u>Neutron Star</u>	<u>White Dwarf</u>
$R_{NS} = 0.114 \frac{\hbar^2}{G m_p^{8/3}} M^{-1/3}$	$R_{WD} = 0.114 \frac{\hbar^2}{G m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} M^{-1/3}$

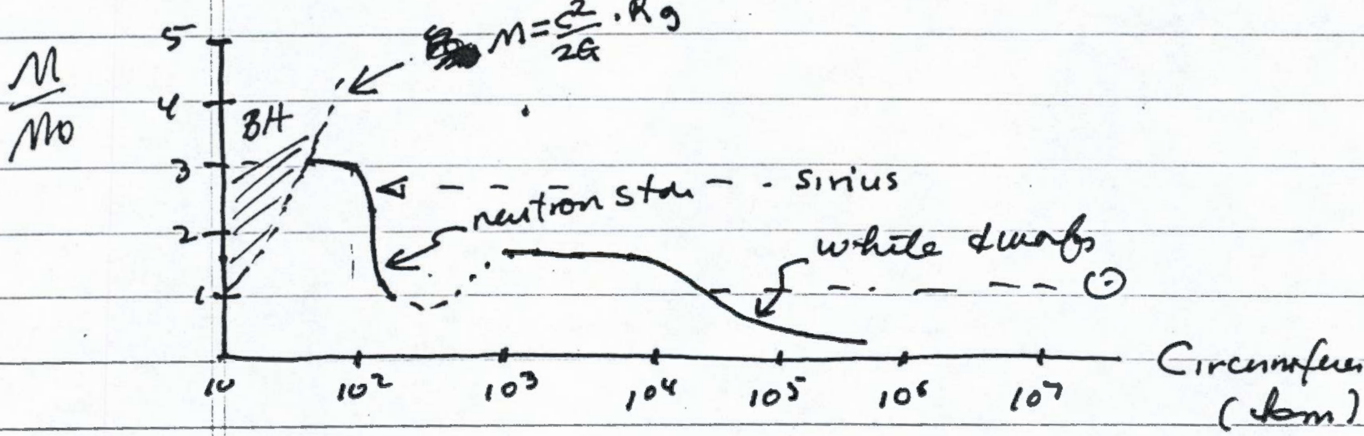
$$\frac{R_{NS}}{R_{WD}} = \left(\frac{Z}{A}\right)^{-5/3} \left(\frac{m_e}{m_p}\right) = 2^{+5/3} (1/1800) \simeq 1.7 \times 10^{-3}$$

$R_{NS} \simeq 15$ km for $M = 1.4 M_\odot$, $\rho_{NS} \geq 10^{14}$ g cm^{-3}



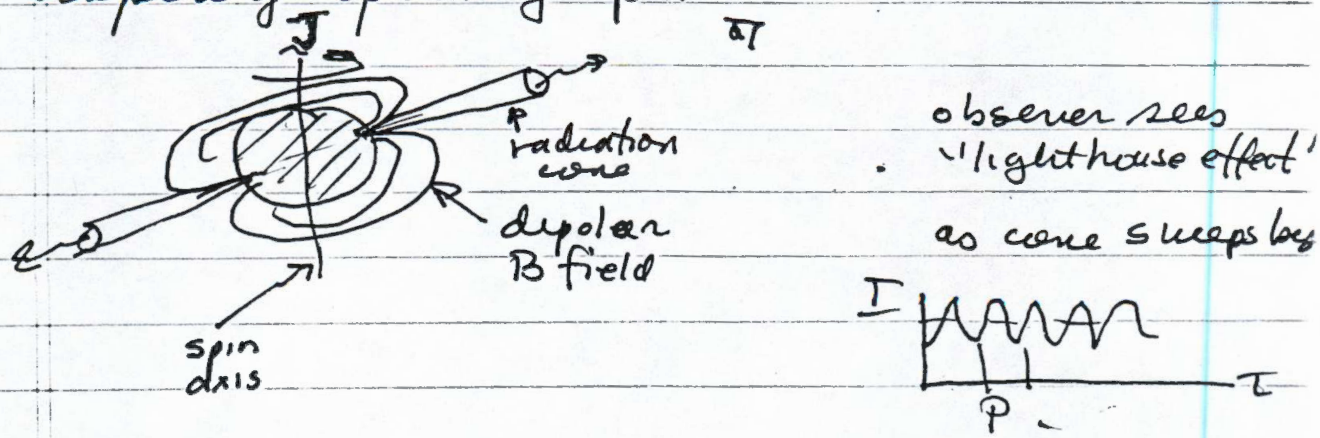
Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270
H1
H2
He3
H3
Li6
Li7
He4
C12
O16
Fe56
U235
U238

<u>Mass Limit</u>: More difficult to compute than for white dwarfs because equation of state involves strong interactions between nuclei. Best bet is that analogue for Chandrasekhar mass for neutron stars is about $\simeq 3 M_\odot$. Neutron stars more massive than this cannot be supported against gravitational collapse by degenerate neutron pressure.



Thus if massive stars leave cores exceeding $3 M_\odot$, core will collapse to black hole

- <u>Data</u>: Neutron stars exist! Observed as rapidly spinning pulsars.



Pulse Periods range from: few$\times 10^{-6}$s $\rightarrow$ 3 sec

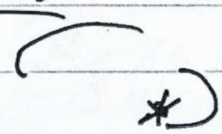
How do neutron stars form?

Evidence is that they are by products of supernova explosions. Spectacular outbursts of energy which signal the end of massive stars. A single star becomes for a few weeks $\rightarrow$ 1 month as luminous as the Milky Way galaxy which contains $\approx 10^{11}$ stars.

Milky Way:

In 1054 Chinese astronomers observed a SN in Crab constellation. Today we see detritus of this explosion: Ionized gas SN remnant. At center of Crab is a pulsar. We can date it by rate at which it slows down: $t_{age} \simeq P/(dP/dt) \simeq 900$ years!

External Galaxies



Physical Mechanism : Fe core collapse

High mass stars can fuse elements heavier than ${}_6C^{12}$, ${}_8O^{16}$, etc. in their cores since they have such high temperatures that QM tunneling through Coulomb barrier is possible. In fact origen of all elements heavier then $_2He^4$ up to $_{26}Fe^{56}$ is in such stars. But not heavier since fusion reactions become endothermic; i.e., require energy (since $Q<0$)

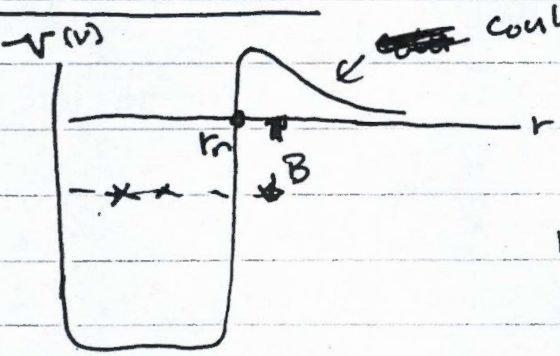
$$m_a c^2 + m_b c^2 \rightarrow m_c c^2 + Q$$

If $Q<0$ we can rewrite

$$|Q| + m_a c^2 + m_b c^2 \rightarrow m_c c^2$$

energy input required to make reaction go.

• $_{26}Fe^{56}$ Peak



Potential energy curve of nucleus

$r_n \simeq 10^{-13}$ cm

Binding energy: Energy required to unbind nucleus

Energy of a nucleus $\overbrace{\;}^{m_a c^2}$ consisting of N nucleons.

$$m_a c^2 = E = N \cdot m_p c^2 - B$$

$m_p c^2$: rest-mass of nucleus; B: Potential energy is <0

Reaction

a	b	$\rightarrow$	c
N nucleons	M nucleons		M+N nucleons

$$m_a c^2 + m_b c^2 = m_c c^2 + Q$$

$$N m_p c^2 - B_a + M m_p c^2 - B_b = (M+N) m_c c^2 - B_c + Q$$

$$\Rightarrow -B_a - B_b = -B_c + Q$$

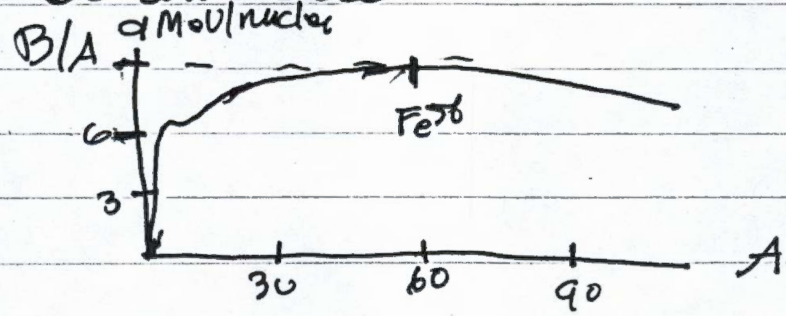
$\therefore \quad Q = B_c - (B_a + B_b)$

Thus for exothermic, $Q > 0$, reaction we must have $B_c > (B_a + B_b)$

Binding energy of product must exceed that of interactants. Rewrite in terms of B/A, binding energy per nucleon

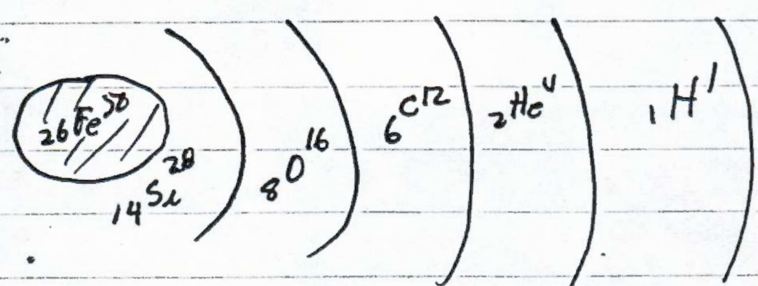
$$\frac{B_c}{A} > \frac{B_a}{2(\frac{A}{2})} + \frac{B_b}{2(\frac{A}{2})} \simeq \frac{1}{2}\left(\frac{B_a}{A_a}+\frac{B_b}{A_b}\right) = \left\langle\frac{B_a}{A}\right\rangle$$

So binding energy per nucleon must increase with increasing A



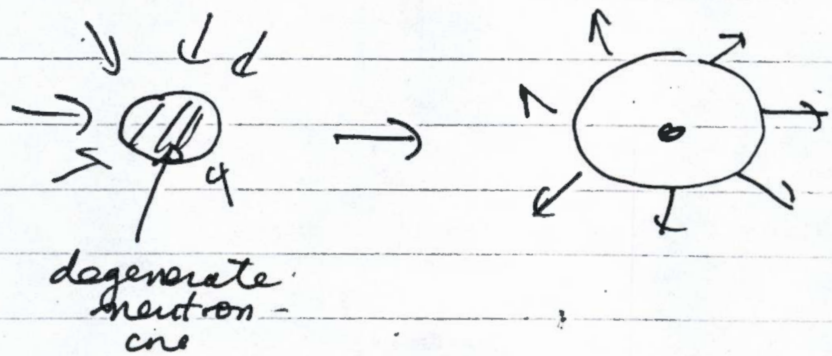
Peak occurs at $_{26}Fe^{56}$. So fusion reactions to heavier elements are endothermic

So Endpoint of massive star looks like this



$_{26}Fe^{56}$ core contracts, but contraction does not raise T, because energy absorbed not released. If $_{26}Fe^{56}$ core Mass is $< 3 M_\odot$, degenerate pressure can halt collapse, but if $M_{core} > 3 M_\odot$ nothing can prevent collapse to black hole.

For $M_{core} \lesssim 3 M_\odot$: layers bounce and rebound off neutron core



Since progenitor stars $M \simeq 10-15 M_\odot$, most of the mass is ejected in the SN explosion leaving less than $3 M_\odot$ in the core.

<u>Summary</u>

(1) <u>Progenitor mass: $0.8 M_\odot < M < 2.3 M_\odot$</u>
result in Degenerate $_2He^4$ cores. ~~Become~~ $_2He^4$ burning occurs in degenerate core resulting in some $_6C^{12}$ production.

(2) <u>Progenitor mass: $2.3 M_\odot < M < 9 M_\odot$</u>
$_2He^4$ burning to $_6C^{12}$ takes place in non-degenerate ~~core~~ $_2He^4$ core

$\rho \propto \frac{M}{R^3}$; But $R \propto M^{0.6}$ in MS stars