Problem 1 (10 pts)
An electron in a hydrogen-like ion is in an orbit of radius $4 a_{0}$, with $a_{0}$ the Bohr radius. When it makes a transition to the ground state it emits a photon of wavelength approximately 10A.
(a) Find the value of $Z$ for this ion and the value of $n$ for this orbit.
(b) Find the speed of this electron $v$, expressed as $v / c$, as a multiple of the fine structure constant $\alpha$.
(c) Find the wavelength (in A) of the longest wavelength photon that this electron can absorb by making a transition to another state.

Problem 2 (10 pts)


An electron is in a box with a lid at height z measured from the bottom of the box, and is in its lowest energy state.
(a) Estimate the kinetic energy of this electron using the uncertainty principle when the lid is at height $\mathrm{z}=\mathrm{z}_{0}=10 \mathrm{~A}$. Give your answer both in eV and in J. Ignore the effect of gravity on the electron, and assume it only moves up and down, not sideways.

The box has a lid of mass M , as shown in the figure, and gravity acts on the lid with force $\mathrm{Mg}\left(\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}\right)$. Assume now the lid can slide freely up and down the z direction making the box bigger and smaller, but is in its equilibrium position at height $\mathrm{z}=\mathrm{z}_{0}=10 \mathrm{~A}$.
(b) Make a qualitative graph of the energy of this system given by the sum of the kinetic energy of the electron and the gravitational potential energy of the lid, as a function of the vertical position of the lid, z . Show where $\mathrm{z}_{0}$ is in your graph.
(c) Calculate the value of the mass of the lid, $M$, using the fact that its equilibrium position is at $\mathrm{z}=\mathrm{z}_{0}=10 \mathrm{~A}$. Give your answer in grams.
Use: $1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}, \hbar^{2} / m_{e}=7.62 \mathrm{eVA}$, with $\mathrm{m}_{\mathrm{e}}$ the electron mass.
Problem 3 (10 pts)
(a) What is the de Broglie wavelength (in A) for an electron in the $\mathrm{n}=3$ state of hydrogen?
(parts (b) and (c) of this problem are unrelated to (a))
(b) For the wavepacket: $y(x, t)=y_{0} \cos \left(11 A^{-1} x-30 s^{-1} t\right)+y_{0} \cos \left(13 A^{-1} x-42 s^{-1} t\right)$ find the phase velocity and the group velocity, in $\mathrm{A} / \mathrm{s}$.
(c) For the wavepacket given in part (b) and for $\mathrm{t}=0$, find the smallest positive position x (in A) where the envelope of the group takes its most negative value.

## Justify all your answers to all problems

