

$$5-3. \quad E_k = eV_o = \frac{p^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} \quad V_o = \frac{1}{e} \frac{(1240eV \cdot nm)^2}{2(5.11 \times 10^5 eV)(0.04nm)^2} = 940V$$

$$5-4. \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}} \quad (\text{from Equation 5-2})$$

$$(a) \text{ For an electron: } \lambda = \frac{1240eV \cdot nm}{[(2)(0.511 \times 10^6 eV)(4.5 \times 10^3 eV)]^{1/2}} = 0.0183nm$$

$$(b) \text{ For a proton: } \lambda = \frac{1240eV \cdot nm}{[(2)(938.3 \times 10^6 eV)(4.5 \times 10^3 eV)]^{1/2}} = 4.27 \times 10^{-4} nm$$

$$(c) \text{ For an alpha particle:}$$

$$\lambda = \frac{1240eV \cdot nm}{[(2)(3.728 \times 10^9 eV)(4.5 \times 10^3 eV)]^{1/2}} = 2.14 \times 10^{-4} nm$$

$$5-5. \quad \lambda = h/p = h/\sqrt{2mE_k} = hc/[2mc^2(1.5kT)]^{1/2} \quad (\text{from Equation 5-2})$$

Mass of N₂ molecule =

$$2 \times 14.0031u(931.5MeV/c^2) = 2.609 \times 10^4 MeV/c^2 = 2.609 \times 10^{10} eV/c^2$$

$$\lambda = \frac{1240eV \cdot nm}{[(2)(2.609 \times 10^{10} eV)(1.5)(8.617 \times 10^{-5} eV/K)(300K)]^{1/2}} = 0.0276nm$$

$$5-11. \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_p E_k}} = 0.25nm$$

Squaring and rearranging,

$$E_k = \frac{h^2}{2m_p \lambda^2} = \frac{(hc)^2}{2(m_p c^2) \lambda^2} = \frac{(1240eV \cdot nm)^2}{2(938 \times 10^6 eV)(0.25nm)^2} = 0.013eV$$

$$n\lambda = D \sin \phi \quad \rightarrow \quad \sin \phi = n\lambda / D = (1)(0.25nm) / (0.304nm)$$

$$\sin \phi = 0.822 \quad \rightarrow \quad \phi = 55^\circ$$

$$5-12. \quad (a) \quad n\lambda = D \sin \phi \quad \therefore \quad D = \frac{n\lambda}{\sin \phi} = \frac{nhc}{\sin \phi \sqrt{2mc^2 E_k}}$$

$$= \frac{(1)(1240 eV \cdot nm)}{(\sin 55.6^\circ) [2(5.11 \times 10^5 eV)(50 eV)]^{1/2}} = 0.210 nm$$

$$(b) \quad \sin \phi = \frac{n\lambda}{D} = \frac{(1)(1240 eV \cdot nm)}{(0.210 nm) [2(5.11 \times 10^5 eV)(100 eV)]^{1/2}} = 0.584$$

$$\phi = \sin^{-1}(0.584) = 35.7^\circ$$

$$5-17. \quad (a) \quad y = y_1 + y_2$$

$$= 0.002m \cos(8.0x/m - 400t/s) + 0.002m \cos(7.6x/m - 380t/s)$$

$$= 2(0.002m) \cos \left[\frac{1}{2}(8.0x/m - 7.6x/m) - \frac{1}{2}(400t/s - 380t/s) \right]$$

$$\times \cos \left[\frac{1}{2}(8.0x/m + 7.6x/m) - \frac{1}{2}(400t/s + 380t/s) \right]$$

$$= 0.004m \cos(0.2x/m - 10t/s) \times \cos(7.8x/m - 390t/s)$$

$$(b) \quad v = \frac{\bar{\omega}}{\bar{k}} = \frac{390/s}{7.8/m} = 50 m/s$$

$$(c) \quad v_s = \frac{\Delta\omega}{\Delta k} = \frac{20/s}{0.4/m} = 50 m/s$$

(d) Successive zeros of the envelope requires that $0.2\Delta x/m = \pi$, thus

$$\Delta x = \frac{\pi}{0.2} = 5\pi m \text{ with } \Delta k = k_1 - k_2 = 0.4 m^{-1} \text{ and } \Delta x = \frac{2\pi}{\Delta k} = 5\pi m.$$

$$5-22. \quad (a) \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2 E_k}} = \frac{1240 eV \cdot nm}{[2(0.511 \times 10^6 eV)(5 eV)]^{1/2}} = 0.549 nm$$

$d \sin \theta = \lambda/2$ For first minimum (see Figure 5-17).

$$d = \frac{\lambda}{2 \sin \theta} = \frac{0.549 nm}{2 \sin 5^\circ} = 3.15 nm \text{ slit separation}$$

$$(b) \quad \sin 5^\circ = 0.5 cm / L \text{ where } L = \text{distance to detector plane } L = \frac{0.5 cm}{2 \sin 5^\circ} = 5.74 cm$$

- 5-23. (a) The particle is found with equal probability in any interval in a force-free region.

Therefore, the probability of finding the particle in any interval Δx is proportional to

Δx . Thus, the probability of finding the sphere *exactly* in the middle, i.e., with

$\Delta x = 0$ is zero.

- (b) The probability of finding the sphere somewhere within 24.9cm to 25.1cm is proportional to $\Delta x = 0.2\text{cm}$. Because there is a force free length $L = 48\text{cm}$ available

to the sphere and the probability of finding it somewhere in L is unity, then the

probability that it will be found in $\Delta x = 0.2\text{cm}$ between 24.9cm and 25.1cm (or any

interval of equal size) is: $P\Delta x = (1/48)(0.2\text{cm}) = 0.00417$.

- 5-24. Because the particle must be in the box $\int_0^L \psi^* \psi dx = 1 = \int_0^L A^2 \sin^2(\pi x/L) dx = 1$

Let $u = \pi x/L$; $x = 0 \rightarrow u = 0$; $x = L \rightarrow u = \pi$ and $dx = (L/\pi) du$, so we have

$$\int_0^\pi A^2 (L/\pi) \sin^2 u du = A^2 (L/\pi) \int_0^\pi \sin^2 u du = 1$$

$$(L/\pi) A^2 \int_0^\pi \sin^2 u du = (L/\pi) A^2 \left[\frac{u}{2} - \frac{\sin 2u}{4} \right]_0^\pi = (L/\pi) A^2 (\pi/2) = (LA^2)/2 = 1$$

$$\therefore A^2 = 2/L \rightarrow A = (2/L)^{1/2}$$

- 5-25. (a) At $x = 0$: $Pdx = |\psi(0,0)|^2 dx = |Ae^0|^2 dx = A^2 dx$

(b) At $x = \sigma$: $Pdx = |Ae^{-\sigma^2/4\sigma^2}|^2 dx = |Ae^{-1/4}|^2 dx = 0.61A^2 dx$

(c) At $x = 2\sigma$: $Pdx = |Ae^{-4\sigma^2/4\sigma^2}|^2 dx = |Ae^{-1}|^2 dx = 0.14A^2 dx$

- (d) The electron will most likely be found at $x = 0$, where Pdx is largest.

$$5-27. \quad \Delta E \Delta t \approx \hbar \rightarrow \Delta E \approx \hbar / \Delta t = \frac{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-7} \text{ s} (1.609 \times 10^{-19} \text{ J/eV})} \approx 6.6 \times 10^{-9} \text{ eV}$$

5-35. The size of the object needs to be of the order of the wavelength of the 10MeV neutron.

$\lambda = h / p = h / \gamma mu$. γ and u are found from:

$$E_k = m_n c^2 (\gamma - 1) \text{ or } \gamma - 1 = 10 \text{ MeV} / 939 \text{ MeV}$$

$$\gamma = 1 + 10/939 = 1.0106 = 1 / (1 - u^2 / c^2)^{1/2} \text{ or } u = 0.14c$$

$$\text{Then, } \lambda = \frac{h}{\gamma mu} = \frac{hc}{[\gamma mc^2 (u/c)]} = \frac{1240 \text{ eV}\cdot\text{nm}}{[(1.0106)(939 \times 10^6 \text{ eV})(0.14)]} = 9.33 \text{ fm}$$

Nuclei are of this order of size and could be used to show the wave character of 10MeV neutrons.

$$5-39. \quad \Delta E \Delta t \approx \frac{\hbar}{2} \rightarrow \Delta t = \frac{\hbar}{2\Delta E}$$

$$\Delta t = \frac{6.58 \times 10^{-16} \text{ eV}\cdot\text{s}}{2 \times 250 \times 10^6 \text{ eV}} = 1.32 \times 10^{-24} \text{ s}$$

5-40 (a) For a proton or neutron:

$$\Delta x \Delta p \approx \frac{\hbar}{2} \text{ and } \Delta p = m \Delta v \text{ assuming the particle speed to be non-relativistic.}$$

$$\Delta v = \frac{\hbar}{2m\Delta x} = \frac{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}{2(1.67 \times 10^{-27} \text{ kg})(10^{-15} \text{ m})} = 3.16 \times 10^7 \text{ m/s} \approx 0.1c \text{ (non-}$$

relativistic)

$$(b) \quad E_k \approx \frac{1}{2} m v^2 = \frac{(1.67 \times 10^{-27} \text{ kg})(3.16 \times 10^7 \text{ m/s})^2}{2} = 8.34 \times 10^{-13} \text{ J} = 5.21 \text{ MeV}$$

(c) Given the proton or neutron velocity in (a), we expect the electron to be relativistic,

$$\text{in which case, } E_k = mc^2 (\gamma - 1) \text{ and}$$

$$\Delta p = \frac{\hbar}{2\Delta x} \approx \gamma m v \quad \rightarrow \quad \gamma v \approx \frac{\hbar}{2m\Delta x}$$

For the relativistic electron we assume $v \approx c$

$$\gamma \approx \frac{\hbar}{2mc\Delta x} = \frac{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}{2(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})(10^{-15} \text{ m})} = 193$$

$$E_k = mc^2(\gamma - 1) = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2(192) = 1.58 \times 10^{-11} \text{ J} = 98 \text{ MeV}$$