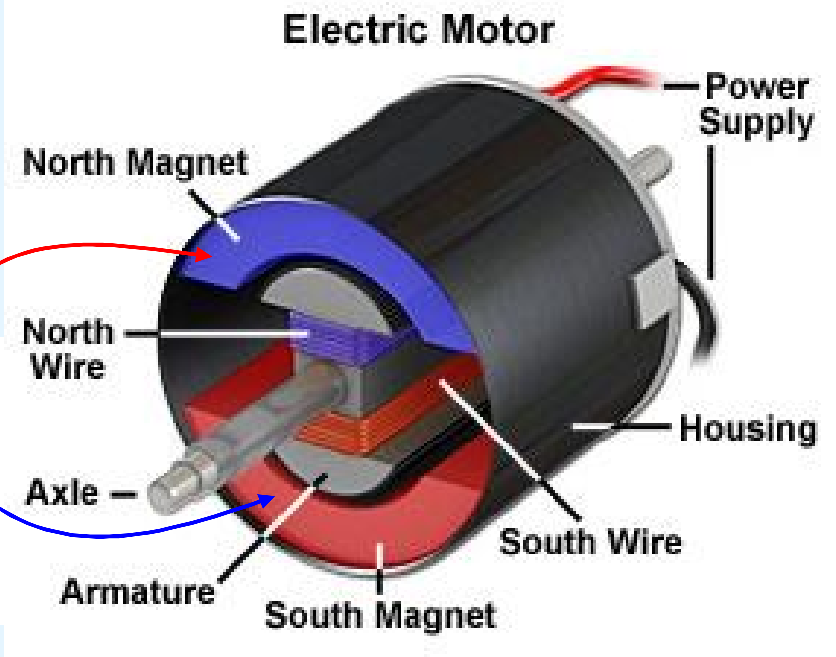
Physics 1B

Part II: Magnetism

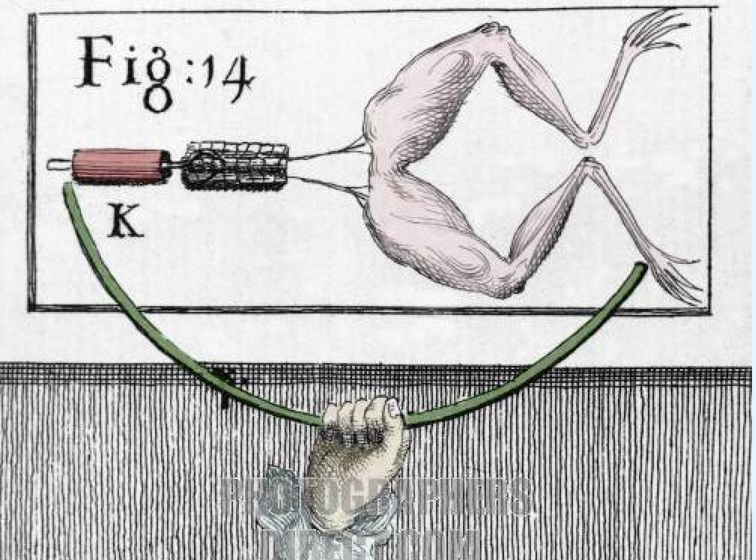
colors reversed (color code varies)



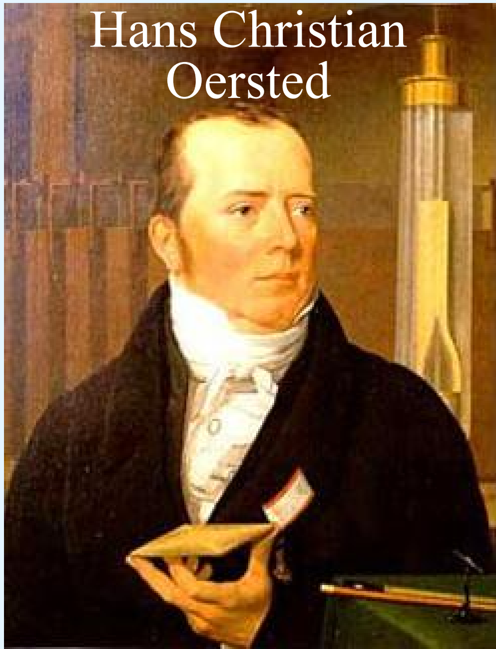
We start with the macroscopic


Michael Faraday

- What did historical people observe?
  - How do magnets behave?
  - Is electricity related to magnetism?
    - If so, how?



- Then we proceed to the microscopic
  - How do particles behave?
  - Lorentz magnetic force

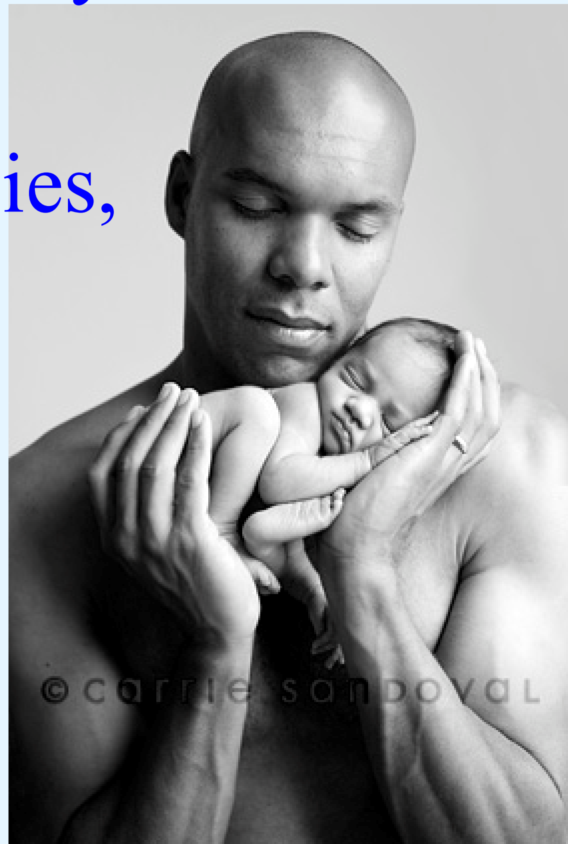

Hans Christian Oersted

The nature of research

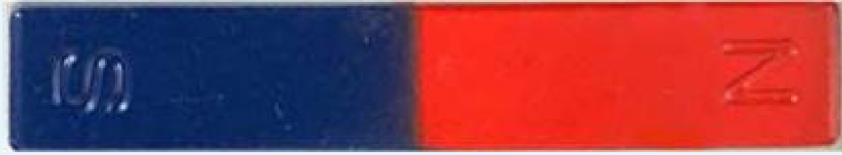
"But Mr. Faraday, of what use is all this?"
- unknown woman

"Madam, of what use is a newborn baby?"
- Michael Faraday

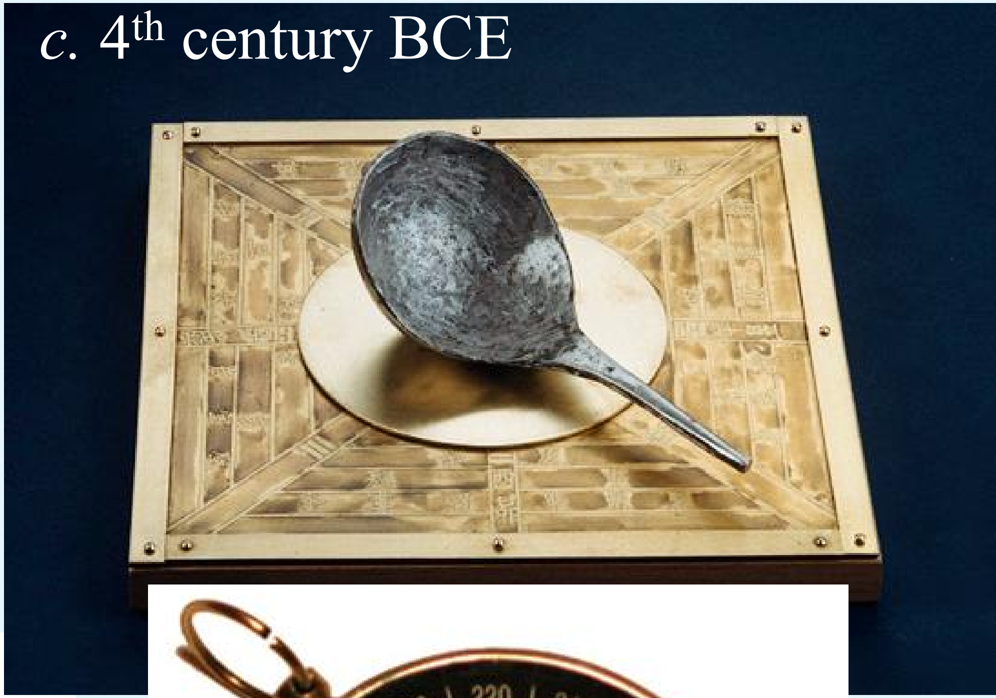
"With electromagnetism, as with babies, it's all a matter of potential."
- Bill Nye, the Science Guy



Compass



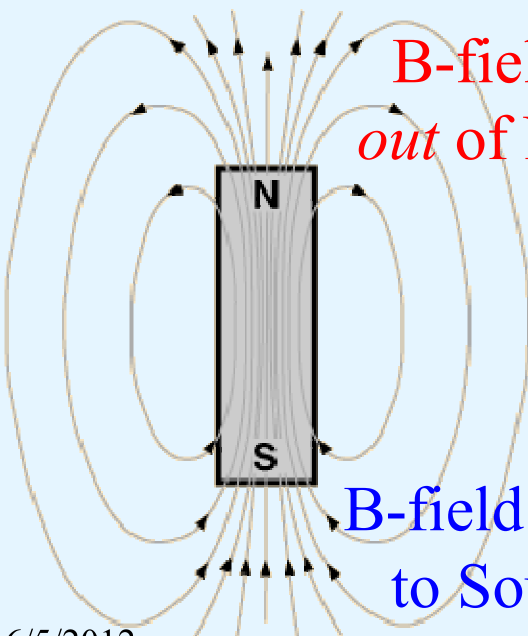
- Two thousand years ago:
  - Hang lodestone from string: it point north
  - Magic!
  - But useful



c. 4th century BCE

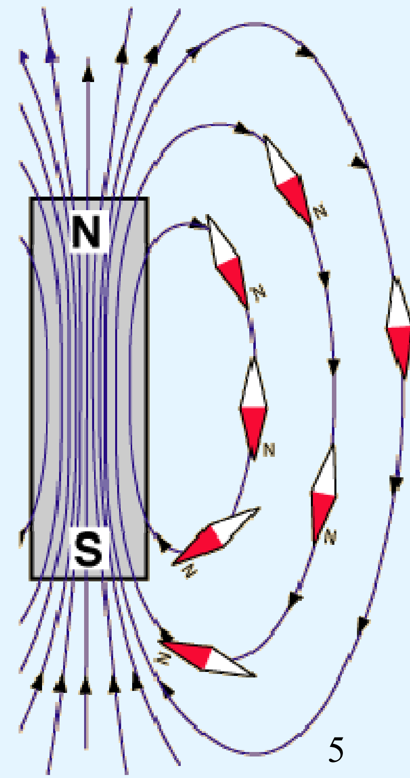
Magnets reinforce each other

- Magnets align to create a *stronger* field
  - Magnets move to *increase* B-field
- This is opposite of electric dipoles
  - Charges move to *reduce* E-field

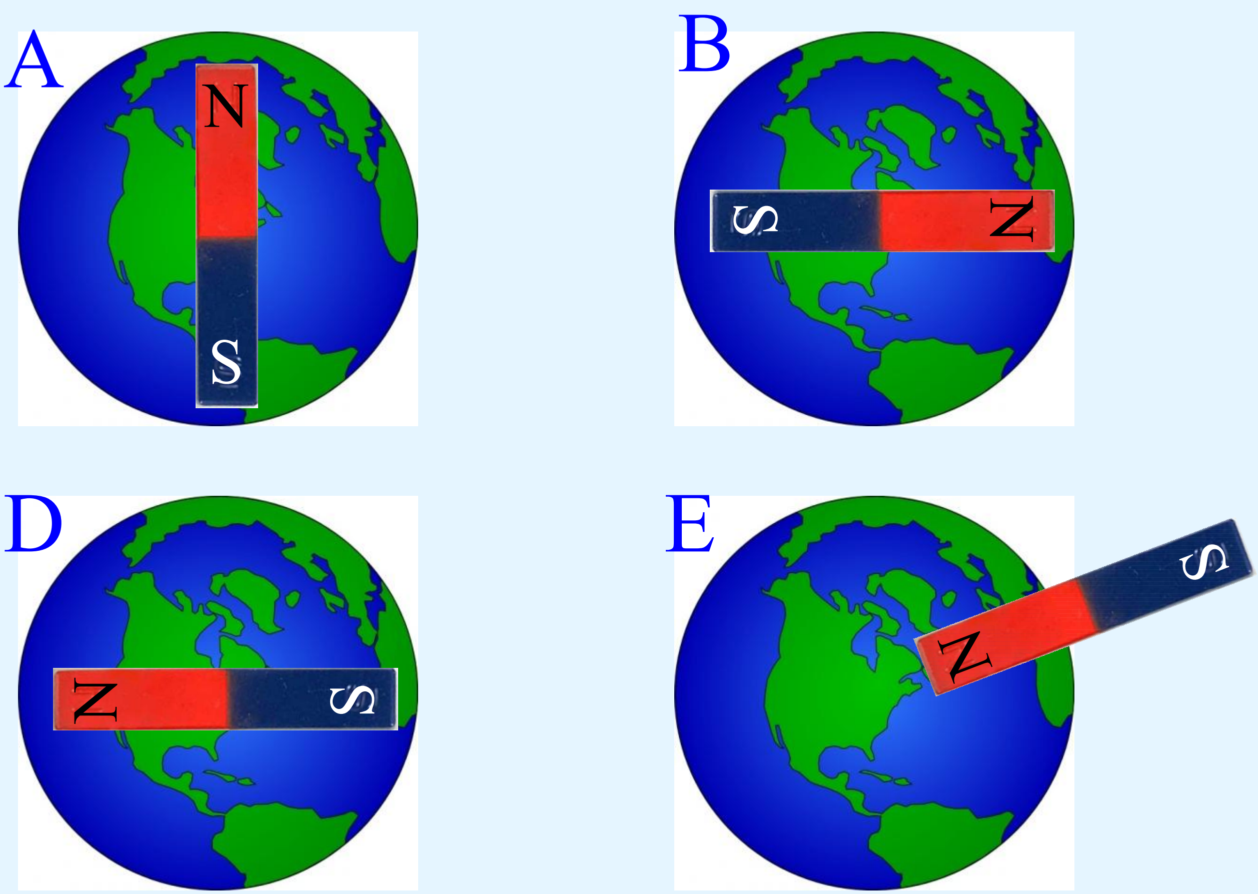


B-field points *out* of North end

B-field points *in* to South end



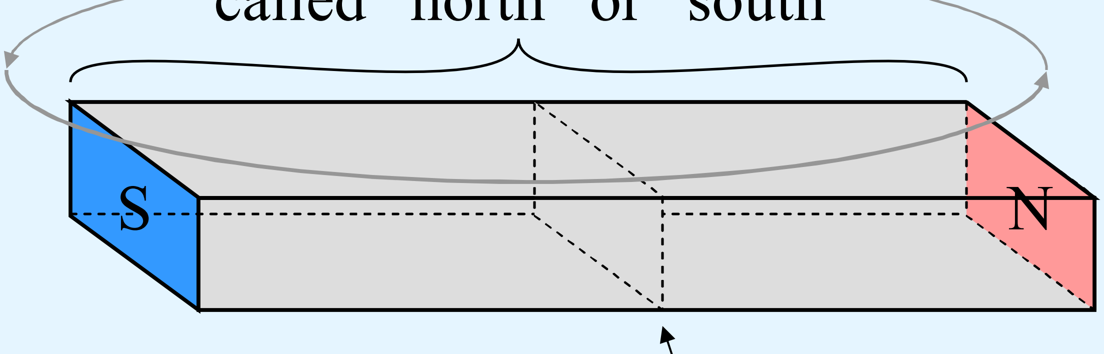
Where is the Earth's magnet?



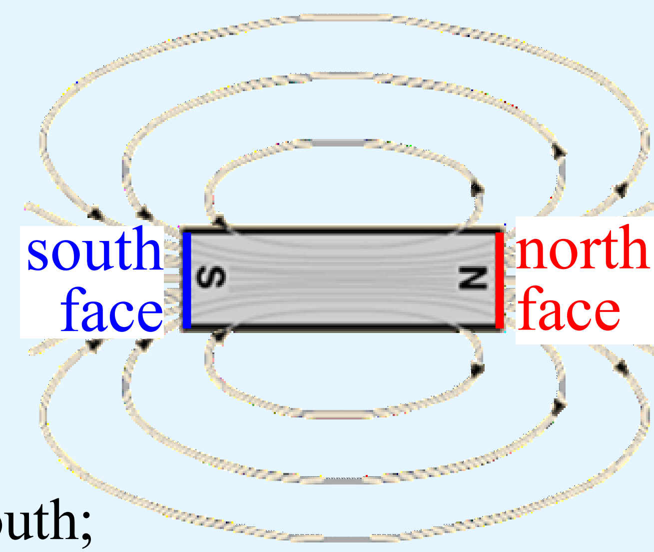
The north face

- Bulk materials are neither "north" or "south"
- Only faces are
  - **faces** have magnetic lines of force piercing them
  - north faces attract south ends of compasses (B-field comes out)
  - south faces attract north ends of compasses (B-field goes in)

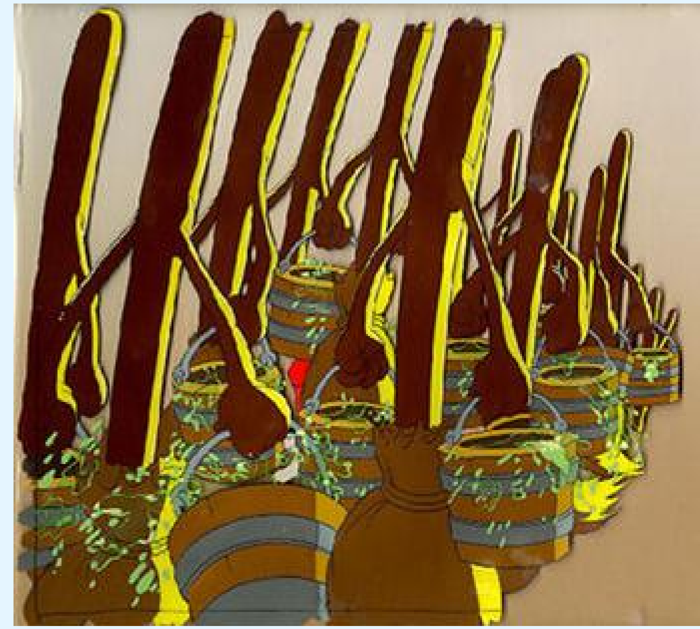
The bulk volume cannot be called "north" or "south"



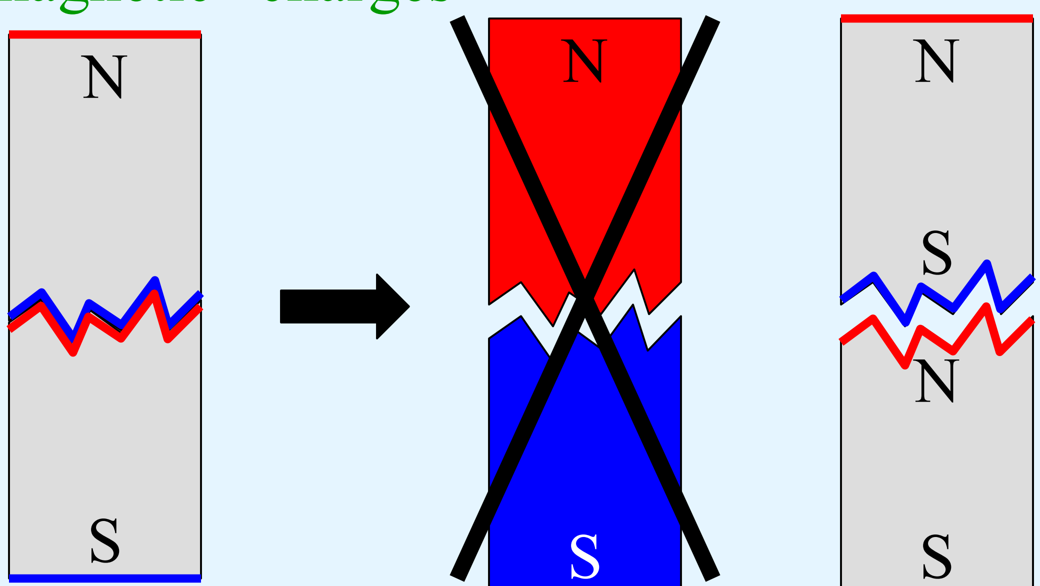
Before I break it, the face looking left is already south; the face looking right is already north



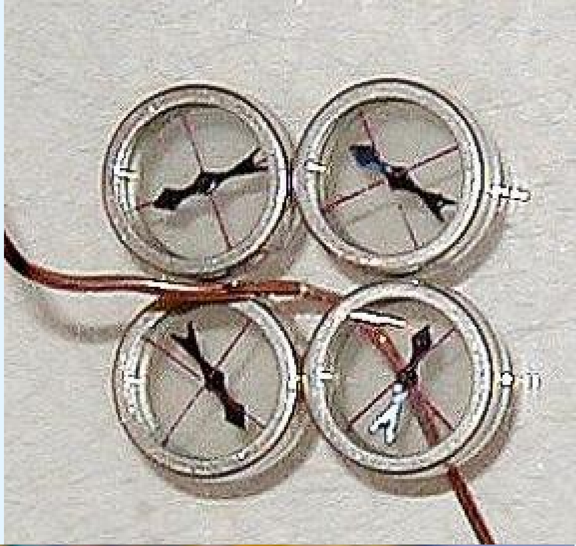
Dipolar disorder



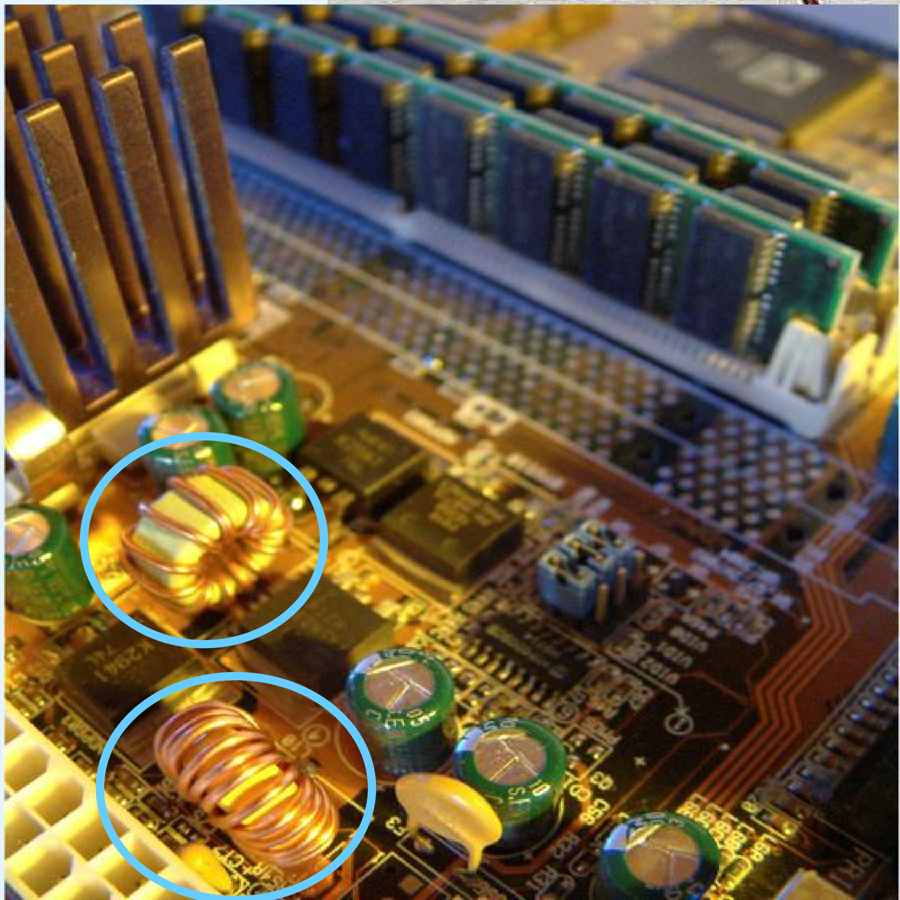
- Break an electric dipole in two …
  - You get one + and one -
  - Two monopoles
    - Only possible because lines *terminate*
- Break a magnet in two …
  - You get two *dipole* magnets
  - Magnetic lines *never* terminate
  - There are no magnetic "charges"



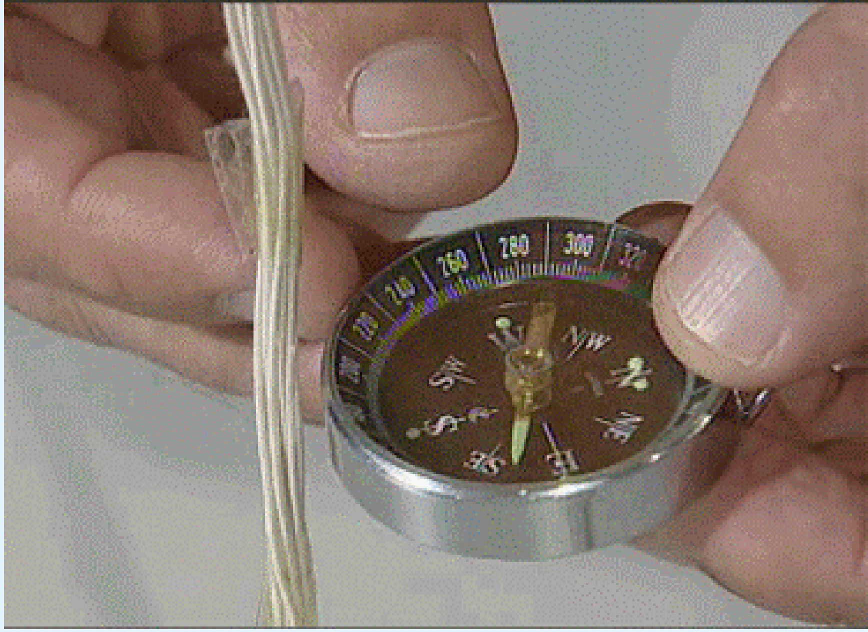
Electric charge doesn’t interact with magnets



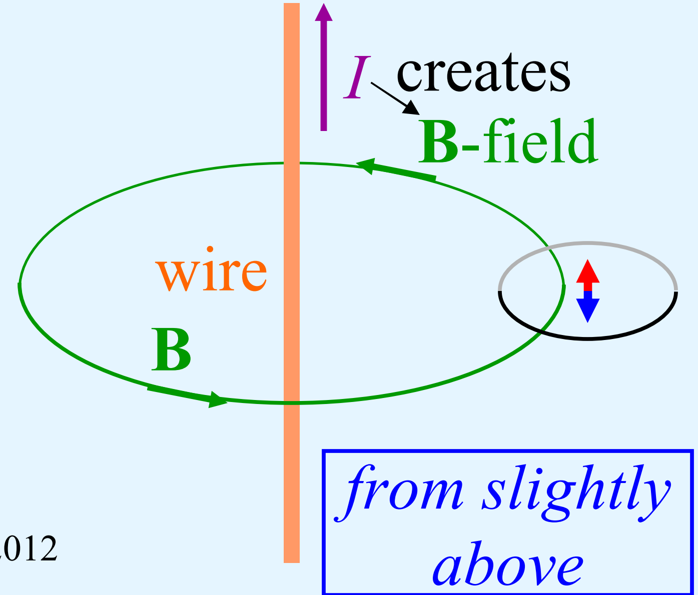
- But electric current does!
- There is a connection between electricity and magnetism

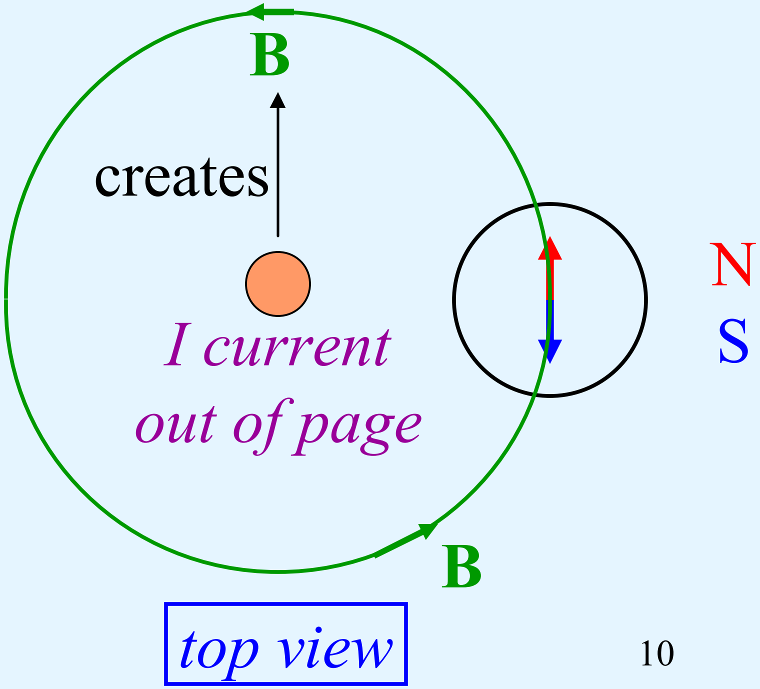


Electrical connection



- A connection between electricity and magnetism
- Compass points perpendicular to radius
  - Tangent to circle around wire





A current carrying wire lies in the plane of the compass. How does the needle respond?

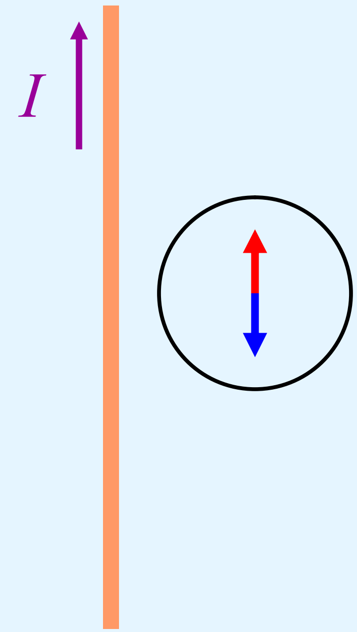
A nothing

B N points left

C N points down

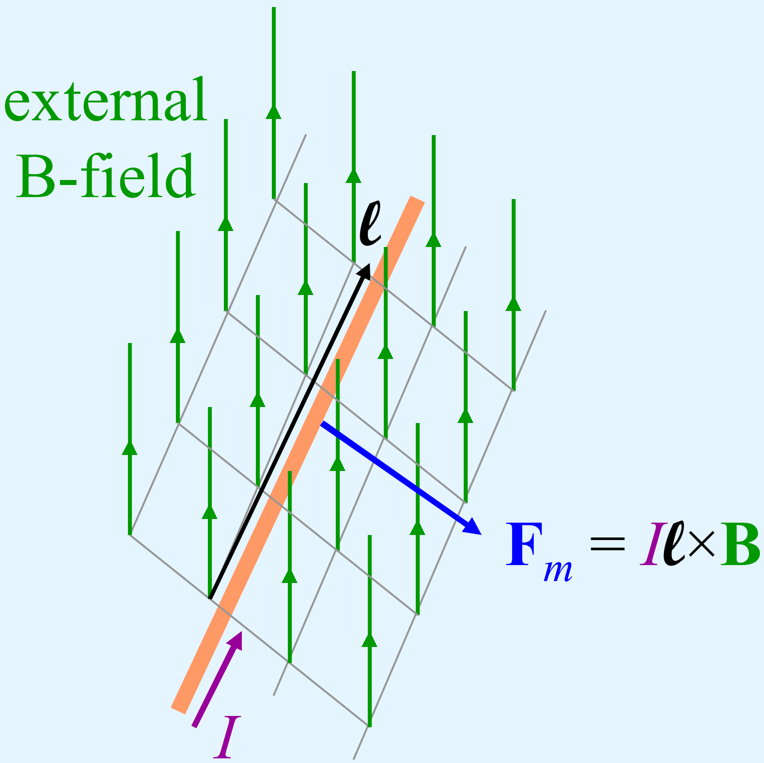
D N points right

E the compass explodes

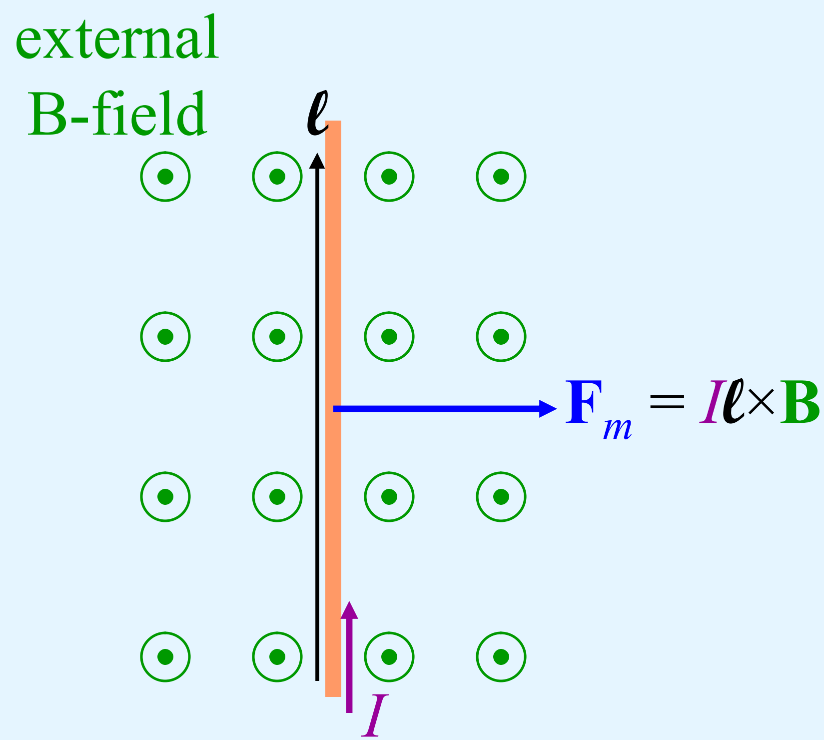


Force on a length of current

- *I* and $\boldsymbol{\ell}$ must have consistent signs



from slightly above



top view

Units of **B**

- $\mathbf{F}_m = I\boldsymbol{\ell}\times\mathbf{B}$ =>
  N = A-m[B] => [B] = N/(A-m) or **tesla**, T
  - Fundamentally, [B] = kg-m/s^2/(C/s-m) = kg/(C-s)
    - But we don't care about fundamental units here

What is the net effect of the B-field on the current loop?

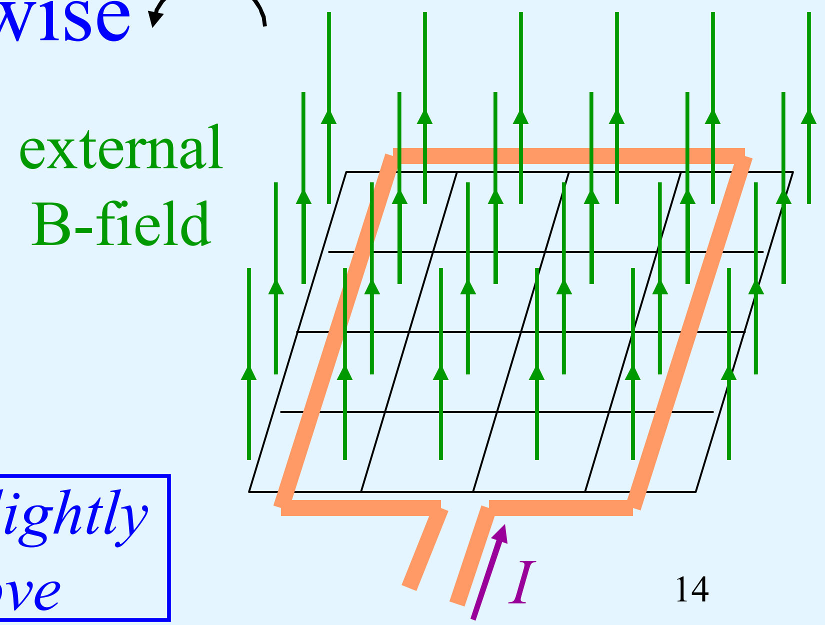
A net force up

B net force down

C net torque clockwise

D net torque counter-clockwise

E nothing



from slightly above

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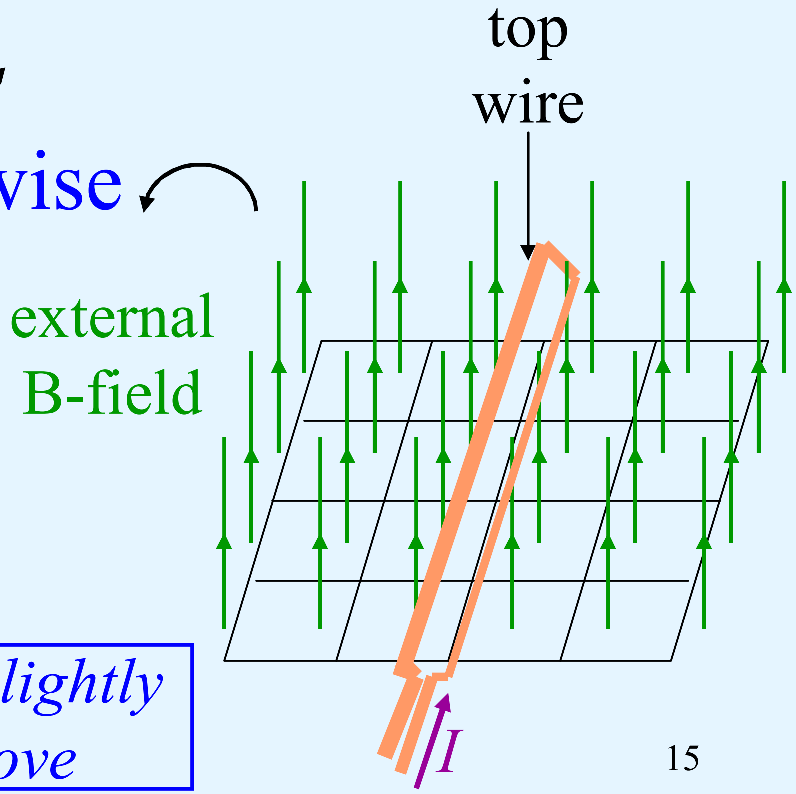


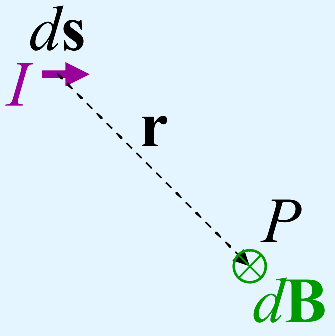
from slightly above

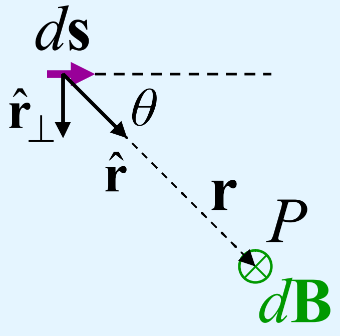
Magnets from electricity: Biot-Savart

- *Current* generates B-field
  - Voltage has no effect
  - Biot-Savart is the "Coulomb's Law" of electromagnets

$$d\mathbf{B}(\text{at } P) = k_m \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}, \qquad \Rightarrow \qquad \left|d\mathbf{B}\right| = k_m \frac{I\left|d\mathbf{s}\right|\left|\hat{\mathbf{r}}_\perp\right|}{r^2} = k_m \frac{I\left|d\mathbf{s}\right|\sin\theta}{r^2}$$

$$k_m \equiv 10^{-7} \text{ T-m/A}$$





What is the B-field from the given current element at *P*?

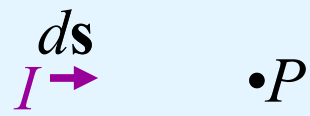
A zero

B into the page

C out of the page

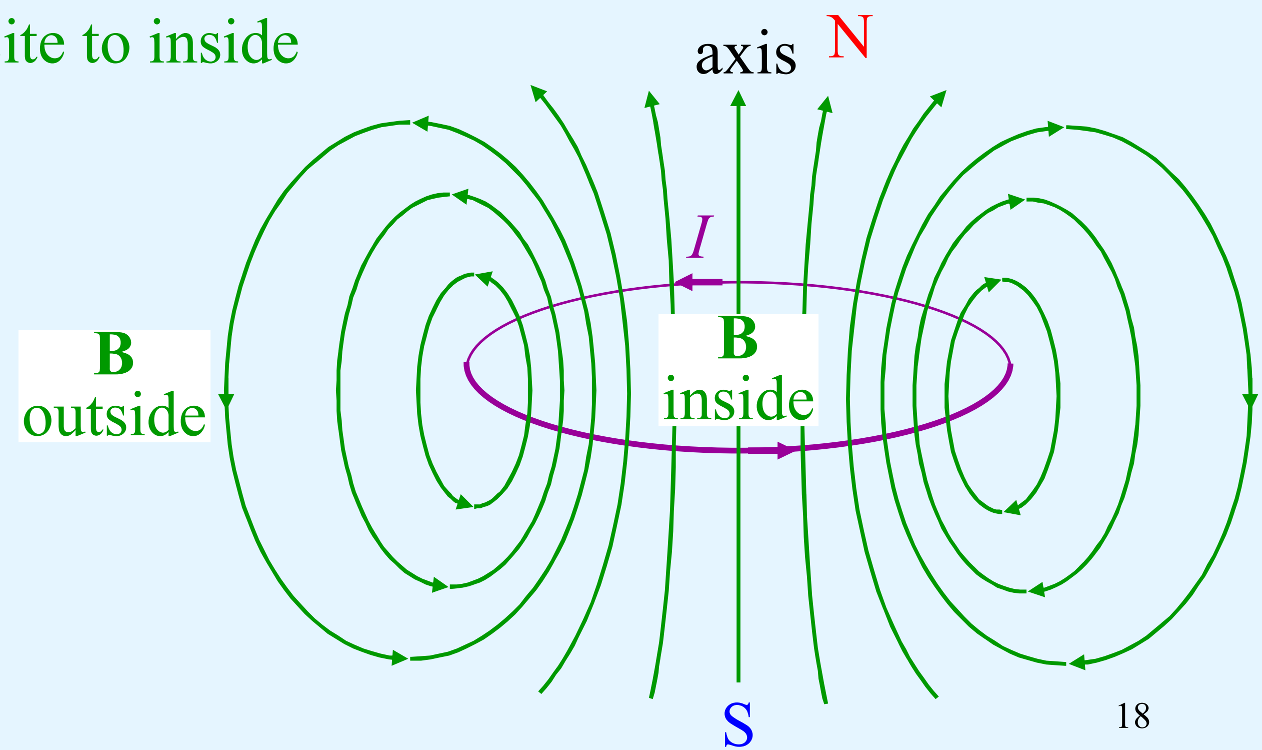
D up

E down



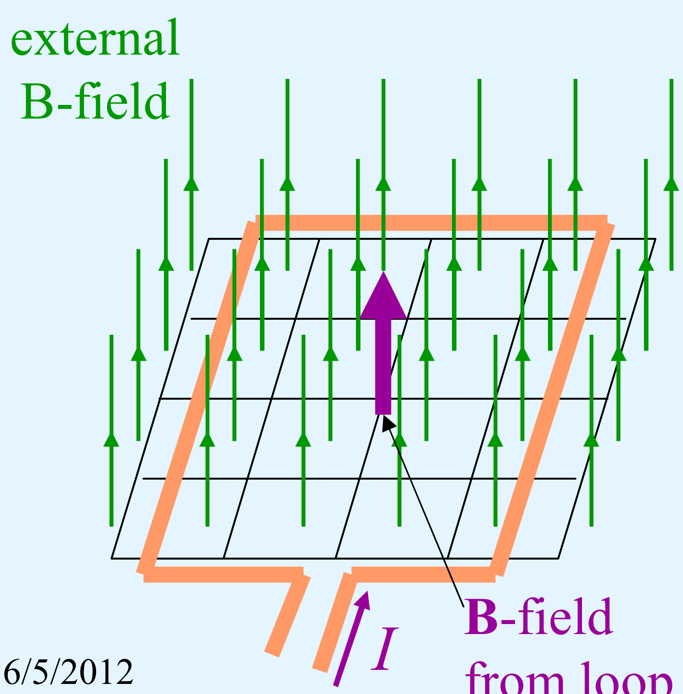
Creating a magnetic dipole: a current loop

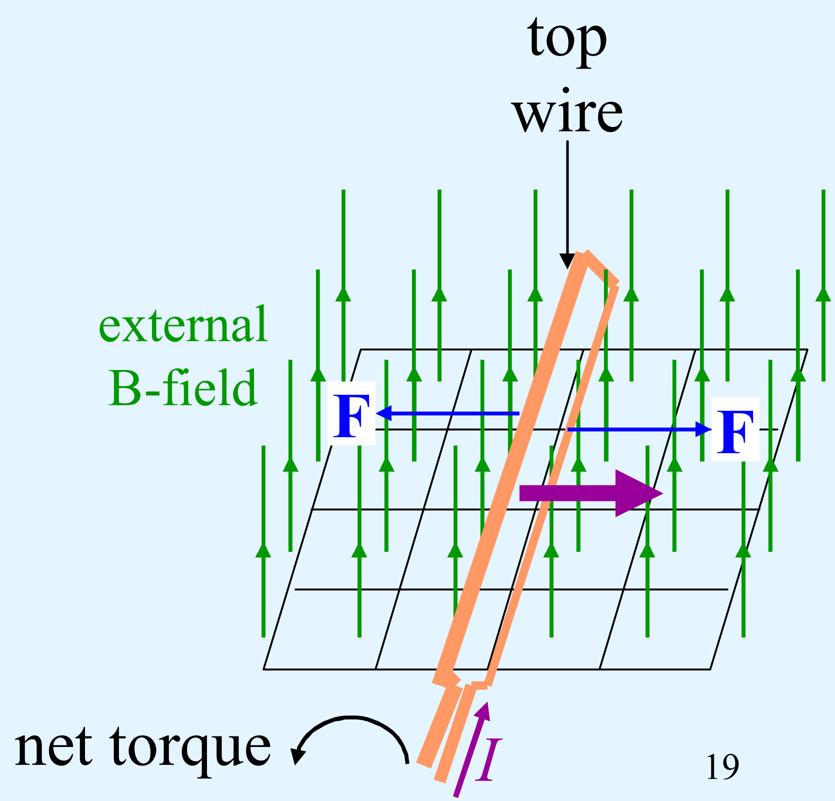
- Current flows in loops: creates a magnetic field
- Magnetic flux always forms closed loops
- **B**-field *inside* the loop follows the right-hand rule
  - Outside, in the equatorial plane, **B** points opposite to inside



Current loop in a B-field: redux

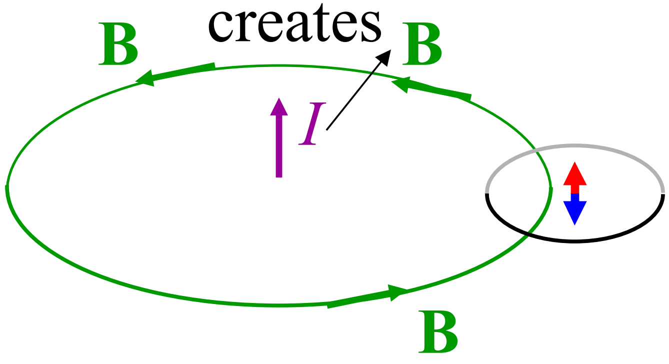
- How are the given B-field and that produced by the loop related?
  - Magnetic forces pull & twist to *increase* the magnetic field





Our 2.5 right-hand rules (RHRs)

RHR #1: **B** from current element



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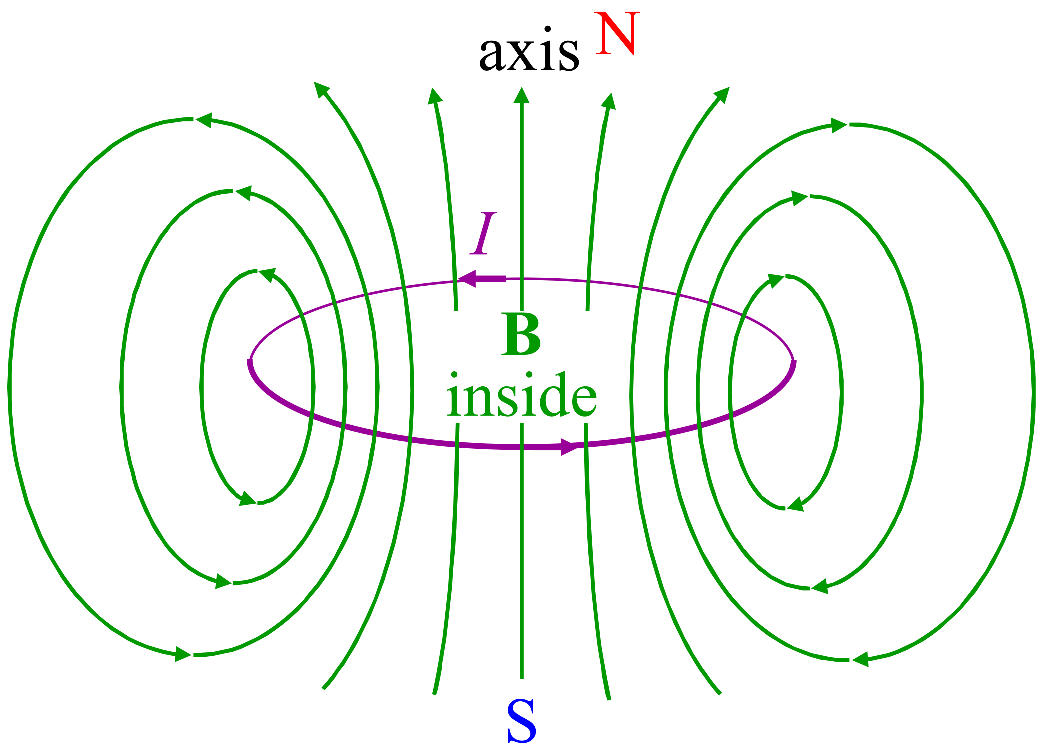
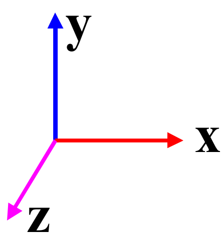
from #1 we can derive #3

RHR #3: **B** inside from a current loop



RHR #2: cross product

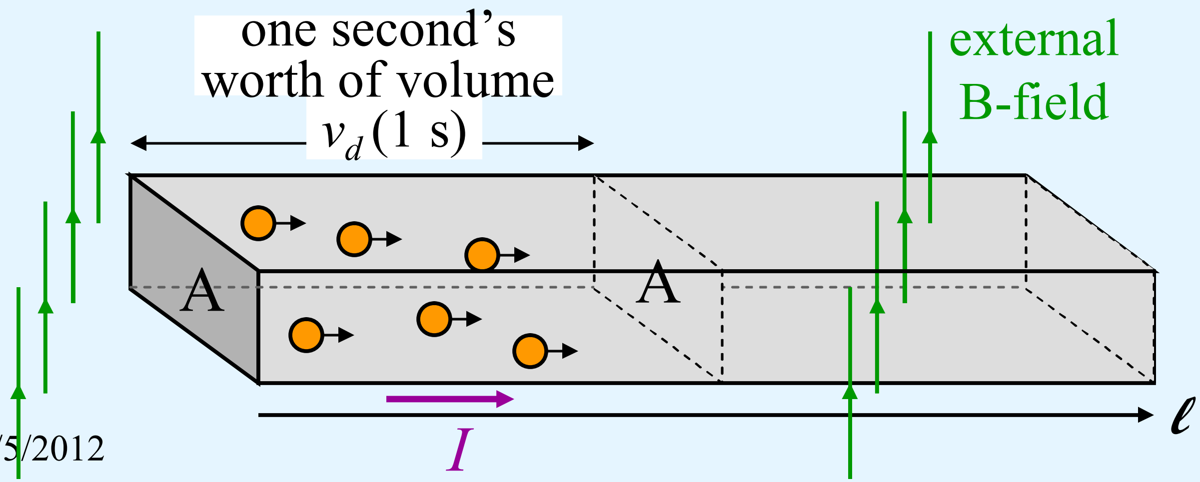


$$\mathbf{x} \times \mathbf{y} = \mathbf{z}$$

Particles

- Force of B-field on a conductor depends on *current* only, independent of the conductor
  - But different conductors have different mobile charge densities and average speeds: this tells us something
  - Consider 1-second's worth of mobile charge in a conductor
    - It's v_d m long
  - Total mobile charge in the volume is $Q = qn(v_d A)(1\text{ s})$
  - Current through conductor of area A is: $\boxed{I = \frac{Q}{t} = \frac{Q}{1\text{ s}} = qnv_d A}$
  - Magnetic force on any current is:

$$F_m = I\ell B = qnv_d A\ell B$$



Particles (2)

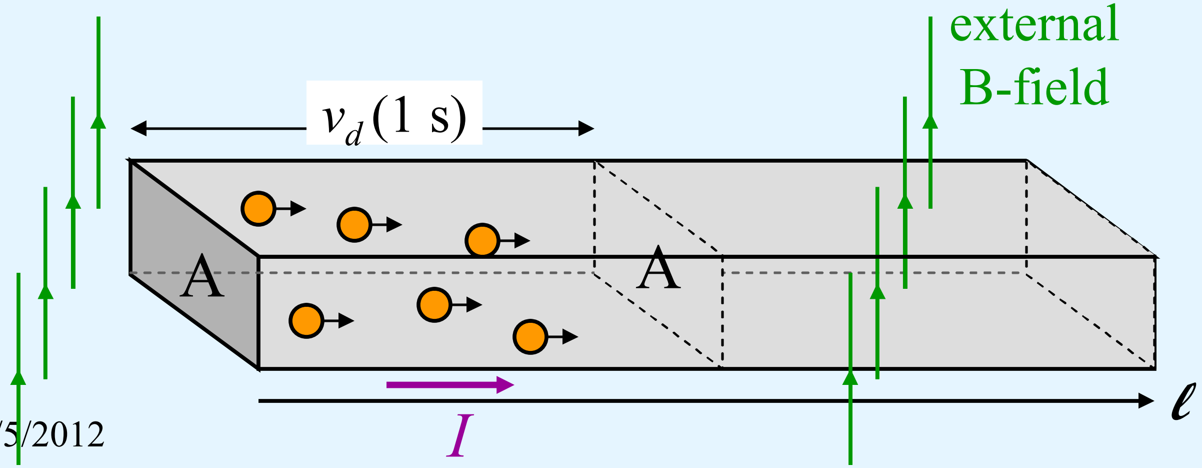
- Magnetic force on a wire is due to magnetic force on mobile charges (individual particles) in it:

$$F_m = I\ell B = qnv_d A\ell B \quad \Rightarrow \quad F_m = qv_d\left(nA\ell\right)B = qv_d NB$$

- Lorentz magnetic force on a single particle:

$$F_m = qvB \quad \Rightarrow \quad \mathbf{F}_m = q\mathbf{v}\times\mathbf{B}$$

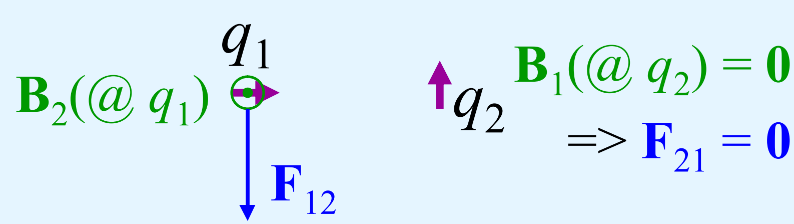
  - The *macroscopic* magnetic force tells us about the *microscopic* magnetic force
- Total electromagnetic (EM) force
  - Force is a vector: vectors add: $\mathbf{F}_{total} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = q\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)$



Newton's 3rd law?

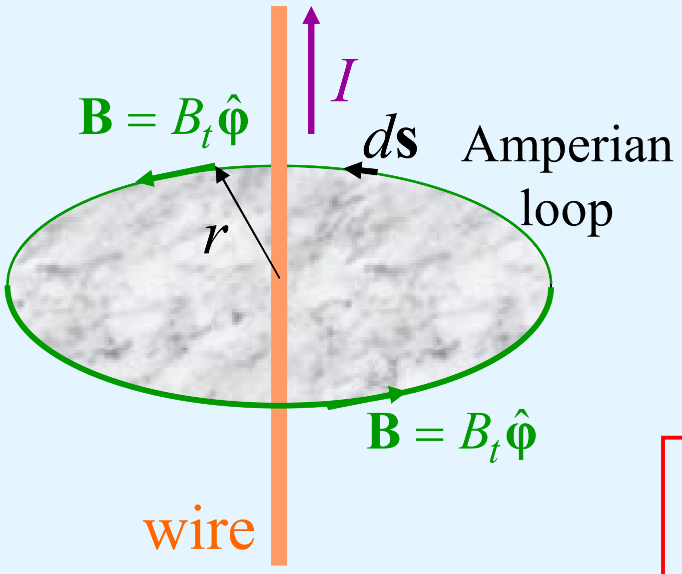
- "And thirdly, the code is more what you'd call 'guidelines' than actual rules."
  - Magnetic forces do *not* obey Newton's 3rd guideline
- But golly, professor, what of conservation of momentum?
  - Electromagnetic waves carry off the remaining momentum, and total momentum *is* conserved
  - Between isolated particles, Coulomb forces dominate
  - Magnetic forces are only significant at relativistic speeds

http://www.youtube.com/watch?v=bplEuBjppTw



Ampere's Law

- Symmetry simplifies the B-field from a current
  - For any 2D surface: $\oint_{around} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{through}$
  - Follows from Biot-Savart law
    - Similar to Gauss' Law for any volume: $\oiint_{surface} \mathbf{E} \bullet d\mathbf{A} = \frac{q_{in}}{\varepsilon}$



Example: B from a wire

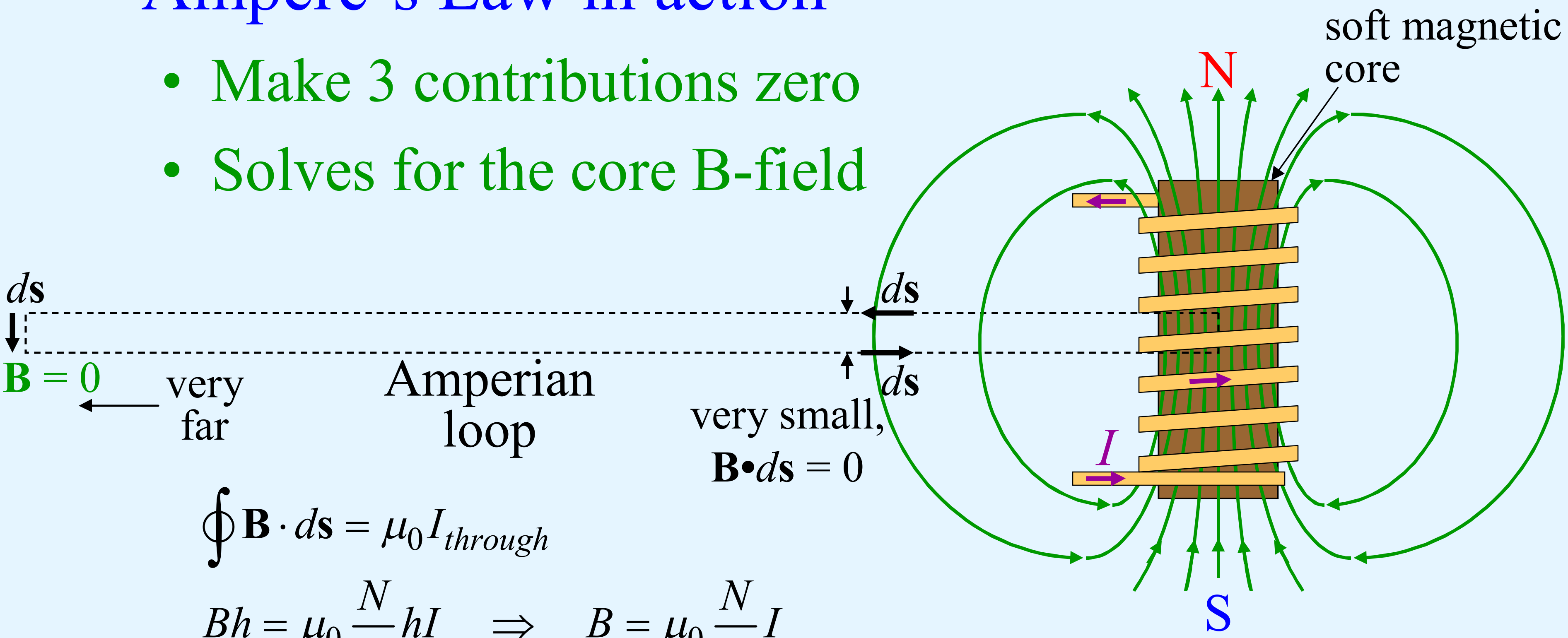
$$\oint \mathbf{B} \bullet d\mathbf{s} = B_t 2\pi r = \mu_0 I_{through}$$

$$B_t = \frac{\mu_0 I}{2\pi r}$$

Confusion over $d\mathbf{s}$ (should use $d\mathbf{r}$ in Ampere's):
In Biot-Savart, $d\mathbf{s}$ is length of current element.
In Ampere's Law, $d\mathbf{s}$ is displacement in space.

Solenoid: A better electromagnet

- Multiple turns increase B-field
- Permeable (e.g. iron) core increases B-field
- Ampere’s Law in action
  - Make 3 contributions zero
  - Solves for the core B-field

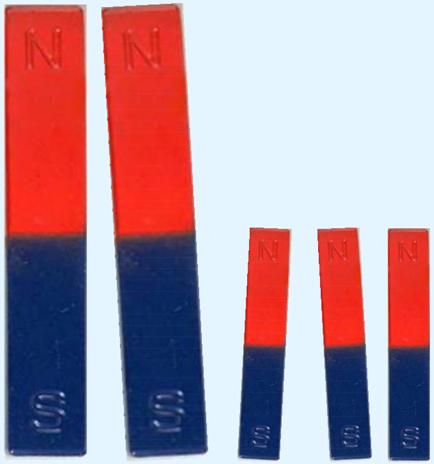


$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{through}$$

$$Bh = \mu_0 \frac{N}{l} hI \quad \Rightarrow \quad B = \mu_0 \frac{N}{l} I$$

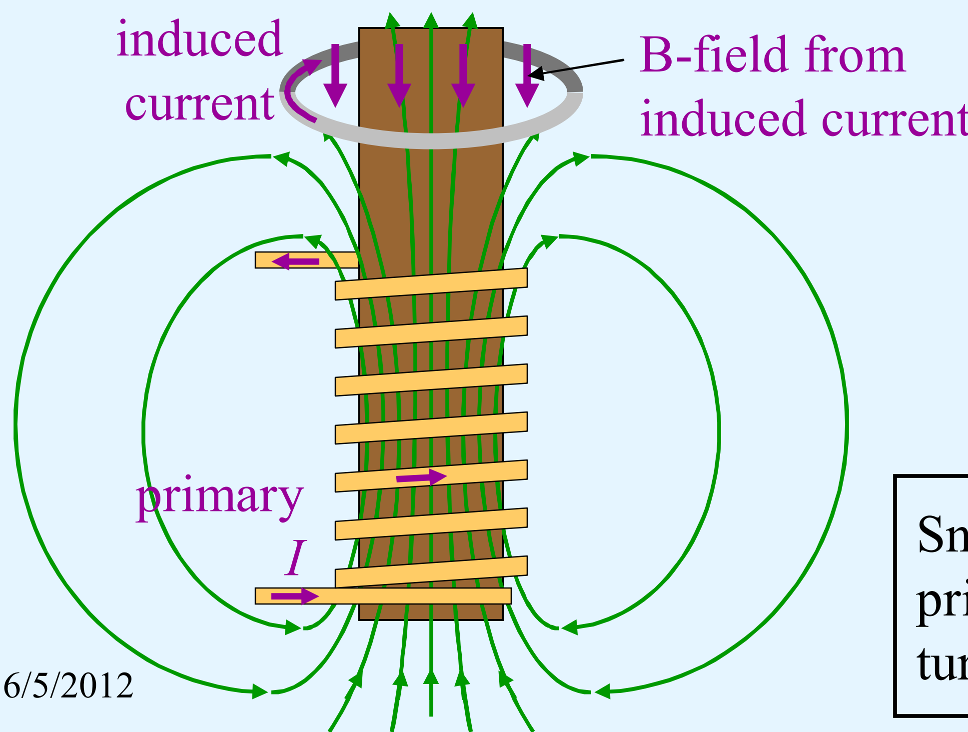
The magnetic facts of life: where do magnets come from?

- They come from currents
- But where do *permanent* magnets come from?
  - The stork brings them?
  - From microscopic currents in the magnet?
    - A teeny bit
  - From the intrinsic magnetic dipole moment of unpaired electrons



Induced magnetic fields *are not necessarily* induced to reinforce

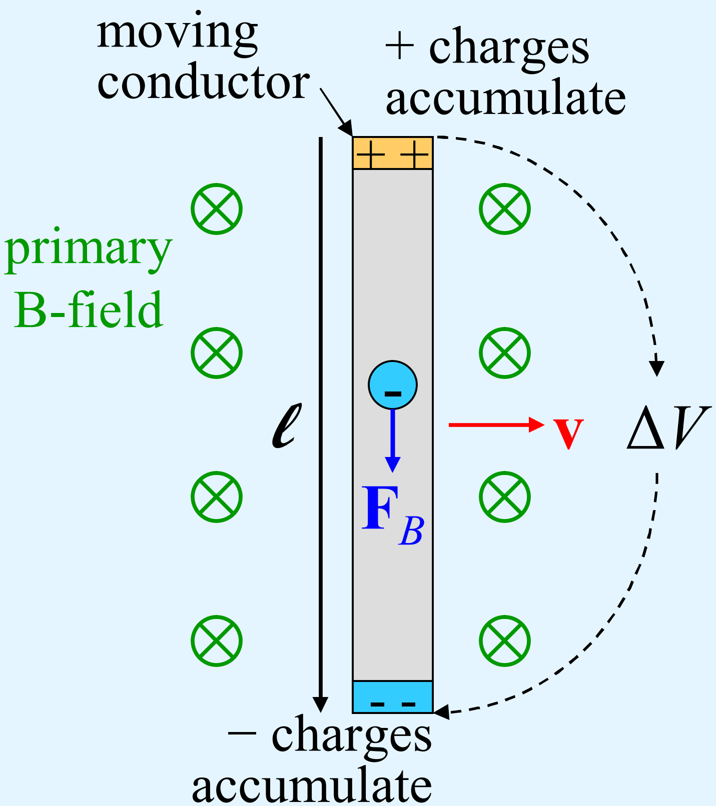
- The induced field *opposes the change* in the primary field
- *Then*, the resulting magnetic fields push and pull to reinforce as best they can
  - Or at least, to minimize cancellation



Snapshot when the primary current is first turned on (*increasing*)

Induced motional voltage (EMF) and current

- We quantify the induced voltage from our existing knowledge
  - The conducting bar moves to the right with velocity, **v**
  - We will return to B-fields and work later



$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}, \quad q<0$$

$$|\Delta U_e| = |q|vB\ell$$

$$|\Delta V| = \left|\frac{\Delta U_e}{q}\right| = vB\ell$$

The book calls this "electro-motive force", or emf: $\boldsymbol{\mathcal{E}}$

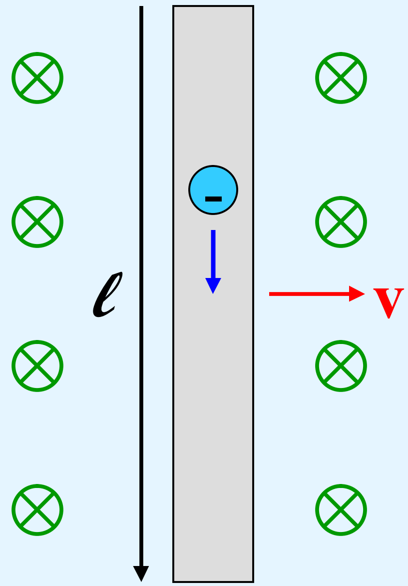
Before equilibrium, which way is the current?

A up

B down

C all around

D zero



$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}, \quad q<0$$

$$|\Delta U_e| = |q| vB\ell$$

$$|\Delta V| = \left|\frac{\Delta U_e}{q}\right| = vB\ell$$

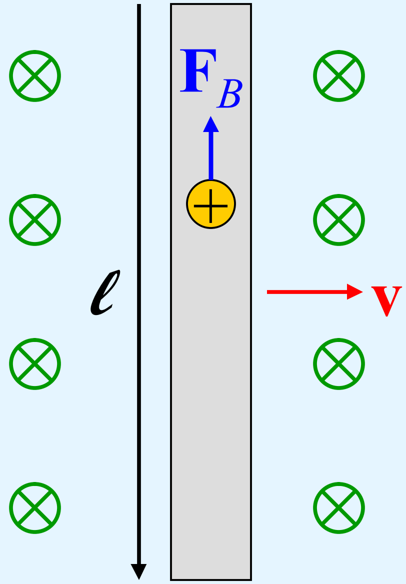
If the mobile charges were positive, then before equilibrium, which way would the current be?

A up

B down

C all around

D zero

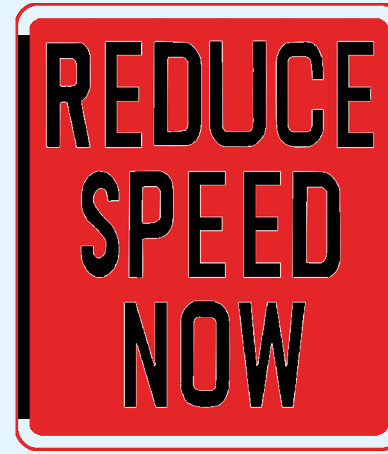


$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}, \quad q > 0$$

$$\left|\Delta U_e\right| = qvB\ell$$

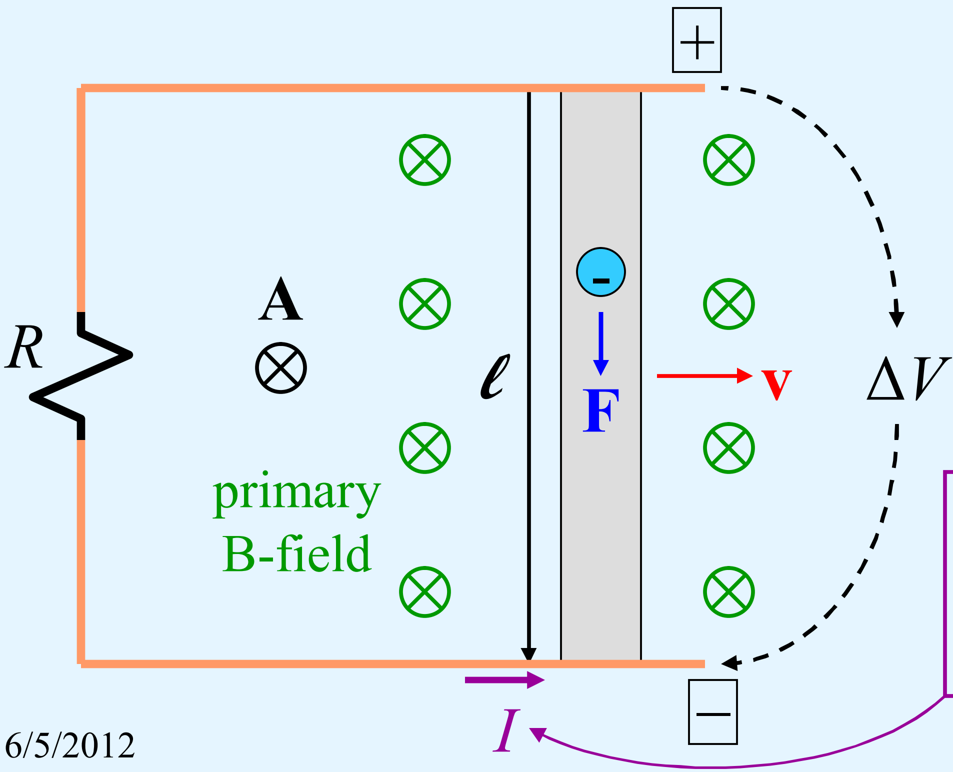
$$\left|\Delta V\right| = \left|\frac{\Delta U_e}{q}\right| = vB\ell$$

Faraday's Law, part 1



- Complete the circuit
- Write the voltage in terms of flux

$$\Phi_B \equiv \iint_{area} \mathbf{B} \bullet d\mathbf{A} \qquad \left(\text{recall: } \Phi_E \equiv \iint_{area} \mathbf{E} \bullet d\mathbf{A}\right)$$



$$\Delta V = -\frac{W_e}{q} = -vB\ell = -B\frac{dA}{dt}$$

$$= -\frac{d}{dt}\mathbf{B} \bullet \mathbf{A} = -\frac{d\Phi_B}{dt}$$

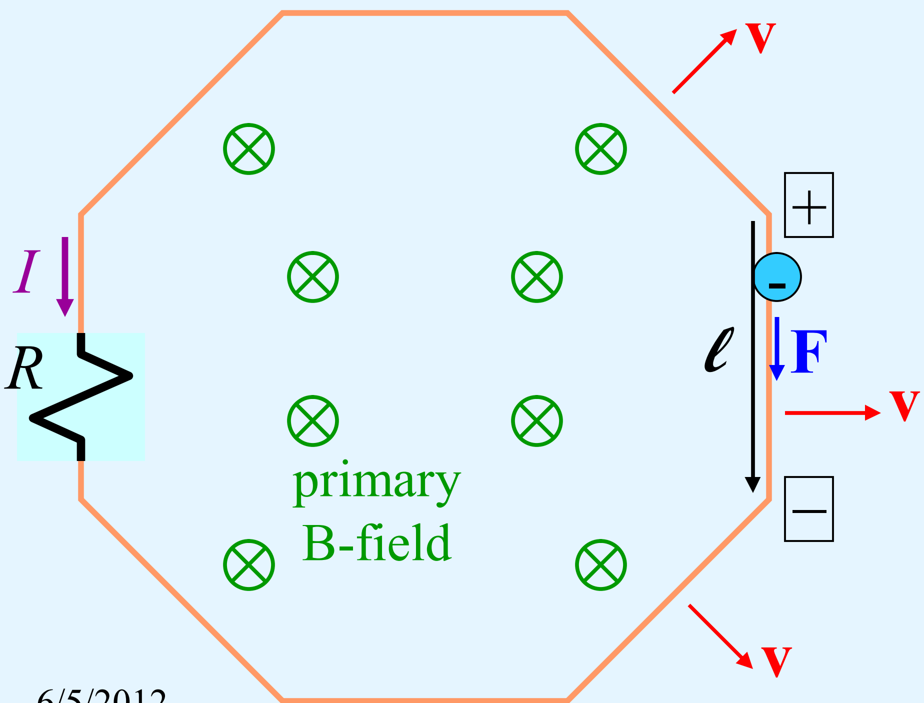
Lenz' Law: Induced current, *I*, creates secondary B-field which opposes the *change* in primary flux, Φ_B

Faraday's Law, the sequel

- If multiple edges move, voltages add

$$\Delta V = \sum_{segments} \Delta V_i = -\frac{d\Phi_B}{dt}$$

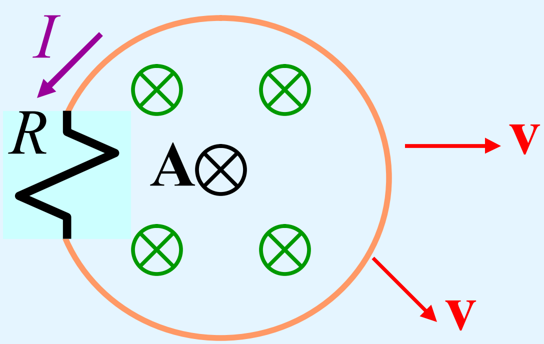
- An arbitrary shape is a sum of short segments:



Faraday's Law:

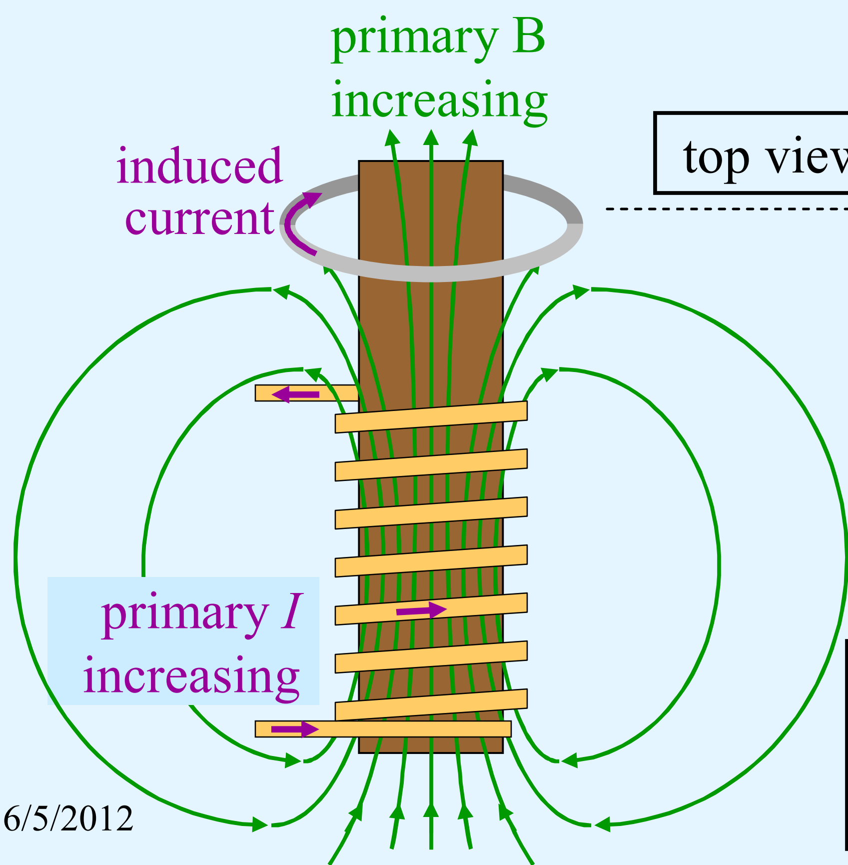
$$\Delta V_{loop,RHR} = -\frac{\partial \Phi_B}{\partial t}$$

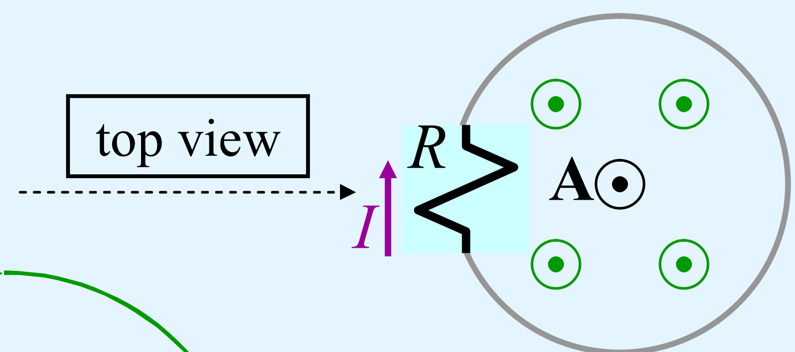
Lenz' Law: current opposes change in flux, Φ_B



Faraday's Law, part trois

- It still holds for stationary wires and changing B-field: Faraday's Law: $\Delta V_{loop,RHR} = -\dfrac{\partial \Phi_B}{\partial t}$
  - Cannot be derived from moving wires





Lenz' Law: current opposes change in flux, Φ_B

Snapshot when the primary current is first turned on (*increasing*)

Work and magnetic fields

- There's a subtlety:
  - Strictly speaking, magnetic fields do no work
    - Because the force is always perpendicular to the motion
- But motors are magnetic, and they certainly do work
  - Strictly speaking, magnetic fields create electric fields, which do work
- The net effect is that magnetic fields *indirectly* do work
  - I have ignored this indirection, and taken the results as work "done" by the magnetic field