PHYS 2D
DISCUSSION SECTION

2012/4/18

- Email me topics/questions you'd like to discuss
- Problem section: Ch.2
- Specific problems?
- Pick up quiz 1 today & at problem section

Topics for Today

- Relativistic Momentum/Energy
- Black-Body Radiation

Relativistic Momentum

- Definition: relativistic momentum $$\vec{\mathbf{p}} \equiv \frac{m\vec{\mathbf{u}}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m\vec{\mathbf{u}}$$
- m: mass of object at rest, **u**: velocity of object
- Reason:

1. We want the same laws of physics in all frames
2. Old form (**p**=m**u**) is not conserved when we Lorentz transform **u**(change of inertial frames)
3. **p**=γm**u** gives energy & momentum conservation under Lorentz transformation
4. Reverts back to classical form when u<<c

Force & Kinetic Energy

Finding relativistic energy from relativistic p:

- Newton's 2nd Law: $\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt = ma(1-u^2/c^2)^{-3/2}$
- Kinetic Energy K:

  Work done in accelerating from velocity 0 to u

- $$\text{Work} = \int_{x1}^{x2} \mathbf{F}\cdot d\mathbf{x} = K = \int_{x_1}^{x_2} \frac{m\,\frac{du}{dt}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\,dx \qquad dx = u\,dt$$

$$= \int_{t_1}^{t_2} \frac{m\,\frac{du}{dt}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\,u\,dt \qquad du = \frac{du}{dt}\,dt$$

$$= \int_0^u \frac{m\,u\,du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}} \qquad y = 1-\frac{u^2}{c^2},\ dy = -2\,\frac{u}{c^2}\,du$$

$$= \int_1^{1-\frac{u^2}{c^2}} \frac{m\left(\frac{c^2}{-2}\right)dy}{y^{3/2}} = \left[y^{-\frac{1}{2}}\,mc^2\right]_1^{1-\frac{u^2}{c^2}}$$

$$= \gamma mc^2 - mc^2$$

Relativistic Energy

- From $p=\gamma mu$, we get $K = \gamma mc^2 - mc^2$
- Define total relativistic energy $E \equiv K + mc^2 = \gamma mc^2$
- When object is at rest, $\gamma=1$, $E=mc^2$
- Rest mass m is a measure of the object's energy at rest, which is the total internal energy

  Ex. Heating up an object increases its rest mass

  Ex. A compressed spring weighs more

Energy Conservation

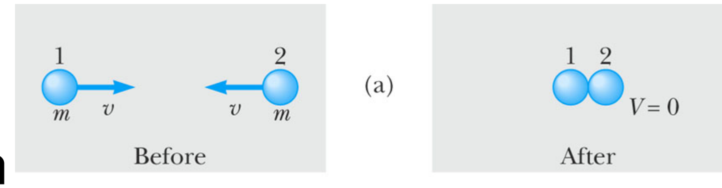
Q: Why these definitions? $p=\gamma mu$, $E=\gamma mc^2$

A: With these forms relativistic energy & momentum are conserved in all frames, and we get classical results when $u<<c$

- $\sum_i E_i^{before} = \sum_i E_i^{after}$, $\sum_i P_i^{before} = \sum_i P_i^{after}$, i: all objects involved

Mass Energy Interchange

- Energy is converted into rest mass in an inelastic collision



- Classically kinetic energy is lost (turned into heat)
- Relativistic energy accounts for everything and is conserved, so

$$\gamma_1 mc^2 + \gamma_2 mc^2 = \gamma_3 Mc^2,\ \gamma_1 = \gamma_2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},\ \gamma_3 = 1$$

- Rest mass of composite: $M = 2\gamma_1 m > 2m$
- M includes rest mass of 1 & 2 & internal energy (heat)
- Kinetic energy converted into rest mass (as heat)

Black-Body Radiation

- Significance: Led to discovery of quantum mechanics
- Blackbody: Anything that absorbs all incident light
- All blackbodies at the same temperature T give out the same spectrum, regardless of material/surface
- Implies something about light itself, independent of the body

Blackbody

- Blackbody example: the small hole (itself) in a cavity

Why?

1. Light entering through hole almost never comes out
2. Light keeps bouncing inside cavity, so it will be absorbed completely with enough time

- Incident light is partially absorbed every time it hits the wall, while the wall emits a certain spectrum
- Light inside cavity keeps interacting with wall and redistribute its frequency (energy) until both reach a state of thermal equilibrium
- Distribution of energy=spectrum

Black-body Radiation

- Thermal equilibrium is characterized by temperature T
- Light leaking out through the hole is a portion of the light inside, and is by definition black-body radiation
- So spectrum of light inside cavity and exiting through hole is the blackbody spectrum, depending only on T
- Spectrum can be calculated with statistical mechanics

- Classical statistical mechanics gives wrong spectrum
- Planck's quantization of light gives correct spectrum
- Light is quantized
- The start of quantum mechanics

- Questions?