

Do the following problems from Shutz Chapter 3: 3,7,9,10,24,30

1. Let  $F^{\alpha\beta}$  be an antisymmetric tensor. Show that

$$F_{\mu}{}^{\alpha}{}_{,\nu} F^{\nu}{}_{\alpha} = -F_{\mu\alpha,\beta} F^{\alpha\beta}$$

2. Is the determinant of the metric tensor,  $g \equiv \det(g_{\mu\nu})$  a scalar, i.e. is it the same in all frames? Check for both the ordinary Lorentz transformation, and the more general coordinate transformation we discussed:

$$\Lambda_{\bar{\alpha}}^{\beta} = \frac{\partial x^{\beta}}{\partial \bar{x}^{\alpha}}.$$

3. A two indexed “object”  $X^{\mu\nu}$  is defined as the “direct sum” of two vectors:  $X^{\mu\nu} = A^{\mu} + B^{\nu}$ . Is  $X^{\mu\nu}$  a tensor? Is there a transformation law to take  $X$  from coordinate frame  $\mathcal{O}$  to a new coordinate frame  $\bar{\mathcal{O}}$ ? i.e. obtain  $X^{\bar{\mu}\bar{\nu}}$ ?