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Page #12 of Lecture #2

Lecture #2:

<-previous page next page->

BH-R

End of simultaneity

Conclusion : Events that are simultaneous from point of view of some observers will not be simultaneous according to other observers. This is a profound conclusion, and one of the deepest insights into the nature of reality ever discovered! Unexpected feature of time follows directly from constancy of speed of light

Notice : If light speed behaved the same way that motions of ordinary objects did, then platform observers would agree with those on train.

Photon still has to travel farther to reach backland president than forward land president. But in that case light speed reaching backward land president gets extra-kick due to train motion: $u_L = c+u$

$$\therefore \; u\Delta t_B + L = \underbrace{(c+u)}_{u_L}\Delta t_B \Rightarrow \Delta t_B = \frac{L}{c}$$

Toward Forward land president light ~~would~~ speed would be reduced to $u_L = c-u$

$$u\Delta t_F + (c-u)\Delta t_F = L \Rightarrow \Delta t_F = \frac{L}{c}$$

But in real world : Light does not speed up or slow down. Platform observers can justifiably claim that Forward land pres. did sign peace agreement first.

<-previous page next page->

If images take too long to load, please inform web guy

Page #13 of Lecture #2

<-previous page next page->

BH-13

Clocks: Constancy of c requires that we give up idea that simultaneity is universal. A universal clock ticking off identical seconds on earth, mars, the center of the Galaxy, in M87 giant E galaxy in Virgo cluster does not exist

Rather, observers in relative motion will not agree on which events occur at the same time. Concepts are rare in every day ~~experi~~ experience. Reason:

$$\frac{\Delta t_B}{\Delta t_F} = \frac{c+u}{c-u} = \frac{1+\frac{u}{c}}{1-\frac{u}{c}}$$

(a) Suppose $u = 10$ mi/hr $\Rightarrow \frac{u}{c} = \frac{10}{670\times10^6} \simeq 1.5\times10^{-8}$

$$\therefore \frac{\Delta t_B}{\Delta t_F} \simeq \frac{1+1.5\times10^{-8}}{1-1.5\times10^{-8}} \approx 1+3\times10^{-8}$$

(b) But if $u = 600\times10^6$ mi/hr, $\frac{u}{c} = \frac{6\times10^8}{6.7\times10^8} = 0.9$

$$\frac{\Delta t_B}{\Delta t_F} = \frac{1+0.9}{1-.9} \approx \frac{1.9}{0.1} \simeq 20!$$

Signal takes 20 times longer to reach backward facing pres. than forward facing president.

<-previous page next page->

Lecture #2:

If images take too long to load, please inform web guy

Page #14 of Lecture #2

<-previous page next page->

BH-14.

More on Time

Lot of discussion that involve abstract definitions

More concrete definition that suits me:

- Definition of Time:
  That which is measured by clocks.
- Definition of Clock:
  Device undergoing repetitive cycles of motion.
- Measure of Time
  Count no. of regular cycles a clock goes through, or fraction of cycles a clock goes through. Good example hands on wrist watch
- Periodic Mechanism responsible for cycles of motion

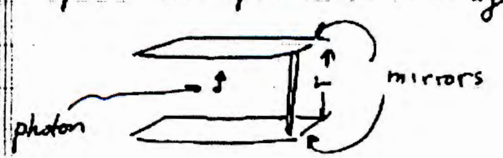
We need something to drive repetitive motions that do not change from cycle to cycle.

- Pendulum
- Atomic clocks based well defined transition ν frequency

- Light clock:

Since we are interested in how motion affects passage of time, we want to know how motion affects "ticking" of clocks, any clock independent of design.

Simplest conceptual clock is light clock.



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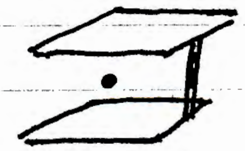
(A) Photon bounces back and forth between mirrors

(B) Ticks : Occur every time photon completes one round trip. $\Delta t_{tick} = \frac{2L}{c}$. If $L = 6''$, $\Delta t_{tick} = \frac{2 \times 6 \times 2.54}{3 \times 10^{10}} = 10^{-9} s$ about a billion ticks per second.

Moving clocks versus stationary clocks

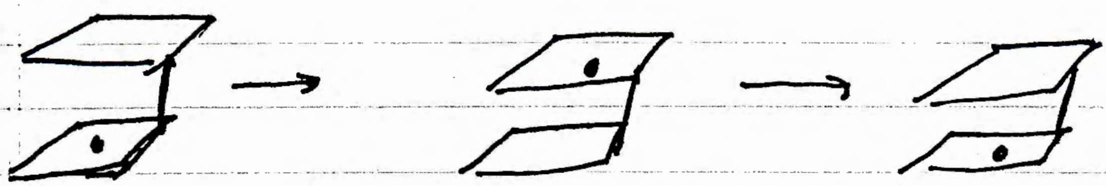
- Observer at rest wrst clock : Θ' observer

Θ'



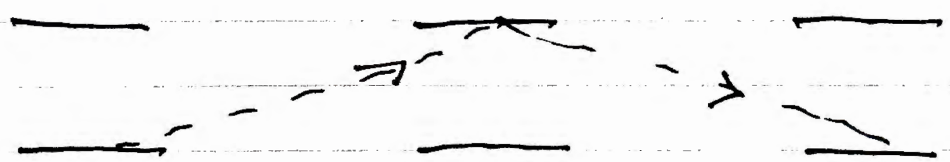
- Observer who sees clock in motion : Θ observer

Θ



Compare whether observers see clocks ticking at same rate. Θ' observer sees vertical motion of photon. But to Θ observer photon motion is diagonal, not vertical

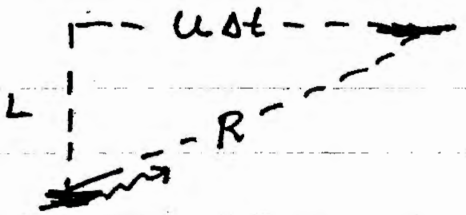
Θ



Implication : Diagonal path seen by Θ is longer than vertical path seen by Θ'. Since SR tells us c is the same in both cases $\Delta t > \Delta t'$

Quantitative :

Θ :



$$R = \sqrt{L^2 + u^2 \Delta t^2}$$

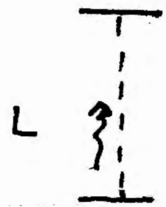
Therefore, for Θ observer, time taken to reach top mirror is given by :

$$\Delta t = \frac{R}{c} = \frac{\sqrt{L^2+u^2\Delta t^2}}{c}$$

$$c^2\Delta t^2 = L^2 + u^2 \cdot \Delta t^2 \quad \Rightarrow \quad \Delta t^2 (c^2 - u^2) = L^2$$

$$\text{or} \qquad \Delta t = \frac{L}{\sqrt{c^2-u^2}} = \frac{L/c}{\sqrt{1-(u/c)^2}}$$

Θ' :



For Θ' observer, time taken to reach top mirror is given by

$$\Delta t' = L/c$$

As a result :

$$\boxed{\Delta t = \frac{\Delta t'}{\sqrt{1-(u/c)^2}}}$$

Time dilation
$\Delta t > \Delta t'$
since $u < c$

Observer who sees clock in motion measures longer time intervals than observer at rest wrt clock ($\Delta t'$ called proper time)

Mechanical Clocks :

Would ticking of wrist watch slow down in same manner?

① Bolt wrist watch to top mirror. Synchronize it with light clock and place it on moving train. If Θ' observer (on train) sees clocks remain in synch, then Θ observer sees them both slow down identically

② Only if Θ' sees them fall out of synch will Θ see them slow down differently. For perfect clocks this $\Rightarrow$ train motion

But, this would mean uniform motion of train made this happen. Not possible since this would violate SR. Why? Because it would allow O' observer to detect his/her motion.

Recall: Equivalence between inertial observers implies that observers cannot do experiments, which result in the detection of their motion.

Consequently: clocks in motion observed to tick slower ~~Implications of Time Dilation.~~ according to O, who sees them in motion, than by O' who sees them at rest.

Implications of Time Dilation: This "slowing down of time" applies not only to light clocks and wrist watches, but also to time measured by heart beats, lifetime of unstable particles, etc...

Muons: Is a light Fermion (Spin = 1/2) with mass $m_\mu = 206 m_e$. But it is unstable

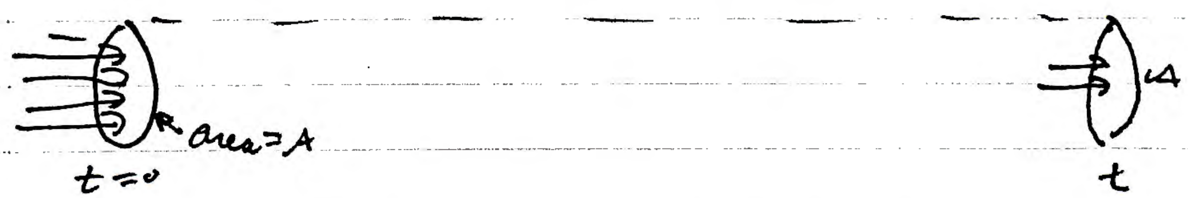
$$\mu^+ \rightarrow e^+ + \nu_e + \tilde{\nu}_\mu$$

Lifetime: $t_\mu = 2\times10^{-6}$ s. What does this mean?

Experiment:

Suppose a beam of muons with incident rate rate

(1) $\frac{dN_\mu(0)}{dt}$ = no. of muons per unit time crossing area A.



(2) Measure no. of muons ~~...~~ crossing A at time t. Experiments show in limit $v \ll c$

$$\frac{dN_\mu}{dt} = \frac{dN_\mu(0)}{dt} \exp\left(-\frac{t}{t_\mu}\right)$$

But when $v \to c$, say $v = 0.995c$, lifetime according to lab observer, increases

$$t_\mu(O) = \frac{t'_\mu(O')}{\sqrt{1-(v/c)^2}} = \frac{t_\mu}{\sqrt{1-(.995)^2}} = 10 t_\mu$$

Cosmic-Ray Muons

since $= \sqrt{1-(v/c)^2} = 1/10$

At top of atmosphere muons produced by decay of pion, π^+, which generated by CR protons

$$\pi^+ \to \mu^+ + \nu_\mu$$

Travel Distance:

(1) O frame: According to earth observer distance traveled by muon:

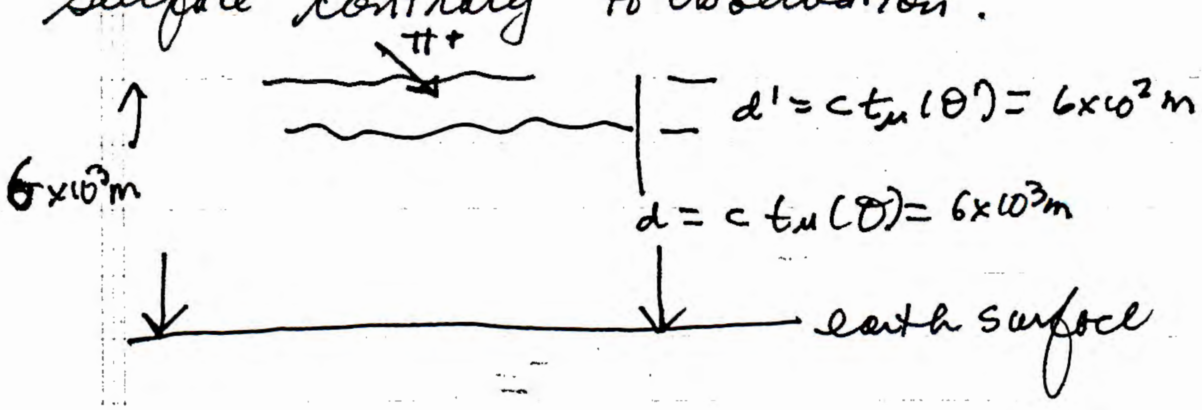
$$d = \overset{.995}{\overbrace{c}}\, t_\mu(O) \approx c \times 10 \times t_\mu = 3\times10^{10} \times 10 \times 2\times10^{-6}$$

$$d = 6\times10^5 \text{ cm} = 6\times10^3 \text{ m} = 6 \text{ km}$$

(2) No time dilation

$$d = c\, t_\mu = 6\times10^2 \text{ m}$$

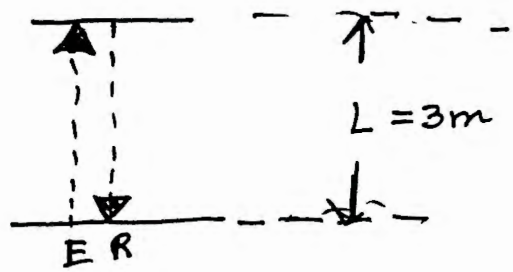
But muons produced at height of $\simeq 6\times10^3$ m. so without time dilation, few would reach earth's surface contrary to observation.



Invariance of the Interval

Let's go back to moving light clock:

Clock Rest Frame



Photon leaves bottom mirror at emission event E, goes upward to top mirror, is reflected back to bottom mirror, where it is recorded at reception event R.

What is the round trip distance traveled by the photon in the clock rest-frame?

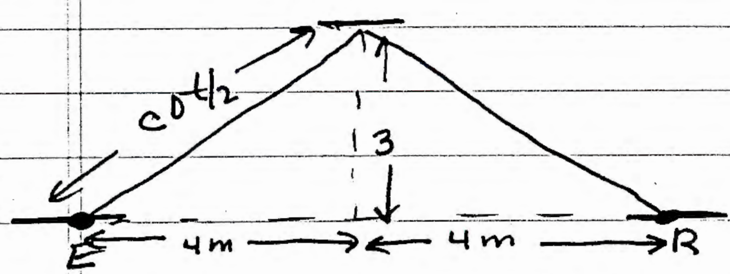
$$c\Delta t' = 2L$$

implies that in light-clock rest frame, total distance traveled is

$$\boxed{c\Delta t' = 6m}$$

what is the round-trip time traveled by photon as measured by lab. observer O ?

Time interval bew E & R events in Lab frame : $c\Delta t$



Because of high speed, train containing light mirror, the bottom mirror travels 8m.

In this case $\left(\frac{c\Delta t}{2}\right)^2 = 4^2 + 3^2 = 25\ m^2$

$\Rightarrow$ $\left(\frac{c\Delta t}{2}\right) = 5m$ or $c\Delta t = 10m$

(comparison $u\Delta t / c\Delta t$ = $8m/10m = 0.8$)

So, there are differences in
(a) Spatial separation and (b) Time separation between events E and R as measured on train and lab.

Interval: But there is a measure of separation that is the same in both frames, i.e., the interval

Interval Definition: $\Delta S^2 = \Delta x^2 - c^2\Delta t^2 = -(c^2\Delta t^2 - \Delta x^2)$

(a) Train : $\Delta x'$ (no motion of mirror); $c\Delta t' = 6m$

$(\Delta S')^2 = -(6^2 - 0) = -36 m^2$

(b) Lab : $\Delta x = 8m$; $c\Delta t = 10m$

$(\Delta S)^2 = -(10^2 - 8^2) = -(100-64) = -36m^2$

Therefore $(\Delta S)^2 = (\Delta S')^2$!

The interval is universal

Interval between 2 spacetime events has same value in all inertial frames that measure these events

Geometrically: $\Delta s^2 = -(2 \times \text{altitude of triangle})^2$

General Form of Interval

In 1-D : $\Delta s^2 = -(c^2\Delta t^2 - \Delta x^2)$

In 3-D : $\Delta s^2 = -(c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2)$

Differential version : $ds^2 = -(c^2dt^2 - dx^2 - dy^2 - dz^2)$

By requiring ds^2 to have same form and value in all inertial frames, we can derive Lorentz transformations, connecting coordinates of given events, between such frames. Generalizations of Galilean transformations discussed previously.

Geometry

Flat Euclidean : Consider dS^2 which is spatial part of ds^2

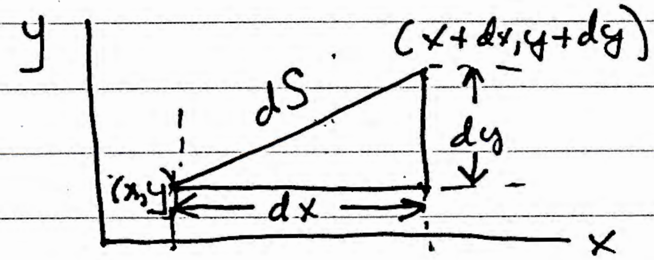
$$dS^2 = dx^2 + dy^2 + dz^2$$

In 2-D

$$dS^2 = dx^2 + dy^2 \quad \text{since } dz = 0$$

BH-21

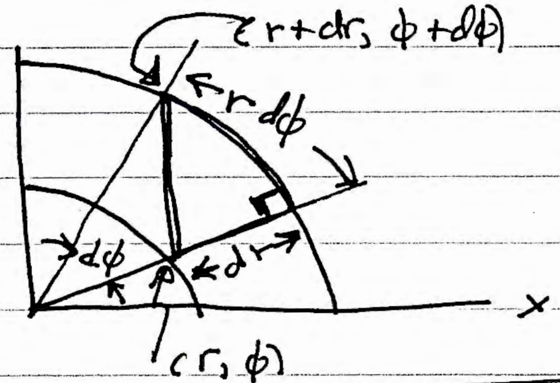
In Cartesian coordinates (x, y), dS is just distance between points (x,y) and $(x+dx, y+dy)$



$$dS^2_{\text{Cartesian}} = dx^2 + dy^2$$

(Pythagorum theorem)

In curvilinear coordinates (r, ϕ) (polar coordinates)



$$dS^2 = dr^2 + (r\,d\phi)^2$$

dS^2 is independent of choice of coordinates

since $x = r\cos\phi$; $y = r\sin\phi$

$dx = dr\cos\phi - r\sin\phi\,d\phi$; $dy = dr\sin\phi + r\cos\phi\,d\phi$

$dx^2 + dy^2 = (dr\cos\phi - r\sin\phi\,d\phi)^2 + (dr\sin\phi + r\cos\phi\,d\phi)^2$

$= (dr\cos\phi)^2 - 2r\,dr\,d\phi\cos\phi\sin\phi + (r\sin\phi)^2 d\phi^2$

$+ (dr\sin\phi)^2 + 2r\,dr\,d\phi\sin\phi\cos\phi + (r\cos\phi)^2 d\phi^2$

$dx^2 + dy^2 = dr^2(\cos^2\phi + \sin^2\phi) + r^2(\sin^2\phi + \cos^2\phi)\,d\phi^2$

$dx^2 + dy^2 = dr^2 + r^2 d\phi^2$

Therefore $dS^2_{\text{cartesian}} = dS^2_{\text{polar}}$ (same frame, different coordinates)

(Principle: Newtonian Physics assumes Space is Euclidean)

Eq. of free particles in inertial frame: $\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0$

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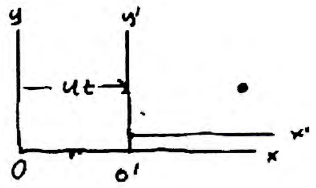
Page #6 of Lecture #3

Lecture #3:

<-previous page next page->

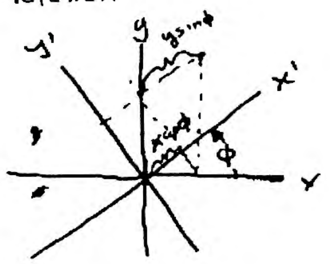
3 ways 2 inertial frames can be connected B-22

(1) Uniform motion



$x' = x - ut$, $y' = y$, $z' = z$, $t' = t$ crucial

(2) Rigid rotation



(3) displacement same as (1), but replace $d \equiv ut$

$$x' = (\cos\phi)\,x + (\sin\phi)\,y$$
$$y' = -(\sin\phi)\,x + (\cos\phi)\,y$$
$$z' = z$$
$$t' = t$$

Invariance

~~[illegible]~~ It turns out that these transformations leave Newton's Laws invariant

(1) $\frac{dx'}{dt'} = \frac{dx}{dt} - u$ $\qquad \frac{dy'}{dt'} = \frac{dy}{dt}$

$\frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2}$ $\qquad \frac{d^2y}{dt'^2} = \frac{d^2y}{dt^2}$, etc.

<-previous page

Particle accelerations the same in ~~both~~ frames separated by uniform motion

next page->

In rotated frame

$$\frac{dx'}{dt} = \cos\phi \frac{dx}{dt} + \sin\phi \frac{dy}{dt} \;:\; \frac{d^2x'}{dt^2} = \cos\phi \frac{d^2x}{dt^2} + \sin\phi \frac{d^2y}{dt^2}$$

$$\frac{dy'}{dt} = -\sin\phi \frac{dx}{dt} + \cos\phi \frac{dy}{dt} \;:\; \frac{d^2y'}{dt^2} = -\sin\phi \frac{d^2x}{dt^2} + \cos\phi \frac{d^2y}{dt^2}$$

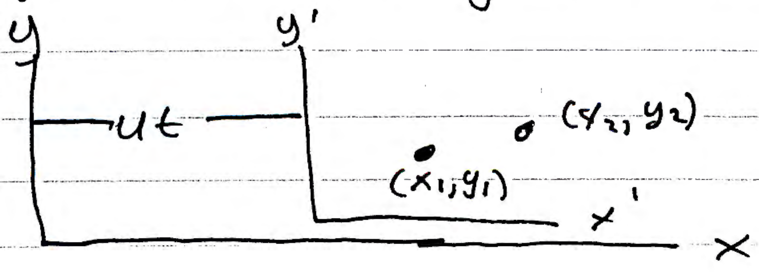
Therefore: if particle is free in unprimed frame; i.e.,

$$\frac{d^2x}{dt^2} = 0 \;;\; \frac{d^2y}{dt^2} = 0$$

then $$\frac{d^2x'}{dt^2} = 0 \;;\; \frac{d^2y'}{dt^2} = 0$$

Eqs. of motion the same in rotated frame

Symmetry of nature holds because Euclidean space shares same symmetries as Newtonian Physics: ds^2 invariant under same Galilean transformation leaving Newton's laws invariant



Two particles at rest in O' frame, separated by $ds^2 = dx'^2 + dy'^2$

$$x_1 = x_1' + ut \;;\; x_2 = x_2' + ut$$

$$dx = x_2 - x_1 = x_2' - x_1' + u(t-t) = x_2' - x_1'$$

$$dy = dy' \Rightarrow ds^2 = dx^2 + dy^2$$

Lecture #3:

If images take too long to load, please inform web guy

Page #9 of Lecture #3

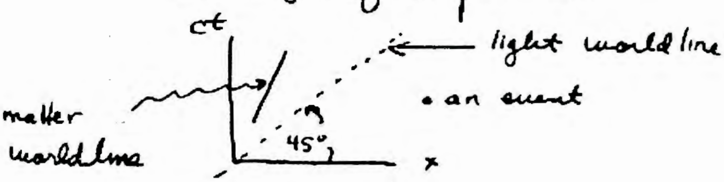
<-previous page next page->

BH-24

Spacetime Diagrams and lightcones

Spacetime diagrams show events in (ct, x) plane

- a single point is an event
- a line $x = x(t)$ or $t = t(x)$ is world line (history) of a particle



Slope $\dfrac{d(ct)}{dx} = c\dfrac{dt}{dx} = \dfrac{1}{(v/c)}$

- Light : $v = c \Rightarrow \dfrac{d(ct)}{dx} = 1$; slope $= 45°$
- matter : $v < c \Rightarrow \dfrac{d(ct)}{dx} > 1$; slope $> 45°$
- Tachyons? : $v > c \Rightarrow \dfrac{d(ct)}{dx} < 1$; slope $< 45°$
- Particle at rest : $v = 0 \Rightarrow \dfrac{d(ct)}{dx} = \infty$

<-previous page next page->

Lecture #3:

Page 1
Page 2
Page 3
Page 4
Page 5
Page 6
Page 7
Page 8
Page 9
Page 10
Page 11
Page 12
Page 13
Page 14

If images take too long to load, please inform web guy

Page #8 of Lecture #3

<-previous page next page->

8 - 23

Principle of Relativity - relating form of laws of physics in inertial frames differing by displacement and rotations is only possible because geometry of Euclidean space shares those symmetries. Laws of physics (in this form) would not be invariant under displacements and rotations if geometry of space were curved.

Geometry:

$$dS^2 = dx^2 + dy^2 + dz^2$$

$dS^2 = dS'^2$ under same transformations that leave Newton's eqs. invariant.

So there is a deep underlying connection between Euclidean Space and Newtonian physics

Back to SR

$$ds^2 = -(c\,dt)^2 + dx^2 + dy^2 + dz^2$$

We will discuss Lorentz transformations that connect inertial frames and leave ds^2 invariant. But in this case geometry of spacetime is non-Euclidean because of $-(cdt)^2$ term, but spatially flat

Newtonian: $ds^2 = (cdt)^2 + dx^2 + dy^2 + dz^2$

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spacetime is Euclidean in Newtonian picture

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Page #10 of Lecture #3

<-previous page next page->

BH-25

Interval Separation between spacetime events

$$ds^2 = -(c\,dt)^2 + dx^2 + dx^2 + dy^2$$

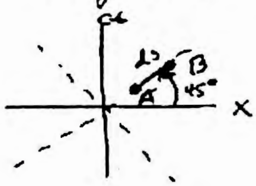
1-D For $dy = dz = 0$ $\quad ds^2 = -(c\,dt)^2 + dx^2$

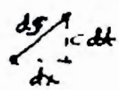
Lightlike : $dx/dt = \pm c$

Since $c\,dt = \pm dx$

implies: $ds = 0$

for differential line element separating lightlike events





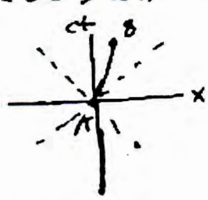
Interval doesn't look zero. But recall we are not using rules of Euclidean geometry here

$$ds^2 = -c^2dt^2 + dx^2 \quad (\text{Not } \oplus c^2dt^2 \oplus dx^2)$$

this is why $ds = 0$

Timelike

$$c\,dt > dx \Rightarrow ds^2 < 0$$



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If images take too long to load, please inform web guy

Page #11 of Lecture #3

<-previous page next page->

BH26

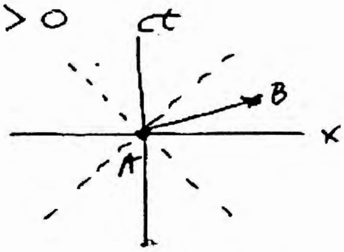
All particles with non-zero rest mass move along timelike world lines within light cones

This includes particle at rest ($dx = 0$)

Spacelike

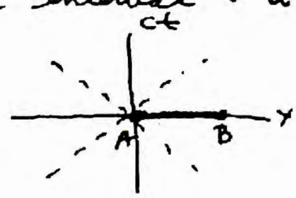
$c\,dt < dx$

$ds^2 > 0$



Such particles move with $v > c$ None have ever been observed.

Note: 2 ends of a ruler are separated by spacelike interval : $dt = 0$



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If images take too long to load, please inform web guy

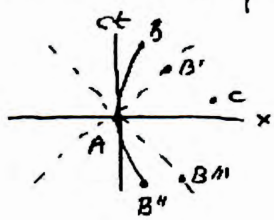
Page #12 of Lecture #3

<-previous page next page->

BH-27

Causality

Light cones define loci of causal connections between events (i.e., points) in spacetime.



Event A can influence

- event B (within its future lightcone)
- " B' (along its " " ")

Event A cannot influence events outside its future light cone (C)

Event A cannot influence any event in its past (B''), but can be influenced by events within its past light cone (B'') or along its past lightcone (B''').

<-previous page next page->

If images take too long to load, please inform web guy

Page #13 of Lecture #3

Lecture #3:

<-previous page next page->

3-28

Proper Time & Time Dilation

Definitions

Clock: A device measuring timelike distances
Ruler: A device measuring spacelike distances

Measure of distance along particle world line:
Define

$$\boxed{d\tau^2 \equiv -ds^2/c^2}$$

Timelike worldlines: $ds^2 < 0 \Rightarrow d\tau^2 > 0$
As a result $d\tau$ is real. This is time measured by clock carried along world line

Recall:

$ds^2 = -(c\,dt')^2 + dx'^2$ where prime is moving coordinate system; i.e., a spacecraft. If clock is at rest wrst spacecraft, $dx' = 0$ → But $ds^2 = -c^2 d\tau^2$

Therefore $ds^2 = -c^2 dt'^2$ ⇒ $d\tau = dt'$

This is why $d\tau$ is called proper time interval — measured by observer at rest wrst clock

Lab Frame Again

ds^2 invariant interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 = -c^2 d\tau^2$$

$$\therefore d\tau^2 = dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2)$$

So, proper time interval separating events A and B

$$\tau_{AB} = \int_A^B d\tau = \int_{t_A}^{t_B} \sqrt{dt^2 - \frac{(dx^2+dy^2+dz^2)}{c^2}}$$

<-previous page next page->

If images take too long to load, please inform web guy

Page #14 of Lecture #3

<-previous page

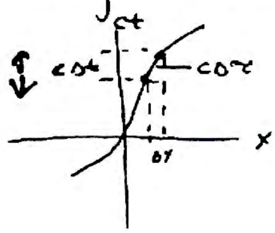
BH-29

$$\tau_{AB} = \int_{t_A}^{t_B} dt \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}$$

$$\boxed{\tau_{AB} = \int_{t_A}^{t_B} dt' \sqrt{1 - \left(\frac{\underline{V}(t')}{c}\right)^2}}$$

Proper Time interval τ_{AB} is shorter than Lab time interval $t_B - t_A$ because $|\underline{V}| < c$.

Differential form $d\tau = dt\sqrt{1 - \left(\frac{v}{c}\right)^2}$



$c^2\Delta\tau^2 = c^2\Delta t^2 - \Delta x^2$ (again $c\Delta\tau$ looks like it is longer than $c\Delta t$. But it is shorter because spacetime geometry is non-Euclidean.

<-previous page

If images take too long to load, please inform web guy

Page #1 of Lecture #4

next page->

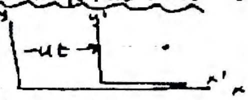
BH-29a

Recap

(1) Connections between Geometry and Physics

- Newtonian Physics : Assumes Space is Euclidean

  Physics (a) Galilean Transformations between inertial frames



$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

  leave Newton's laws invariant; i.e., particle travelling in straight line in one frame does so in all frames. Concept of straight line (shortest distance between 2 points) has meaning only in Euclidean Space

  Geometry (b) Galilean Transformations :
  leave Spatial Interval $ds^2 = dx^2 + dy^2 + dz^2$ invariant; i.e., $dS^2 = dS'^2$
  assume $dt' = dt$: connection geometry ⟺ physic

- Special Relativity

  Physics (a) Lorentz Transformations = New (as yet unspecified) transformations between inertial frames leave Maxwell's eqs. for E&M waves invariant, so c same in all inertial frames.
  (Galilean transforms require speed of light to change, so are rejected by Einstein). Newton's Laws change!

  Geometry (b) Lorentz transformations :
  leave space-time interval,
  $ds^2 = -(c\,dt)^2 + dx^2 + dy^2 + dz^2$, invariant : $ds^2 = ds'^2$

  Spacetime Geometry non-Euclidean

next page->

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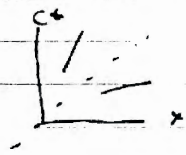
Page #2 of Lecture #4

<-previous page next page->

BH-296

(2) I introduced concept of spacetime diagrams



- world lines
- events
- light cones divide space-like from timelike worldlines

(3) I introduced concept of proper time: Time interval in coordinate frame in which clock is at rest: $d\tau^2 = -ds^2/c^2$.

$$d\tau = dt\sqrt{1-(v/c)^2}$$

where dt is time interval in Lab frame that sees clock move with speed v

<-previous page next page->

Lecture #4:

Page 1
Page 2
Page 3
Page 4
Page 5
Page 6
Page 7
Page 8
Page 9
Page 10
Page 11
Page 12
Page 13
Page 14
Page 15

If images take too long to load, please inform web guy

Page #4 of Lecture #4

<-previous page next page->

Suppose Alice travels at uniform speed $v = \frac{3}{5}c$ until $t_{mid} = \frac{1}{2}(t_1 + t_2)$. She stops instantaneously at t_{mid}, reverses direction and returns to x_p at $t = t_2$, moving again at $v = \frac{3}{5}c$.

Suppose $t_2 - t_1 = 50$ yrs., i.e., Bob has aged 50 years.

$$\Delta\tau_{mid} = \int_{t_1}^{t_{mid}} dt \sqrt{1 - \frac{v(t)}{c^2}} = \sqrt{1 - \left(\frac{v}{c}\right)^2} \int_{t_1}^{t_{mid}} dt \qquad (\text{since } v = \text{const.})$$

$$\therefore \Delta\tau_{mid} = \sqrt{1 - \left(\frac{v}{c}\right)^2} \left[\frac{1}{2}(t_1 + t_2) - t_1\right] = \sqrt{1 - \left(\frac{v}{c}\right)^2} \left[\frac{1}{2}(t_2 - t_1)\right]$$

$$\text{or } \Delta\tau_{mid} = \sqrt{1 - \left(\frac{3}{5}\right)^2} \left[\frac{1}{2}(t_2 - t_1)\right] = \frac{4}{5} \cdot \left[\frac{1}{2}(t_2 - t_1)\right]$$

Same result holds from $t_{mid} \to t_2$

Therefore proper time elapsed for round trip is

$$\Delta\tau_{TOT} = 2 \cdot \Delta\tau_{mid} = 2 \times \frac{4}{5} \times \frac{1}{2}(t_2 - t_1)$$

$$\boxed{\Delta\tau_{TOT} = \frac{4}{5}(t_2 - t_1)}$$

Since $t_2 - t_1 = \Delta t_{Bob} = 50$ years, $\Delta\tau_{TOT} = 40$ years

{ Bob aged by 50 years; Alice " " 40 years }

Note: Situation not symmetric! Even though Alice sees Bob moving, they have different histories. Alice is not an inertial observer, whereas Bob is. Her velocity vector changed and as a result she suffered acceleration whereas Bob is an inertial observer. This explains why from Alice's point of view Bob is not younger.

<-previous page next page->

Lecture #4:

If images take too long to load, please inform web guy

Page #5 of Lecture #4

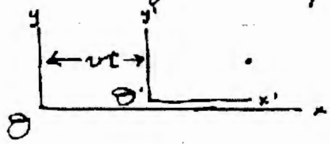
<-previous page next page->

B14-32

What does spacetime look like to other observers?

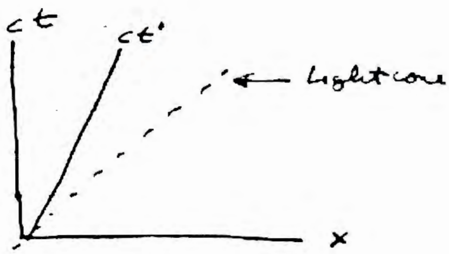
Suppose $\mathcal{O}$ observer uses (ct, x) coordinates and another inertial observer $\mathcal{O}'$ uses (ct', x') coordinates where $\mathcal{O}'$ moves along x axis with velocity v wrst $\mathcal{O}$. Where do (ct', x') coordinates go on spacetime diagram for $\mathcal{O}$?

Spatial



snapshot at $t = \text{const.}$

Spacetime



- ct' axis is locus of all $x'=0$ points (also $y'=z'=0$ points) for all time.
- ct' axis is worldline of $\mathcal{O}'$: at rest in $\mathcal{O}'$, but moves wrst $\mathcal{O}$.

Recall: $\dfrac{d(ct)}{dx} = \dfrac{c}{dx/dt} = \dfrac{1}{(v/c)}$ is slope of $\mathcal{O}'$ worldline.

Question: where is x' axis? We need to determine it using light trajectories

<-previous page next page->

If images take too long to load, please inform web guy

Page #3 of Lecture #4

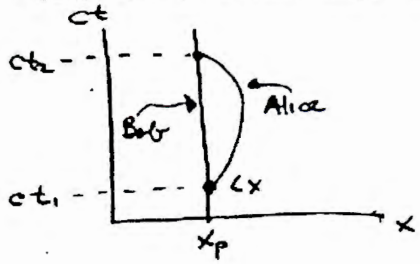
<-previous page next page->

BH-30

Twin Paradox

Equation for T_{AB} ⇒ time registered by clock moving between 2 points depends on route traveled even if it returns to same point it started from.

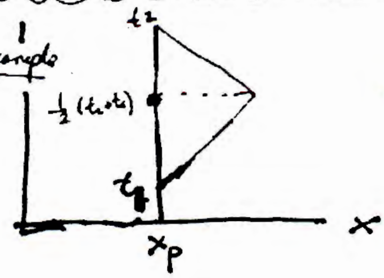
Two twins start out from rest at same spacetime event in an inertial frame. Alice moves away from starting point, but then returns at later time. Each have their own clock.



- Bob's clock. Time elapsed between Alice's departure & return: $t_2 - t_1$
- Alice's clock: less than this, since $\sqrt{1-\frac{v^2}{c^2}} < 1$

Alice ages less than Bob

Numerical Example



returns at midpoint

<-previous page next page->

Lecture #4:

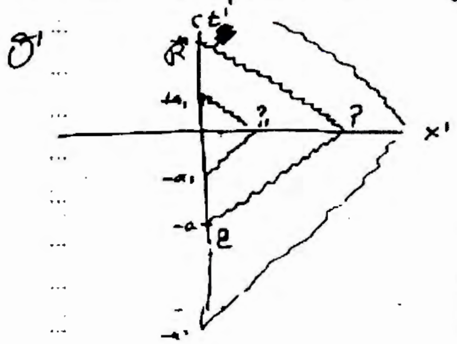
If images take too long to load, please inform web guy

Page #6 of Lecture #4

<-previous page next page->

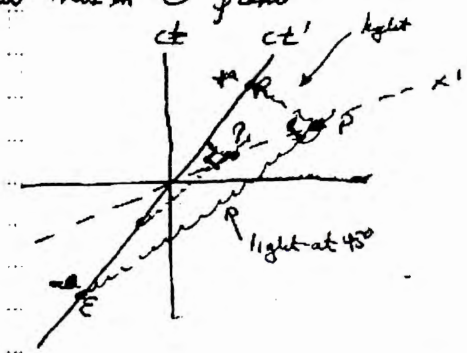
33

Consider O' space time. light mirror again. Photon emitted at event E at $ct' = -a$ reaches x' axis at $t' = 0$ at P. If reflected, it will return to $x' = 0$ at $ct' = +a$ (R).



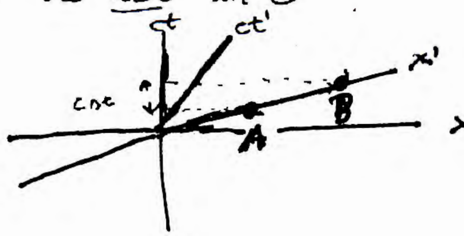
x' axis defined as locus of events that reflect light emitted at $ct = -a$ and return back at $ct = +a$ (for all a) Reflection events synchronized at $ct' = 0$. Definition of x' axis.

Now draw this in O frame



① E & R at (-a,+a) along t' axis
② Let beam emitted from E at 45°
③ Reflected beam must arrive at R along beam with negative 45° slope
④ Intersection is event P

Locus of all P is x' axis ($t' = 0$)
In both diagrams light travels at 45°, while t & t' axes change slope. Events that were simultaneous in O' are not in O



A & B simultaneous in ct' But not in ct

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Lecture #4:

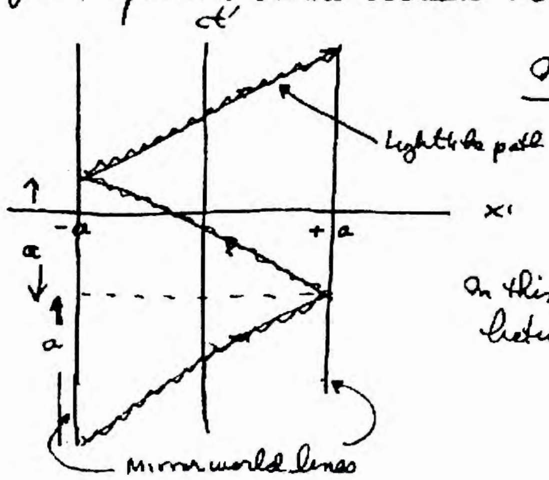
If images take too long to load, please inform web guy

Page #7 of Lecture #4

<-previous page next page->

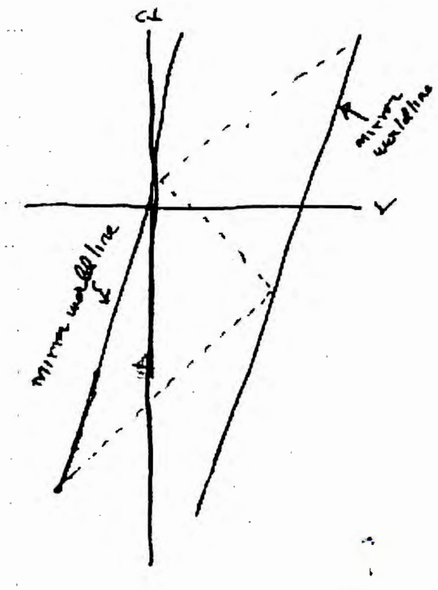
BH-34

Last Exercise gives graphic illustration of breakdown of simultaneity. Suppose we have 2 mirrors at $x' = \pm a$. Let photon bounce between them.



On O' frame

In this case, equal time intervals between ticks



On O frame

In this case unequal time intervals between ticks

<-previous page next page->

If images take too long to load, please inform web guy

Page #8 of Lecture #4

<-previous page next page->

BH-[redacted]35

Invariance of the interval

Consider 2 events in spacetime diagram. The ~~co~~ coordinate differences separating the events are $\Delta t, \Delta x, \Delta y, \Delta z$.
In that case the interval is given by

$$\Delta S^2 = (c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2)$$

Example: Consider events $\mathcal{E}$ & P. world line between these events propagates with speed of light, c.

In $\mathcal{O}$ $\quad \dfrac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta t^2} = c^2 \qquad \therefore \Delta S^2 = 0$

In $\mathcal{O}'$ universality of speed of light implies:

$$\frac{\Delta x'^2 + \Delta y'^2 + \Delta z'^2}{\Delta t'^2} = c^2 \qquad \therefore (\Delta S')^2 = 0$$

As a result $\Delta S = \Delta S' = 0$; interval separating lightlike events invariant for $\mathcal{O} \to \mathcal{O}'$ transform

General ΔS: Previously I argued that ΔS was invariant for interval connecting timelike events $\mathcal{E}, R$ on moving light clock which had speed $v < c$.
this is a timelike

In fact $(\Delta S)^2 = (\Delta S')^2$ for ~~events~~ intervals connected by ~~any~~ any 2 events, even $\mathcal{E}, S$.

<-previous page next page->

Page #9 of Lecture #4

<-previous page next page->

BH-35a

Coordinate Systems & Lorentz transformations

Coordinate Systems look as follows: from O frame we have –

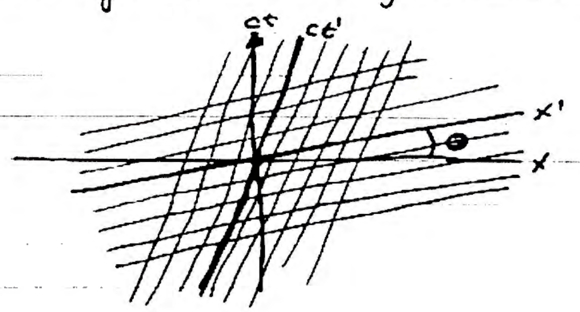


Figure shows (ct', x') coordinate grid plotted on (ct, x) axes. The (ct', x') coordinates are not orthogonal to each other in Euclidean Geometry of the page. But they are orthogonal in geometry of spacetime.

What are the new transformations that preserve the non-Euclidean interval $ds^2 = -(c\,dt)^2 + dS^2$ of 4D spacetime? Consider analogues of rotation in (ct, x) plane (not x, y plane!). ~~[illegible]~~ analogous to rotation but with hyperbolic trig. because of non-Euclidean nature of spacetime.

Euclidean rotation: (mix y, x)

$$x' = (\cos\phi)\,x + (\sin\phi)\,y$$
$$y' = -(\sin\phi)\,x + (\cos\phi)\,y$$
$$z' = z$$

preserves dS^2

SR (non-Euclidean) (mix ct, x)

$$ct' = (\cosh\theta)\,ct - (\sinh\theta)\,x$$
$$x' = -(\sinh\theta)(ct) + (\cosh\theta)\,x$$

$\boxed{\text{Preserves } \Delta S^2}$

$$\cosh\theta = \frac{1}{2}\left(e^{\theta} + e^{-\theta}\right); \quad \sinh\theta = \frac{1}{2}\left(e^{\theta} - e^{-\theta}\right)$$

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a particle at rest at origin of O'; i.e., $x'=0$ in (ct', x') frame has ct' axis as its world line in (ct, x) frame. Particle moves along x axis with speed $v = \text{const.}$ found by setting $x'=0$

or $$0 = -(\sinh\theta)(ct) + (\cosh\theta)x$$
$$0 = -(\sinh\theta)(c\,dt) + (\cosh\theta)dx$$
$$\Rightarrow \frac{dx}{dt} = c \cdot \tanh\theta \quad \text{or} \quad \boxed{v = c\tanh\theta}$$

Lorentz Transformations

Identity: $\cosh^2\theta - \sinh^2\theta = 1$

Divide by $\cosh^2\theta \Rightarrow 1 - \tanh^2\theta = 1/\cosh^2\theta$

or $$\cosh^2\theta = \frac{1}{1-\tanh^2\theta} = \frac{1}{(1-(\frac{v}{c})^2)}$$

But since $\sinh^2\theta = \cosh^2\theta - 1$,
$$\sinh^2\theta = \frac{1}{1-(\frac{v}{c})^2} - 1 = \frac{(v/c)^2}{1-(v/c)^2}$$

therefore:
$$x' = -(\sinh\theta)(ct) + (\cosh\theta)x$$
$$x' = -\frac{(v/c)}{\sqrt{1-(v/c)^2}}(ct) + \frac{1}{\sqrt{1-(v/c)^2}}(x)$$
$$\boxed{x' = \frac{1}{\sqrt{1-(v/c)^2}}(x - vt)}$$

$$ct' = (\cosh\theta)(ct) - (\sinh\theta)x$$
$$ct' = \frac{1}{\sqrt{1-(v/c)^2}}(ct) - \frac{v/c}{\sqrt{1-(v/c)^2}}x$$
$$\boxed{t = \frac{1}{\sqrt{1-(v/c)^2}}\left[t - \frac{vx}{c^2}\right]}$$

Summarize: If $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ y' &= y \\ z' &= z \end{aligned}}$$

Lorentz transformations

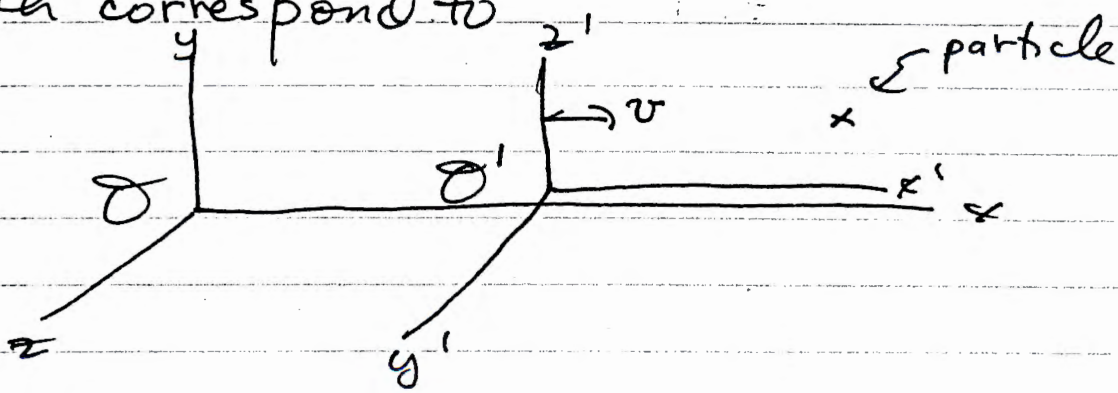
Here I know O coordinates. Find O' coordinates

Inverse Transformations: But suppose I know O' coordinates and I want to find O coordinates

In this case $(t', x', y, z') \to (t, x, y, z)$

$$\boxed{\begin{aligned} x &= \gamma(x' + vt') \qquad & y &= y' \\ t &= \gamma(t' + vx'/c^2) \qquad & z &= z' \end{aligned}}$$

Both correspond to



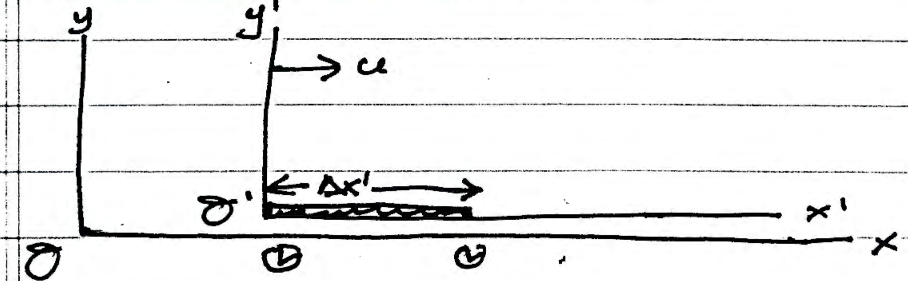
Properties of Lorentz Transformations

Limit $v/c \ll 1$, $\gamma \to 1$:

Lorentz reduces to Galilean transformation

Applications of Lorentz Transformation:

(1) Lorentz length Contraction



Suppose rigid rod with length $\Delta x'$ is placed at rest in moving O' frame. What is the length of the rod as measured by O?

Place clocks along x axis in O frame and measure length simultaneously at both ends, $\Delta t = 0$

Left end $\quad x_1' = \gamma(x_1 - v t_1)$

Right end $\quad x_2' = \gamma(x_2 - v t_2)$

$$\Delta x' = x_2' - x_1' = \gamma\left[x_2 - x_1 - v(t_2 - t_1)\right]$$

But $t_2 - t_1 = 0 \Rightarrow \Delta x' = \gamma \Delta x$

Let $\Delta x' = L_0$ (proper length in O' frame)

$\Delta x = L$ (measured length in O frame)

Therefore $\boxed{L = L_0/\gamma} = \boxed{\sqrt{1-(u/c)^2}\, L_0}$

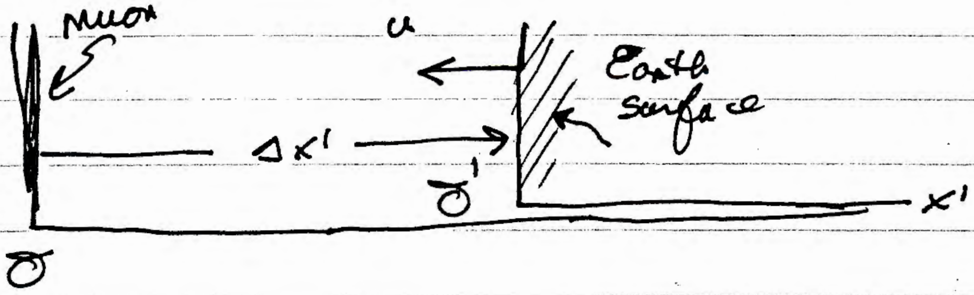
Length of moving body measured in Lab frame is reduced by factor $\sqrt{1-\left(\frac{u}{c}\right)^2}$ in direction of its motion.

Comment: Lorentz contraction is no "illusion". It is real in the same sense that time dilation is real

Example: $\longrightarrow$

Applications

Muon produced 6 km above earth.
Assume Lab frame is muon rest frame $\mathcal{O}$. So earth is $\mathcal{O}'$ moving toward muon



Distance between muon and earth:

(1) Earth frame $\Delta x' = 6$ km

(2) Muon frame $\Delta x = \dfrac{\Delta x'}{\gamma} = \sqrt{1-(v/c)^2}\,\Delta x'$

$\therefore$ According to muon, time it takes for ground to rush up and hit it is

$$\Delta t = \frac{\Delta X}{.9995c} \simeq \frac{\Delta x}{c}\sqrt{1-(.995)^2} = \frac{1}{10}\frac{\Delta x}{c}$$

$$\Delta t = \frac{1}{10} \times \frac{6\times 10^5\text{cm}}{3\times 10^{10}} = 2\times 10^{-6}\text{s}$$

Since this is comparable to t_μ (muon lifetime) muon survives long enough to reach earth.

Page #14 of Lecture #4

<-previous page next page->

BH-39

$$\boxed{ct'_3 = -1}$$

• Shelimar event

$x_{sh} = 0$; $ct_{sh} = -4$

$x'_{sh} = \gamma\left(x_{sh} - \frac{v}{c}(ct_{sh})\right)$

$= 1.25\,(0 - 0.6\,(-4)) = 1.25 \times 2.4$

$$\boxed{x'_{sh} = 3}$$

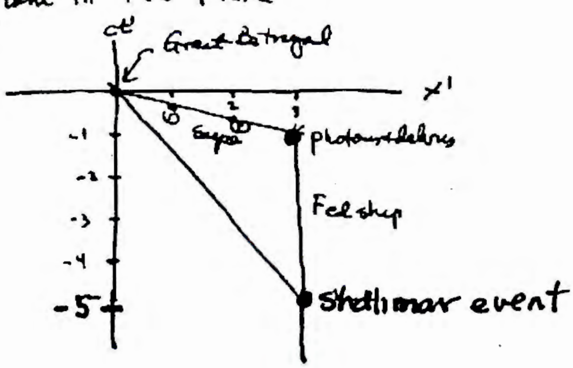
(Note : $x'_{sh} = x'_3$ as it should. Rocket still in its frame

$ct'_{sh} = \gamma\left(ct_{sh} - \frac{v}{c}x_{sh}\right)$

$ct'_{sh} = 1.25(-4 - 0.6 \times 0) = -1.25 \times 4$

$$\boxed{ct'_{sh} = -5}$$

Spacetime diagram in Fed Frame



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Lecture #4:

If images take too long to load, please inform web guy

Page #15 of Lecture #4

<-previous page

BH-40

• Klingon events : Great Betrayal

$$x_{GB} = 0 \; ; \quad ct_{GB} = 0$$

$$x'_{GB} = \gamma\left(x_{GB} - \frac{v}{c}(ct_{GB})\right)$$

$$= \gamma\left(0 - \frac{v}{c} \times 0\right)$$

$$\boxed{x'_{GB} = 0}$$

$$ct'_{GB} = \gamma\left(ct_{GB} - \frac{v}{c}x_{GB}\right) = \gamma\left(0 - \frac{v}{c}0\right)$$

$$\boxed{ct'_{GB} = 0}$$

Interpretation:

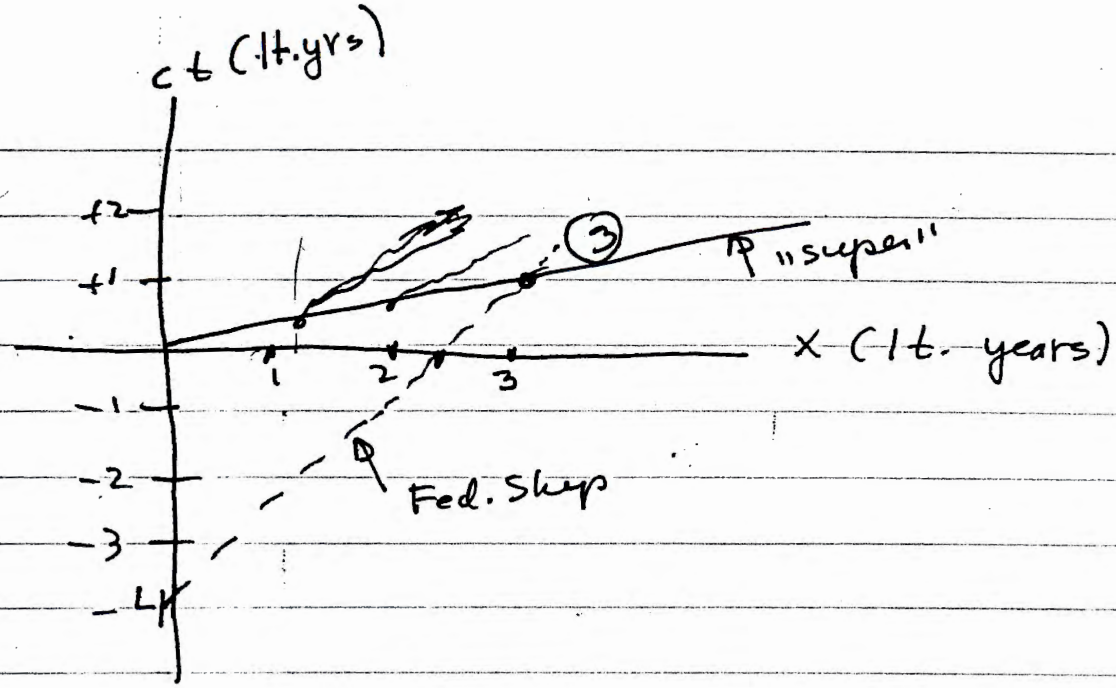
In Federation ship coordinate frame

• worldline of super tilts downward to right. Deaths of Fed negotiator occur at $ct_J' = -1$, 1 year before Klingons launch super.

• One could conclude that super moves from Fed ship toward Klingons at $v = 3c$. Yet Feds created no such weapon. Rather they are destroyed by it. Klingon suffer no such damage

Result. Confusion between cause & effect. So no object can go with $v > c$. If it did, we could influence the past

<-previous page



Klingon Rest Frame:

- Peace treaty of Shalimar signed 4 years before great betrayal

- Peace treaty says: Klingons to stop attacking federation troops in return to access to federation data base

- Federation negotiators leave Shalimar after signing on ship moving at $v=0.6c$

  $\Delta x/\Delta t = 0.6c \Rightarrow \Delta x/c\Delta t = 0.6$

- Within 4 years, Klingons use database to construct a projectile, the "super" with $v>c$ velocity. At $ct=0$, they launch super at $v=3c$ toward fed ship $(\Delta x/c\Delta t)=3$

- At event 3 Super destroys Fed. Ship