Do the following problems from Shutz Chapter 5: 17, 21

Do the following problems from Shutz Chapter 6: 5, 7

1. Prove the following identities:
(a) $g_{\alpha\beta,\gamma} = \Gamma_{\alpha\beta\gamma} + \Gamma_{\beta\alpha\gamma}$.
(b) $g_{\alpha\mu}g^{\mu\beta}_{,\gamma} = -g^{\mu\beta}g_{\alpha\mu,\gamma}$
(c) $g^{\alpha\beta}_{,\gamma} = -\Gamma^{\alpha}_{\mu\gamma}g^{\mu\beta} - \Gamma^{\beta}_{\mu\gamma}g^{\mu\alpha}$.

2. Write the following expressions in index free notation (e.g. $A^{\alpha}B_{\alpha}$ written as $A \cdot B$)
(a) $U_{\alpha;\beta}U^{\beta}U^{\alpha}$
(b) $V^{\alpha}_{;\beta}U^{\beta} - U^{\alpha}_{;\beta}V^{\beta}$
(c) $T_{\alpha\beta;\gamma}V^{\alpha}W^{\beta}U^{\gamma}$