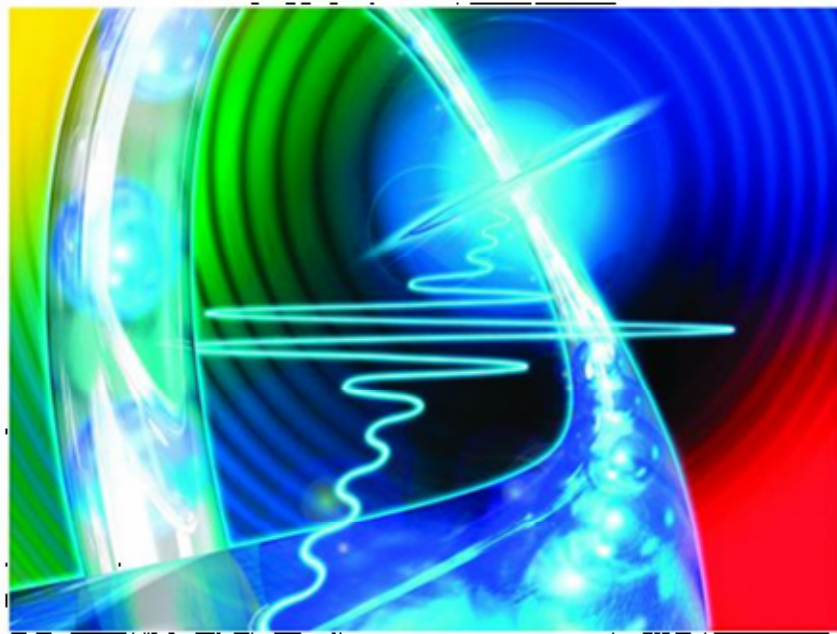


Physics 1B: Electricity and Magnetism



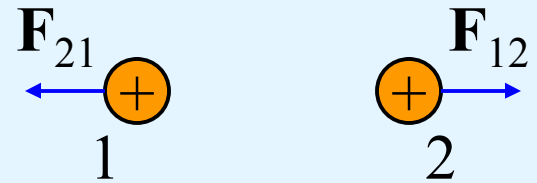
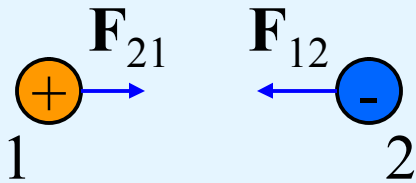
Part I: Electricity

Eric L. Michelsen
emichels at physics etc.



Charges

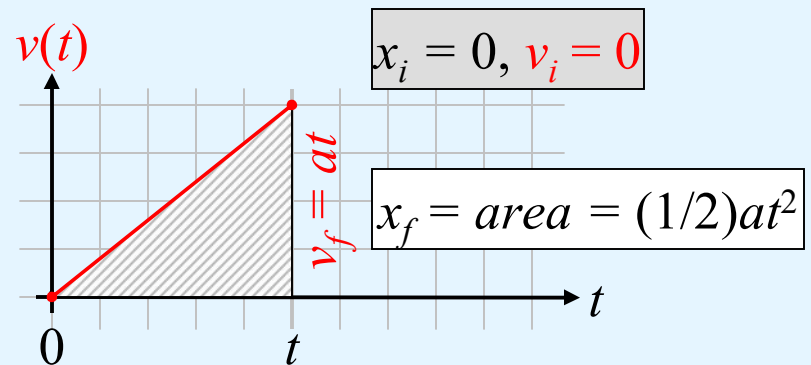
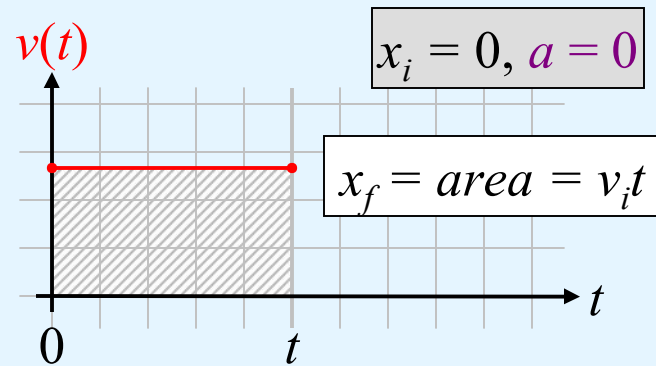
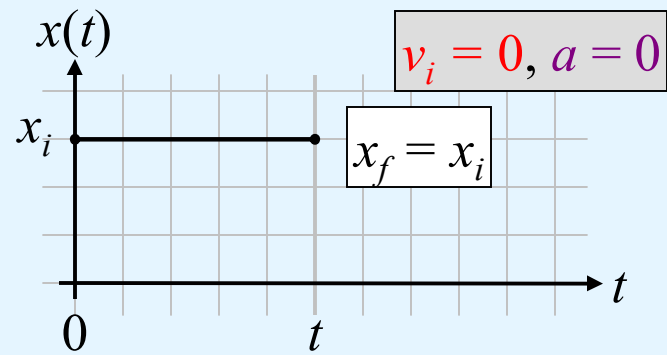
- Do particles push on each other because they are charged, or do we say that particles are charged because they push on each other?



Superposition

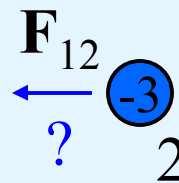
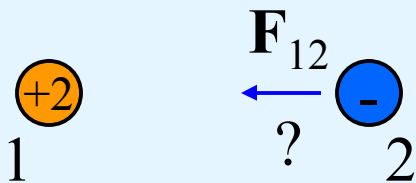
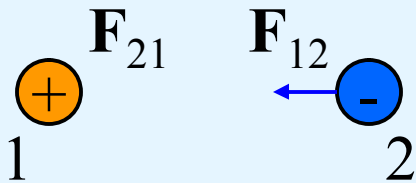
- Kinematic example
 - First: Take conditions one at a time
 - Only initial position
 - Only initial velocity
 - Only initial acceleration
- Superposition:
 - The effect of all the conditions together is the sum of each separately

$$x_f = x_i + v_i t + (1/2)at^2$$



More on charges

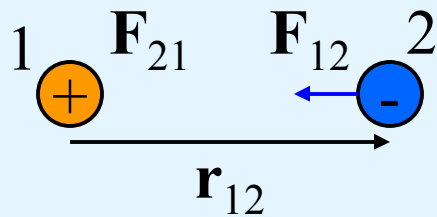
- Charge is measured in **Coulombs (C)**
 - A coulomb is like a “dozen,” only bigger: $6.242e18$ electrons-worth of charge
 - $\sim 10^{-5}$ moles of electrons
 - Or, an electron has $1.602e-19$ C of charge
 - Very small



Force is proportional to *both* charges:
$$|\mathbf{F}| = k q_1 q_2$$

Coulomb's Law

- Force weakens with distance
 - $\propto 1/r^2$ (like gravity)
- Obeys superposition
 - new charges don't interfere with forces from existing charges, so forces simply add vectorially



$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$$

where $r \equiv |\mathbf{r}_{12}|$, $\hat{\mathbf{r}}_{12} \equiv \frac{\mathbf{r}_{12}}{r}$



What are the units of $\hat{\mathbf{r}}_{12} \equiv \frac{\mathbf{r}_{12}}{r}$?

A dimensionless

B m

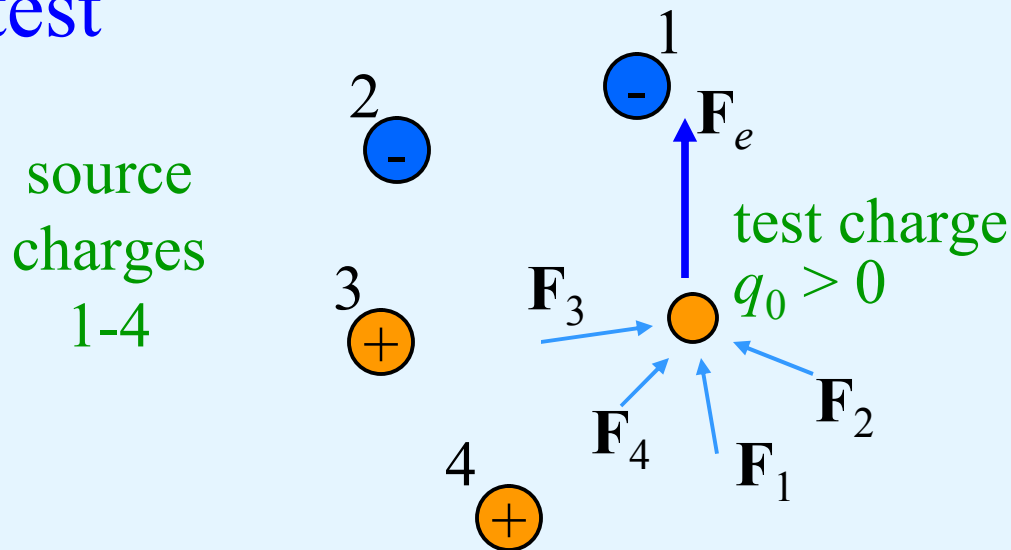
C m^2

D m^{-1}

E m^{-2}

Source charges and test charge

- Source charges usually considered fixed in space
- Test charge moves around
- No physical difference between “source” and “test”



$$\mathbf{F}_e = \sum_{i=1}^4 k_e \frac{q_i q_0}{r_i^2} \hat{\mathbf{r}}_i$$
$$= q_0 k_e \sum_{i=1}^4 \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Electric Fields

- Consider a distribution of **source charges**
- The force on a **test charge** at any point is proportional to the charge:

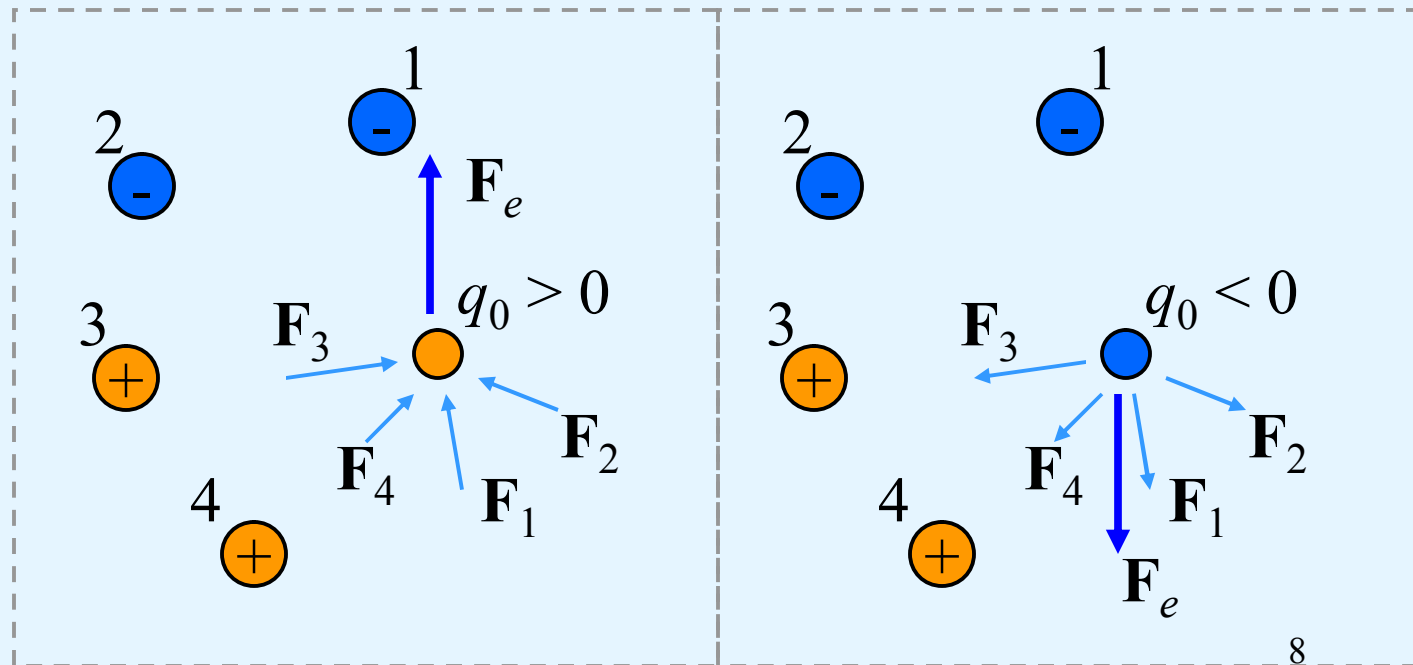
At a point: $\mathbf{F}_e \propto q_0 \Rightarrow \mathbf{F}_e = \mathbf{E}q_0$ where $\mathbf{E} \equiv$ constant vector

Extending to any point in space:

$$\mathbf{F}_e(\mathbf{r}) = q_0 \mathbf{E}(\mathbf{r})$$

units
of \mathbf{E} ?

direction
of \mathbf{E} ?

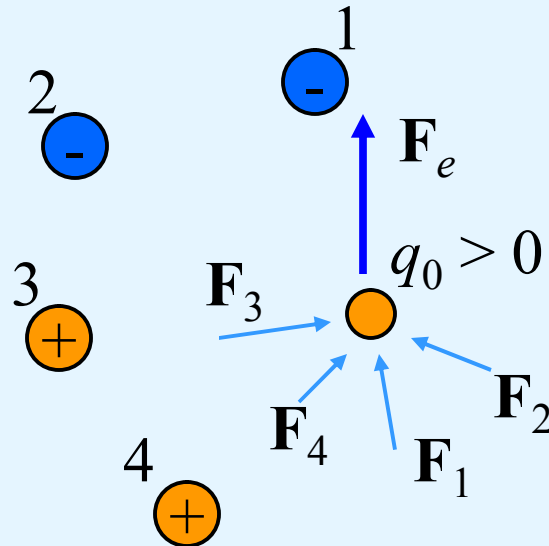


Electric field from source charges

- Source charges create an E-field

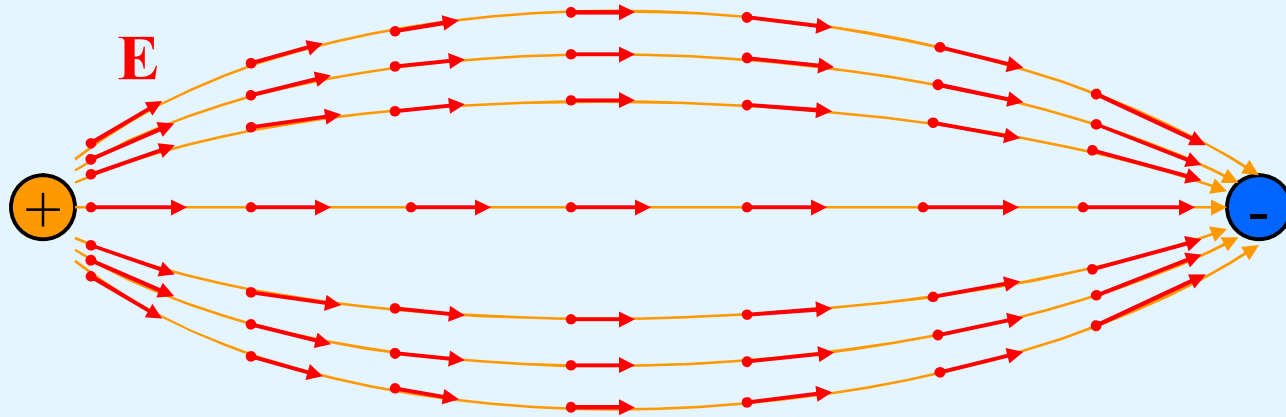
Recall: $\mathbf{F}_e = q_0 k_e \sum_{i=1}^4 \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$, *and* $\mathbf{F}_e = q_0 \mathbf{E}$

$$\Rightarrow \quad \mathbf{E}(\mathbf{r}) = k_e \sum_{i=1}^4 \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$



Lines of Force

- Field of vectors in space: the **E**-field
- Lines of force follow the arrows

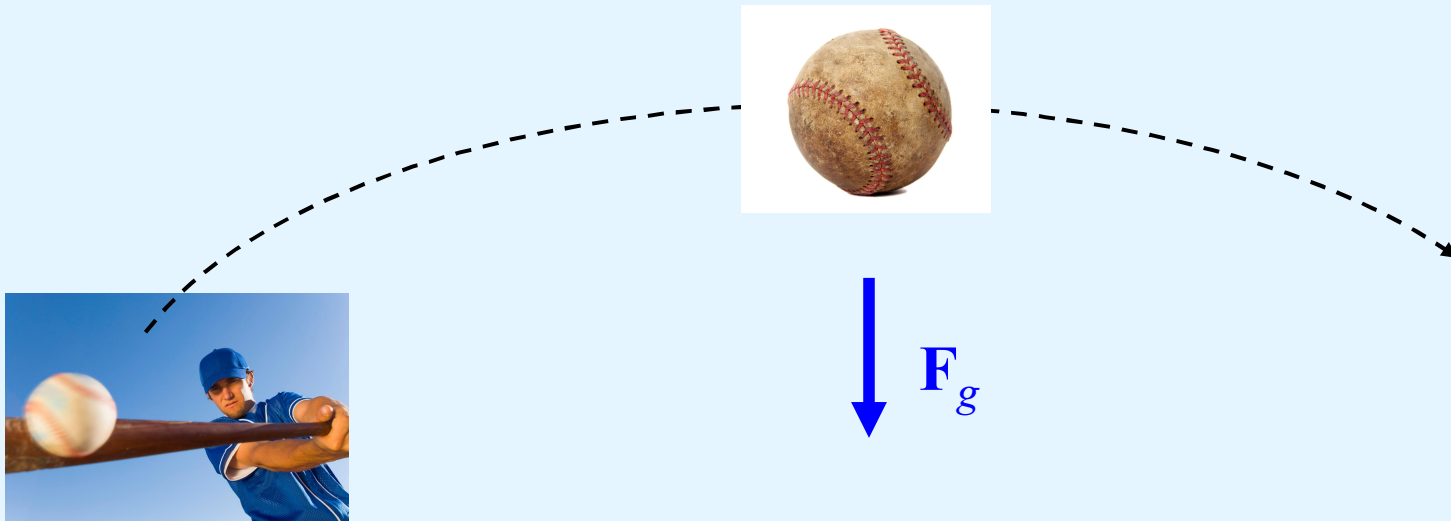


How does lines-of-force density relate to E-field strength?

E-field strength is proportional to lines-of-force density.

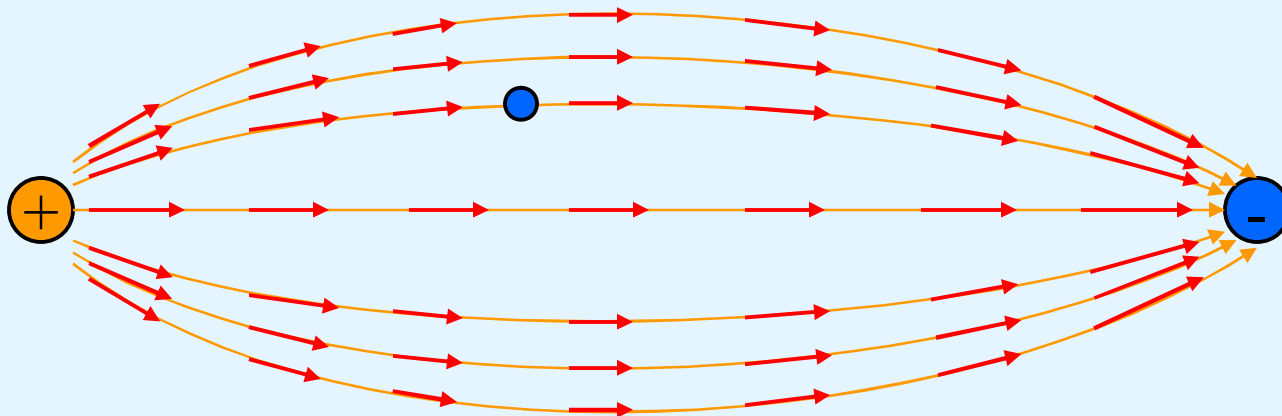
Do objects always move in the direction of the force applied to them?

- No. Objects bend toward the direction of applied force, but they generally don't move parallel to it



If I release a negative test charge at rest on a line of force, how will it move?

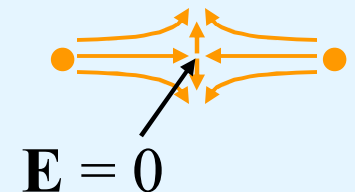
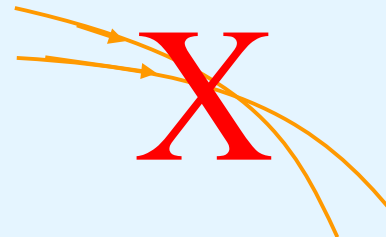
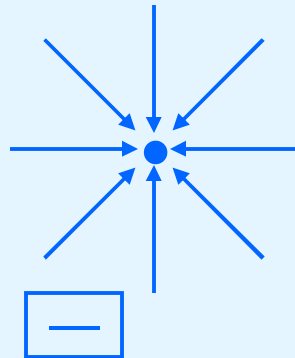
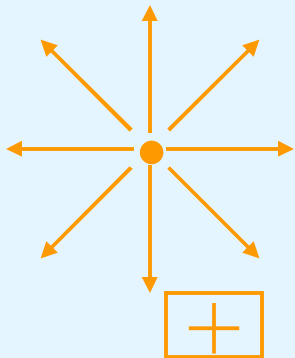
- A To the right, following the line of force
- B To the right, crossing the lines of force
- C It won't move
- D To the left, following the line of force
- E To the left, crossing the lines of force



When *does* the test charge move along the line of force?

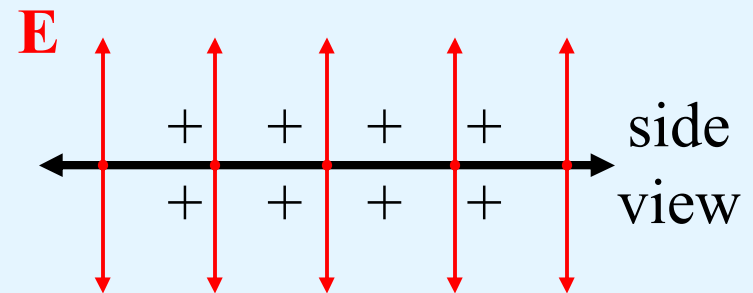
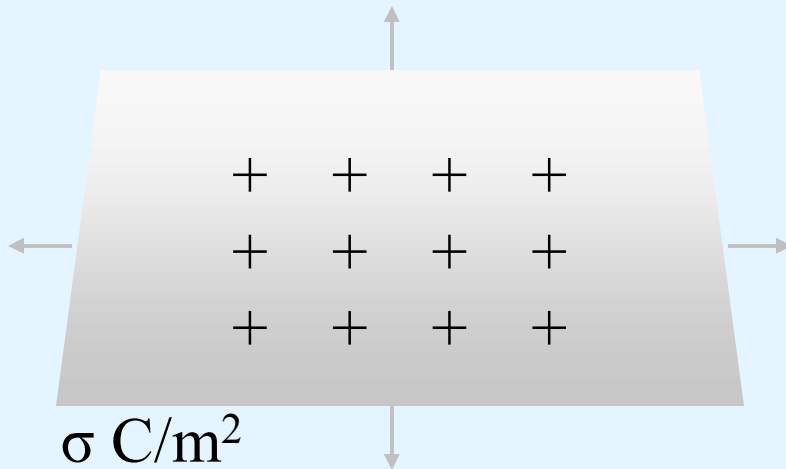
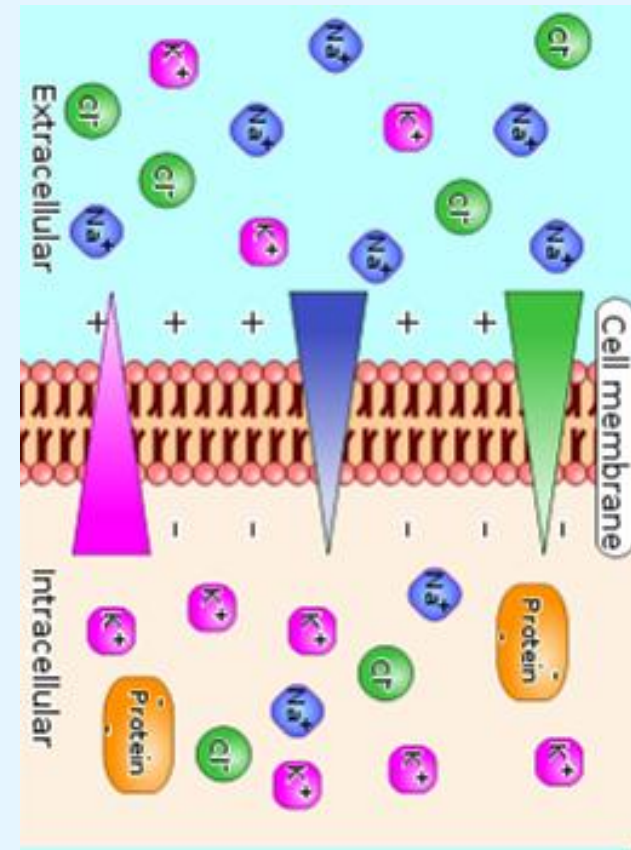
Birth, life, and death of lines of force

- No matter how close we get to a positive charge, lines of force point away from it
 - Lines of force *originate* on positive charges
- No matter how close we get to a negative charge, lines of force point toward it
 - Lines of force *terminate* on negative charges
- Can't cross or split
 - ... because they follow the E-field, which can only point one way from every point
- But they *can* turn sharply at a point of zero field



Why flux?

- Surface charge distributions
- A uniform density of charge in a large flat sheet
- What is the E-field near the surface?
 - Critically important, but ...
 - Can integrate over Coulomb's Law, but it's tedious
- Gauss' Law makes this simple

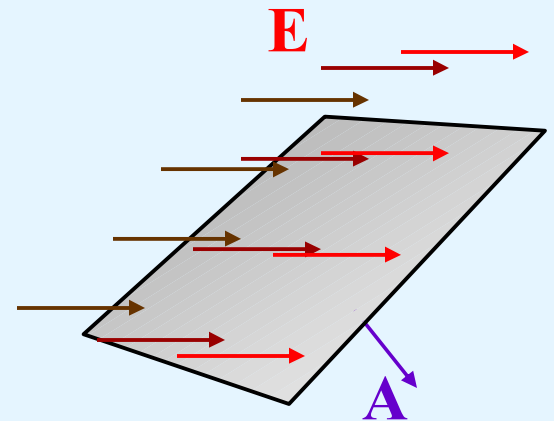
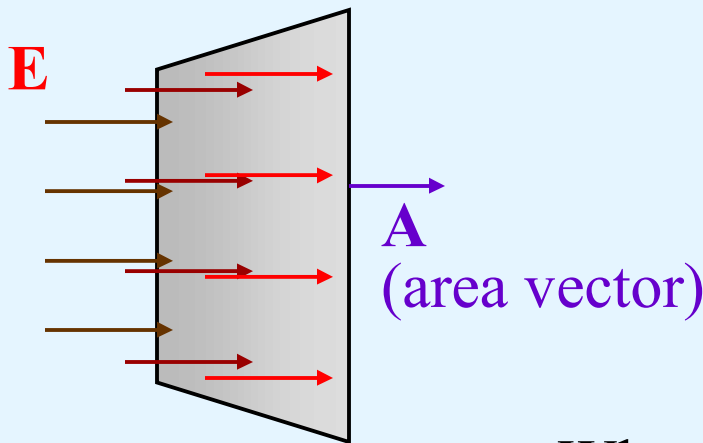


Flux

- Loosely: “counting” lines of force through the area
 - This only works because $\mathbf{E} \propto 1/r^2$
- More precisely: Flux is E-field component through the area, times area:

$$\Phi_E = \mathbf{E} \cdot \mathbf{A} \quad \text{or} \quad \Phi_E \equiv \iint_{\text{area}} \mathbf{E} \cdot d\mathbf{A}$$

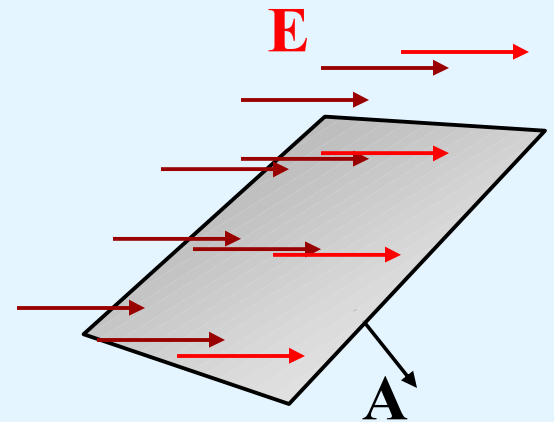
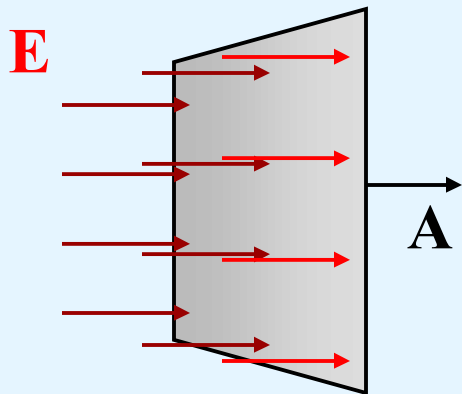
- **Area vector** is perpendicular to the area, so the parallel E-field component gives flux *perpendicular* to the area
 - So flux is a dot-product



What kind of quantity
is Φ ?

For the same $|\mathbf{E}|$ and $|\mathbf{A}|$, how does the flux through the area on the left compare to that on the right?

- A Flux is greater on the left
- B They are the same
- C Flux is greater on right



What is the flux through a sphere?

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

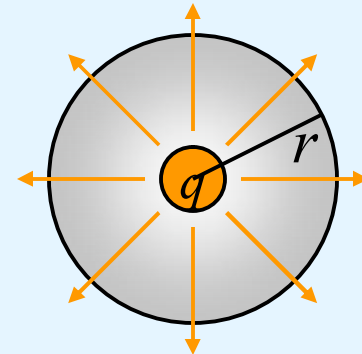
A $\Phi_E = \frac{k_e q}{r^2}$

B $\Phi_E = \frac{k_e q}{r^2} \hat{\mathbf{r}}$

C $\Phi_E = 4\pi r^2 k_e q$

D $\Phi_E = 4\pi k_e q$

E $\Phi_E = 0$



About what is the flux through the funny-shaped surface?

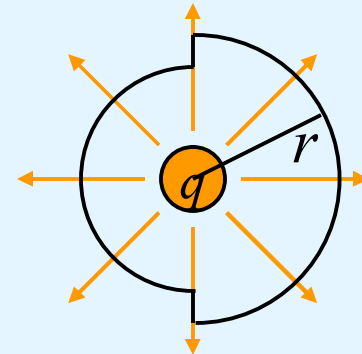
A $\Phi_E < 4\pi k_e q$

B $\Phi_E = 4\pi k_e q$

C $\Phi_E > 4\pi k_e q$

D

E $\Phi_E = 0$



About what is the flux through the bag?

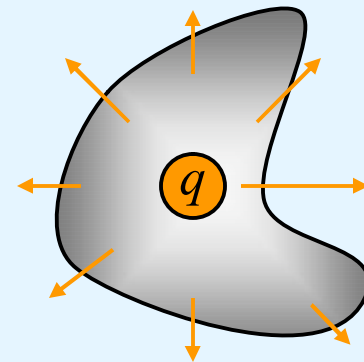
A $\Phi_E < 4\pi k_e q$

B $\Phi_E = 4\pi k_e q$

C $\Phi_E > 4\pi k_e q$

D

E $\Phi_E = 0$

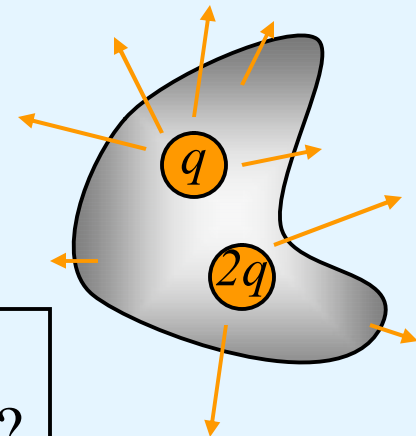


Gauss' Law

- The total *outward* flux from a **closed** surface equals $4\pi k_e$ times all the charge enclosed:

$$\Phi_E = 4\pi k_e q_{in} = \frac{q_{in}}{\epsilon_0} \quad \left(k_e \equiv \frac{1}{4\pi\epsilon_0}, \quad \epsilon_0 \text{ will be handy later} \right)$$

- Does superposition apply?
 - Superposition of what?
- What is q_{in} here?



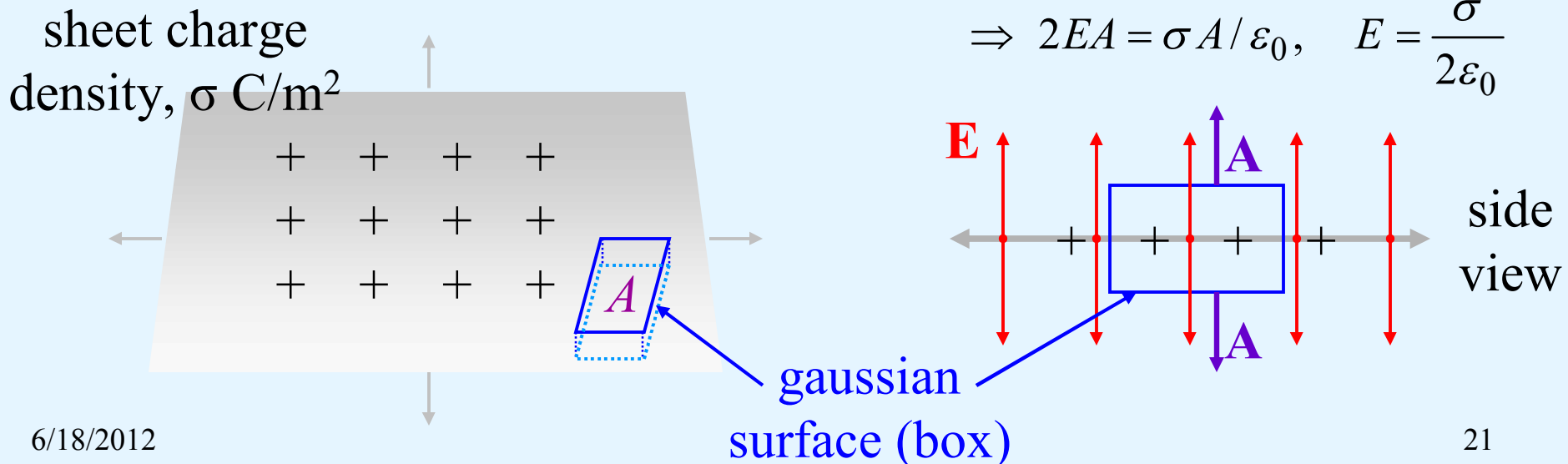
Why are arrows different lengths?

Surface charge distributions: the beauty of Gauss' Law

- A uniform density of charge in a large flat sheet
 - Gazillions of tiny particles (electrons, ions)
- What is the E-field near the surface?
 - What “shape” is it?

$$\text{Gauss' Law: } \Phi_E = q_{in} / \epsilon_0$$

$$\Rightarrow 2EA = \sigma A / \epsilon_0, \quad E = \frac{\sigma}{2\epsilon_0}$$

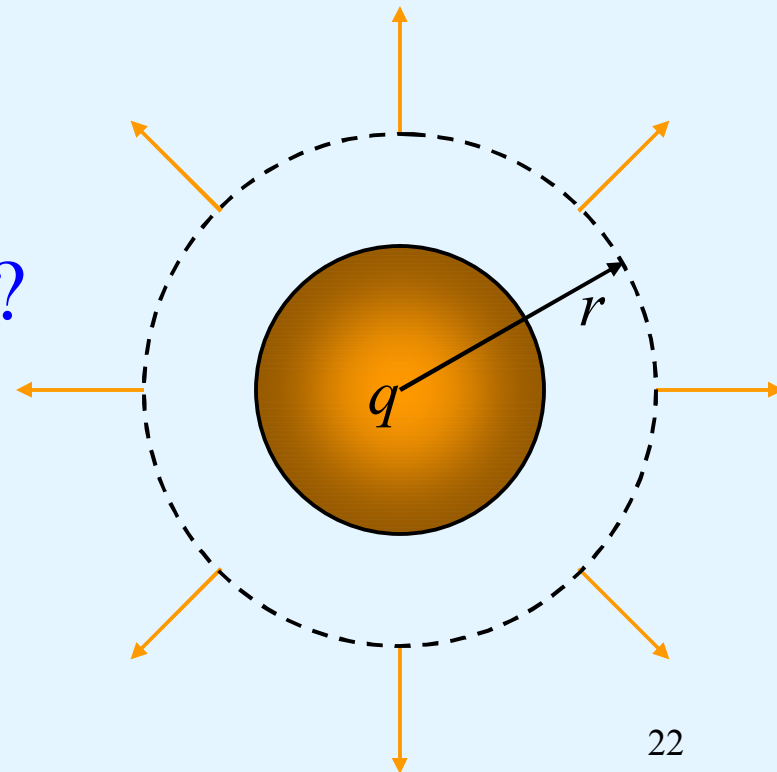


Volume charge distributions

- Consider a spherically symmetric volume distribution, in C/m^3
- What is E-field at distance r , outside the sphere?
- What does Gauss say?
- How does it compare to a point charge at distance r ?

$$\Phi_E = 4\pi k_e q_{in} \Rightarrow E_r \underbrace{4\pi r^2}_{\text{area}} = 4\pi k_e q$$

$$E_r = k_e \frac{q}{r^2}$$

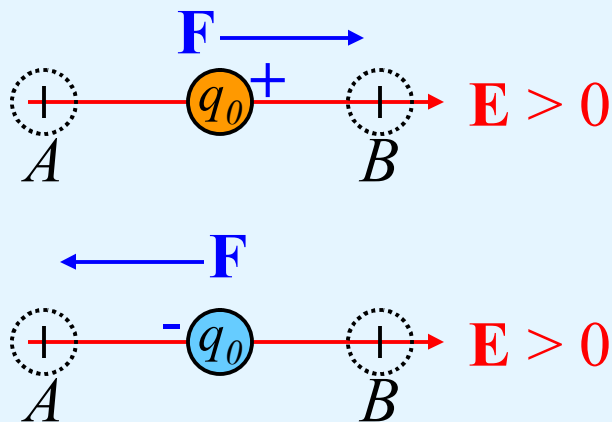


How does the E-field of a spherically symmetric volume distribution compare to that of a point charge?

- A It's bigger, because some of the charge is closer
- B It's the same
- C It's smaller, because more of the charge is farther away

Forces lead to work, potential energy, and voltage (aka potential)

- **1D:** $W = \int_A^B F(x) dx \Rightarrow U(B) - U(A) = -W_e = -\int_A^B q_0 E(x) dx$
 - Independent of the trajectory: time, speed
 - Work can be + or -
- Potential energy is proportional to test charge
- **Voltage** is defined as electric potential energy per unit charge (aka **potential difference**)
 - In volts (V), equivalent to joules/coulomb



$$V(B) - V(A) \equiv \frac{U(B) - U(A)}{q_0} = -\int_A^B E(x) dx < 0$$

“Potential” is different than “Potential Energy”

- Potential = Electric Potential = Voltage
 - Potential is *defined* as electric potential energy **per unit charge**

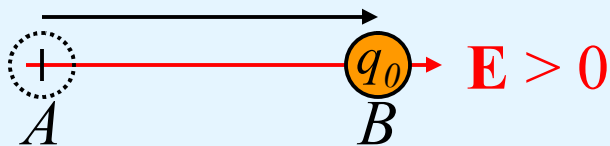
$$V(B) - V(A) \equiv \frac{U_e(B) - U_e(A)}{q_0}$$

- Potential is *independent* of test charge
- “Potential energy” of the test charge *depends* on its charge:

$$U_e(B) - U_e(A) = q_0 [V(B) - V(A)]$$

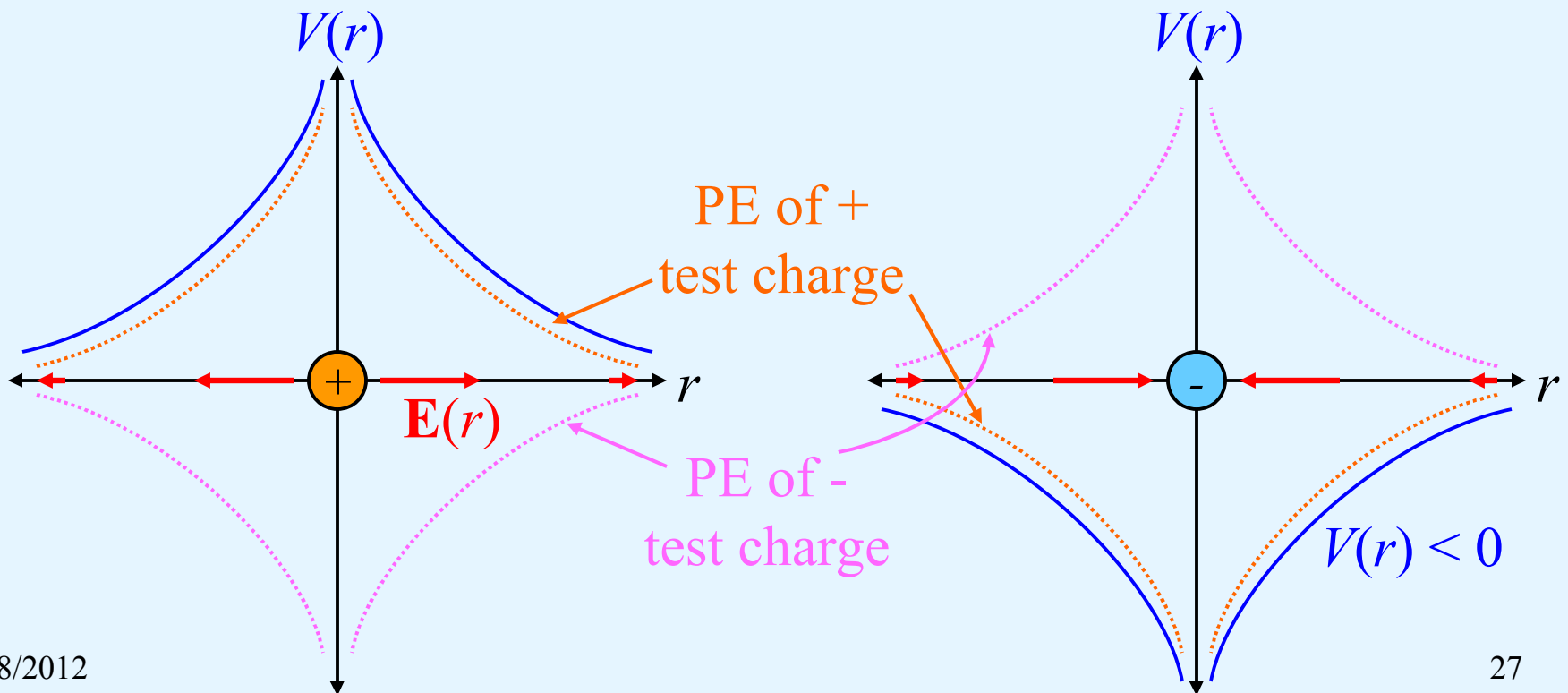
How does the work done (from A to B) by the field on a fast-moving charge compare to the work done on a slow-moving charge?

- A It's greater
- B It's the same
- C It's less
- D Depends on the charge



Potential of a point charge

- In other words, what is the work needed to bring +1 C of charge from far away to a distance r from the source charge?
 - Gets steeper closer in
 - Which way does a positive charge move?

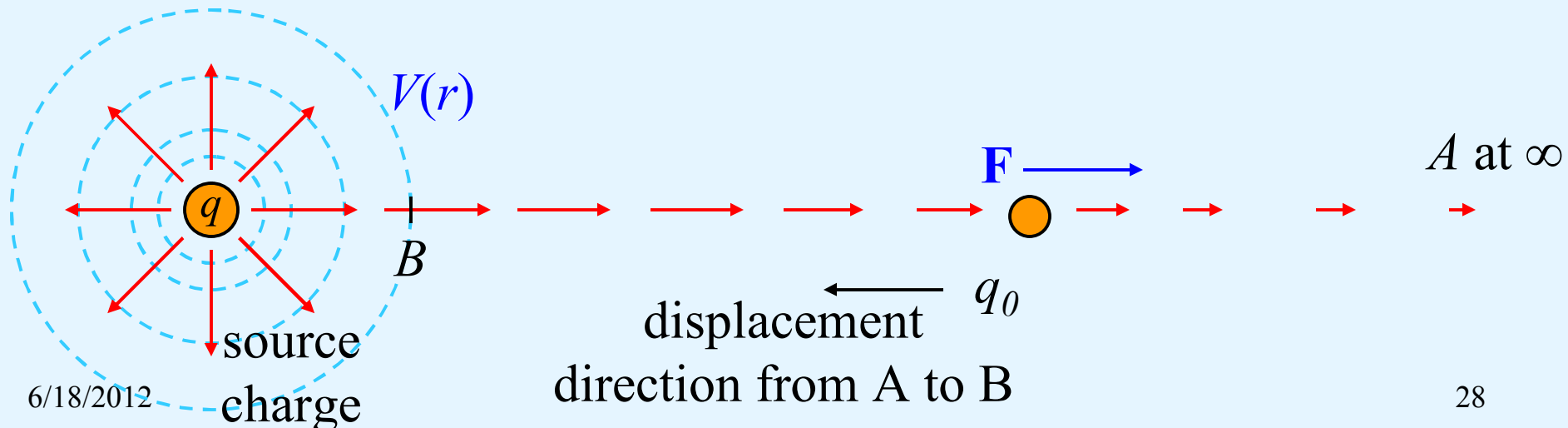


Potential of a point charge

- In other words, what is the work needed to bring +1 C of charge from far away to a distance r from the source charge?
- We can use a 1D analysis

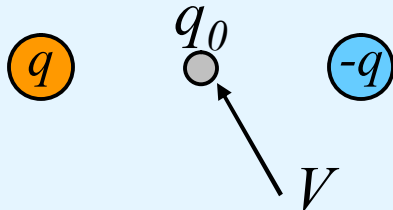
$$V(B) - V(A) = -\int_A^B E(r) dr = -\int_{\infty}^{r_B} k_e \frac{q}{r^2} dr = k_e q \left[\frac{1}{r} \right]_{\infty}^{r_B} = k_e \frac{q}{r_B}$$

- Voltage referenced to ∞ is defined as **absolute potential**
 - Still a function of distance, r : $V(r_B) \equiv \Delta V_{\infty \rightarrow r_B} = k_e \frac{q}{r_B}$
- If source charge is negative, potential is negative



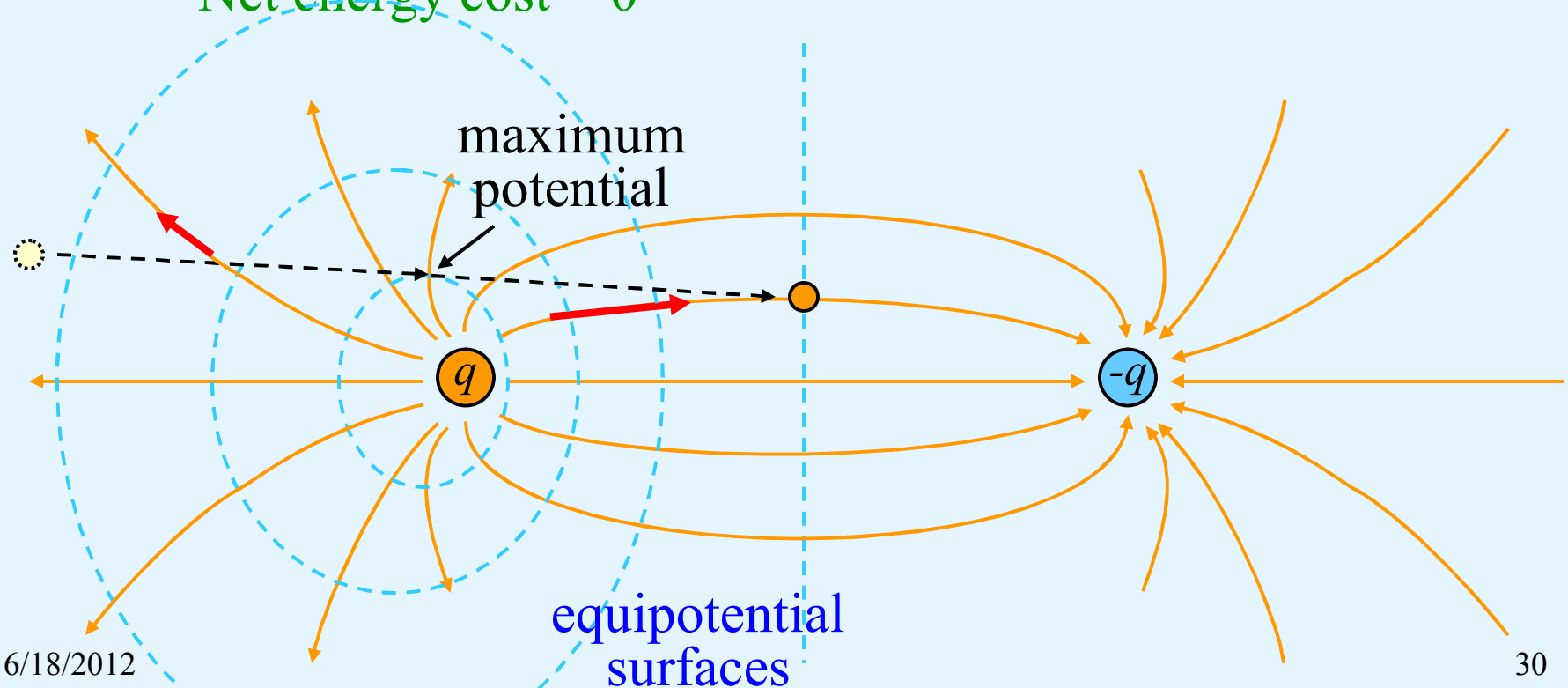
What is the potential midway between two point charges?

- A positive
- B zero
- C negative
- D depends on the test charge



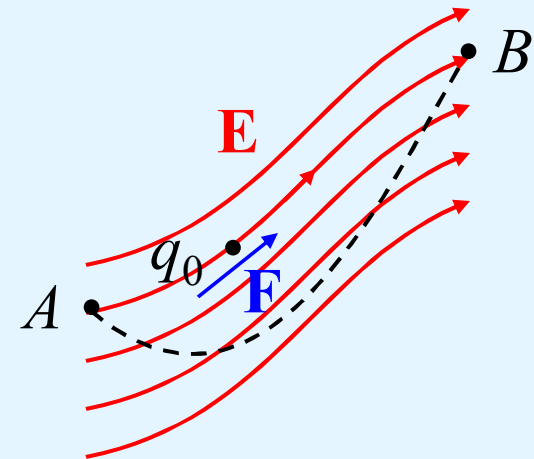
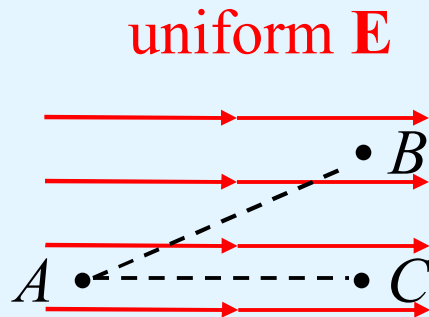
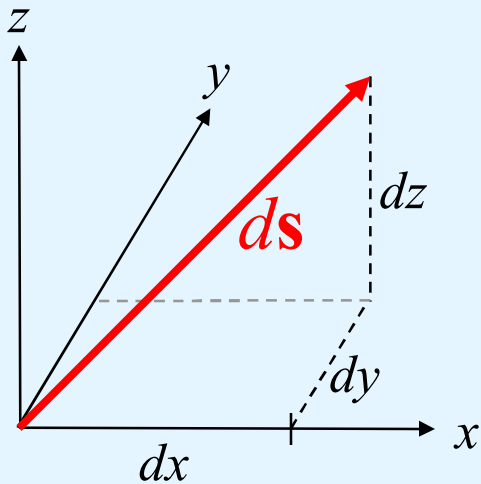
Wouldn't I have to do work to bring a charge from infinity to the midpoint?

- At first, yes
- But once past the + charge, the E-field does as much work (returns as much energy) to finish the job as it cost to bring the charge in from infinity
 - Net energy cost = 0



Forces lead to work, potential energy, and voltage (Part Deux: 3D)

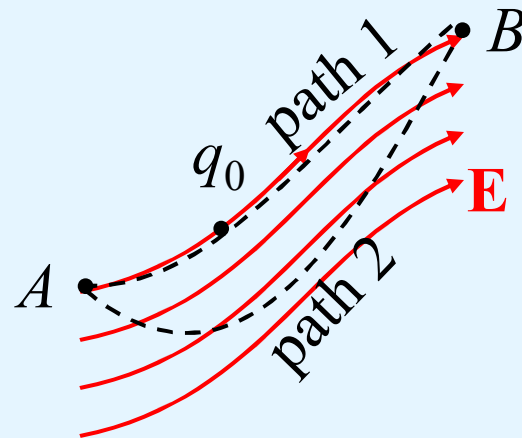
- ds is the same as dr
- How does V_{AB} compare to V_{AC} ?
- 3D: $W = \int_A^B \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} \Rightarrow U(B) - U(A) = -W_e = -\int_A^B q_0 \mathbf{E}(\mathbf{r}) \cdot d\mathbf{r}$
 - Independent of: time, speed
 - What about path?



How does the work done from A to B along path 1 compare to that of path 2?

- A It's more
- B It's the same
- C It's less

What if it were different?



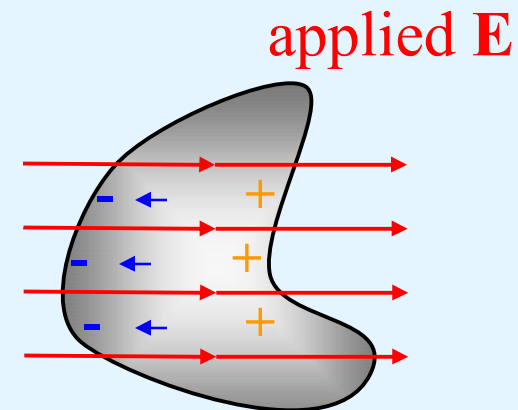
How do negative test charges move in an electric potential?

- A Down the potential hill
- B They don't move
- C Up the potential hill

How do negative charges move in a potential energy field?

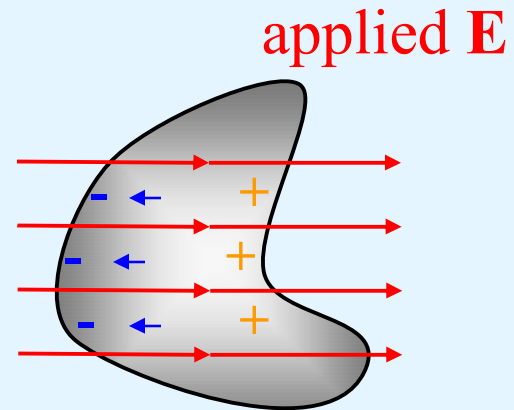
Good conductors

- Good conductors (typically metals) have huge (essentially infinite) numbers of **mobile electrons** ($\sim 10^{23}$ C/cm³)
 - Electrons free to move *within the conductor*
 - But they can't leave the surface
 - E-field drives negative electrons
 - leave behind **positive ions**
 - Allows for a redistribution of charge
 - Charge always moves to reduce E-field
- **Molarity:**
 - $1 \text{ C} = 10^{-5} \text{ mol}$ of electrons
 - mobile density = 10^{18} mol/cm^3
= 100 mol/L



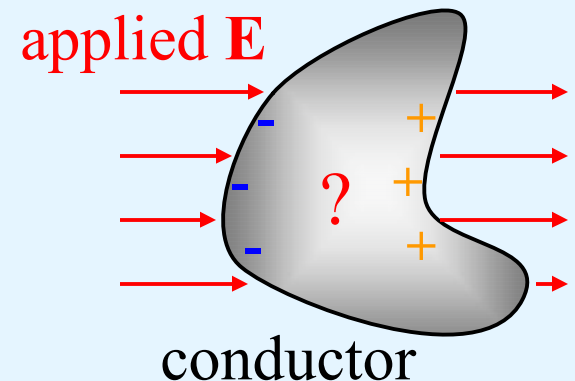
Which way do the positive charges move?

- A left
- B they don't move
- C right
- D depends on the conductor



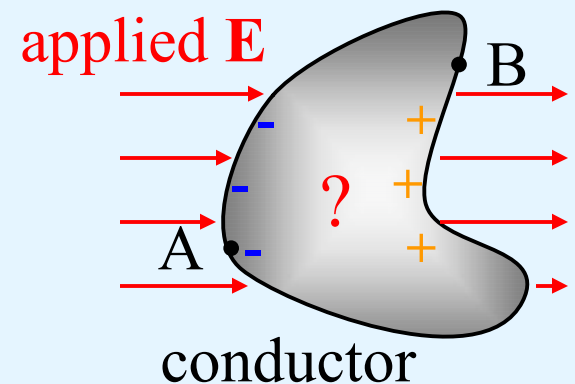
In a static E-field, after equilibrium, what is the E-field inside a good conductor?

- A Depends on the source charges around it
- B Enhances any applied E-field
- C Reduces any applied E-field
- D zero



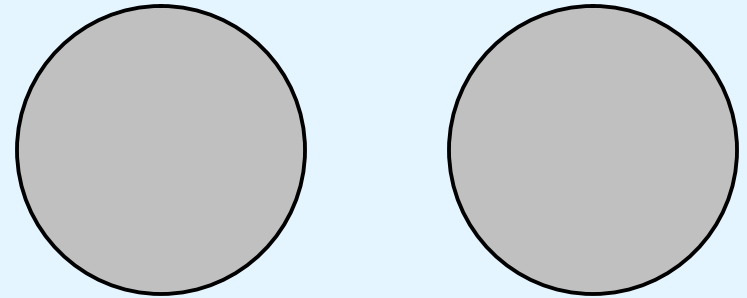
In equilibrium, how does the potential at B compare to that at A?

- A It's less
- B It's the same
- C It's greater
- D Depends on the conductor

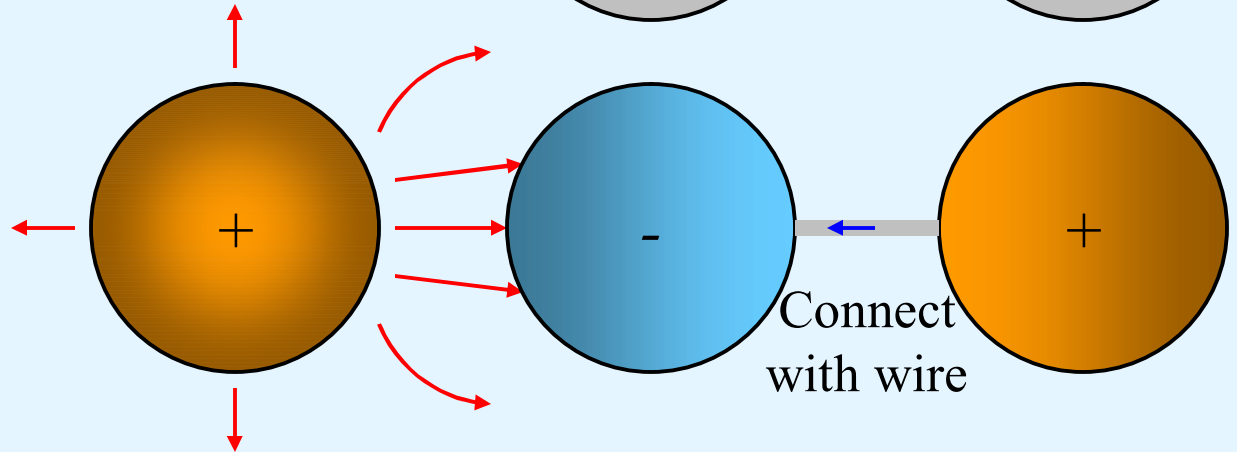


Charging by induction

- Start with neutral conductors

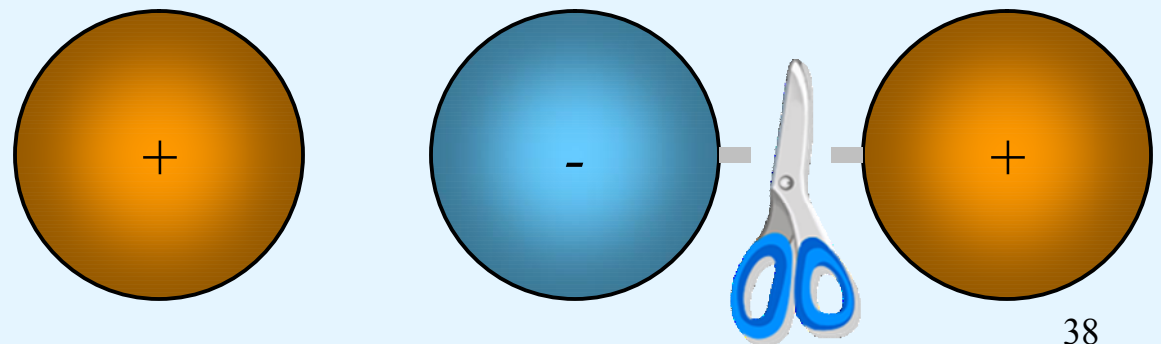


- Induce



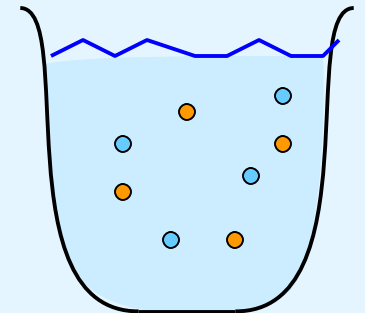
- Separate

- (E-field not shown)



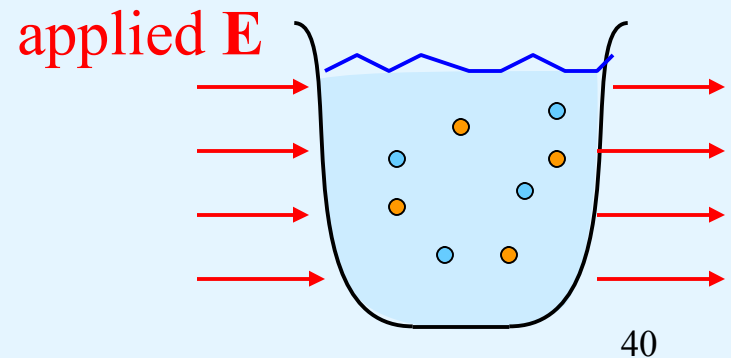
Fluid conductors

- Have smaller number of mobile charges
 - Mobile charges may be electrons or ions
- Conductor may be liquid or gas
 - Plasma (ion/electron gas, not blood)
 - Electrolyte



Which way do the positive charges move?

- A left
- B they don't move
- C right



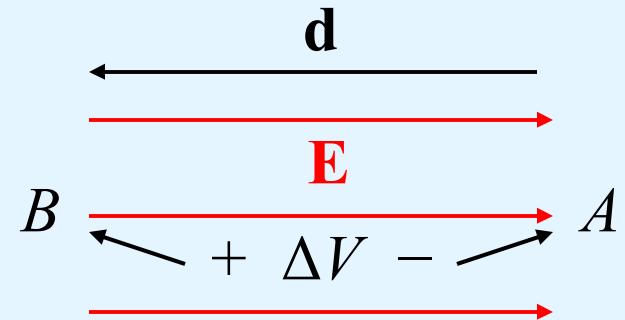
Alternate units for E

- $[\mathbf{E}] = \text{V/m}$
 - Equivalent to N/C

$$\Delta V = -\mathbf{E} \cdot \mathbf{d}, \quad \text{or from} \quad \Delta V = -\int \mathbf{E} \cdot d\mathbf{x}$$

- Can view \mathbf{E} two ways:
 - For simplicity, consider a uniform E-field:

$$\Delta U = -q_0 \overbrace{\mathbf{E} \cdot \mathbf{d}}^{\text{voltage}} = -\underbrace{\mathbf{F}_e}_{\text{force}} \cdot \mathbf{d} = q_0 \overbrace{\Delta V}^{\text{potential energy per unit charge}}$$



Sign convention for ΔV : **positive** end of ΔV has **higher** PE for **positive** test charge

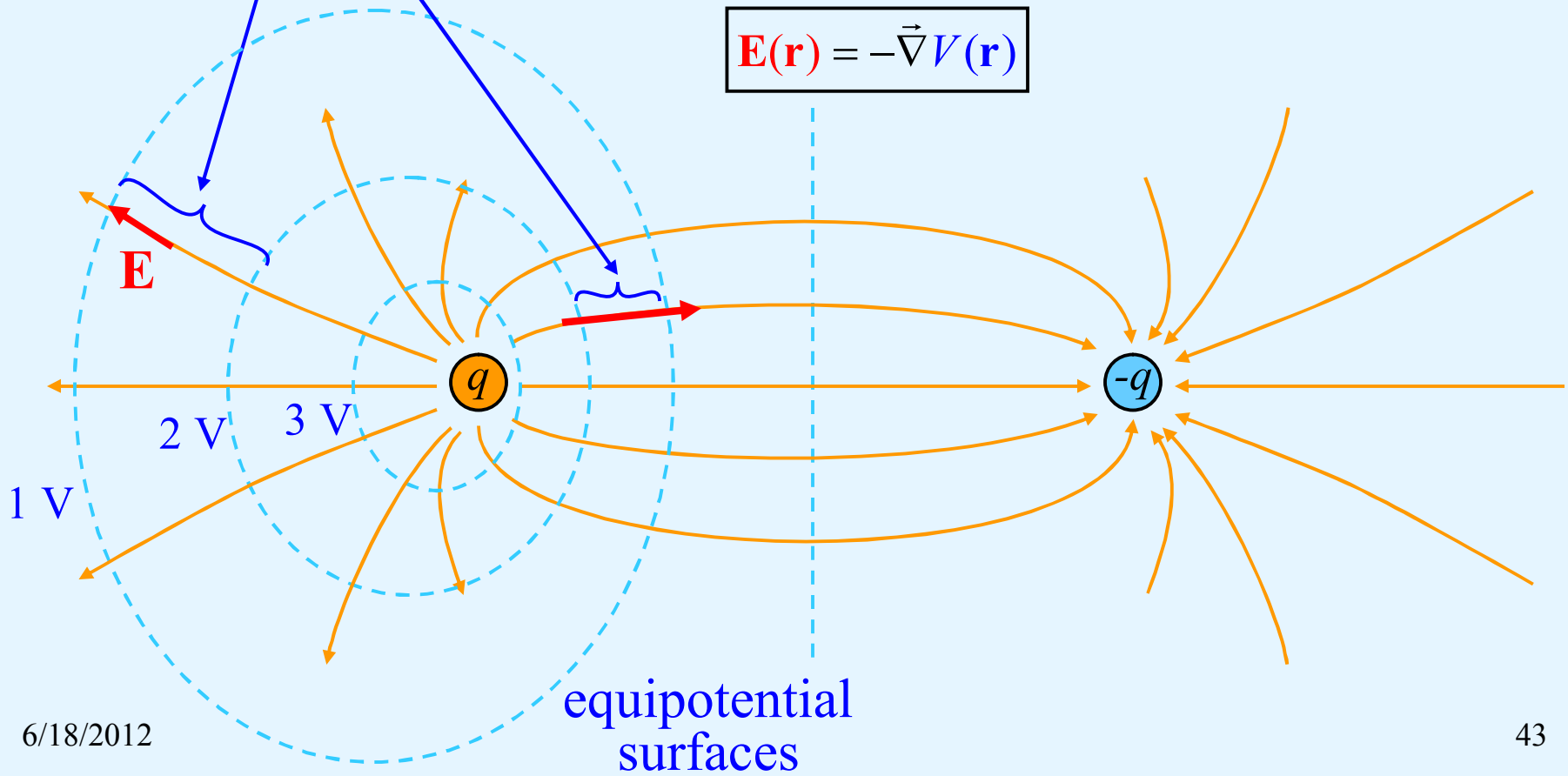
E and V

- **E** is to F as V is to U_e
 - Electric field **E** is force per unit charge
 - V is potential energy per unit charge

electric force per unit charge $\mathbf{E}(\mathbf{r}) \equiv \frac{\mathbf{F}_e(\mathbf{r})}{q_0}$, $V(\mathbf{r}) \equiv \frac{U_e(\mathbf{r})}{q_0}$ electric potential energy per unit charge

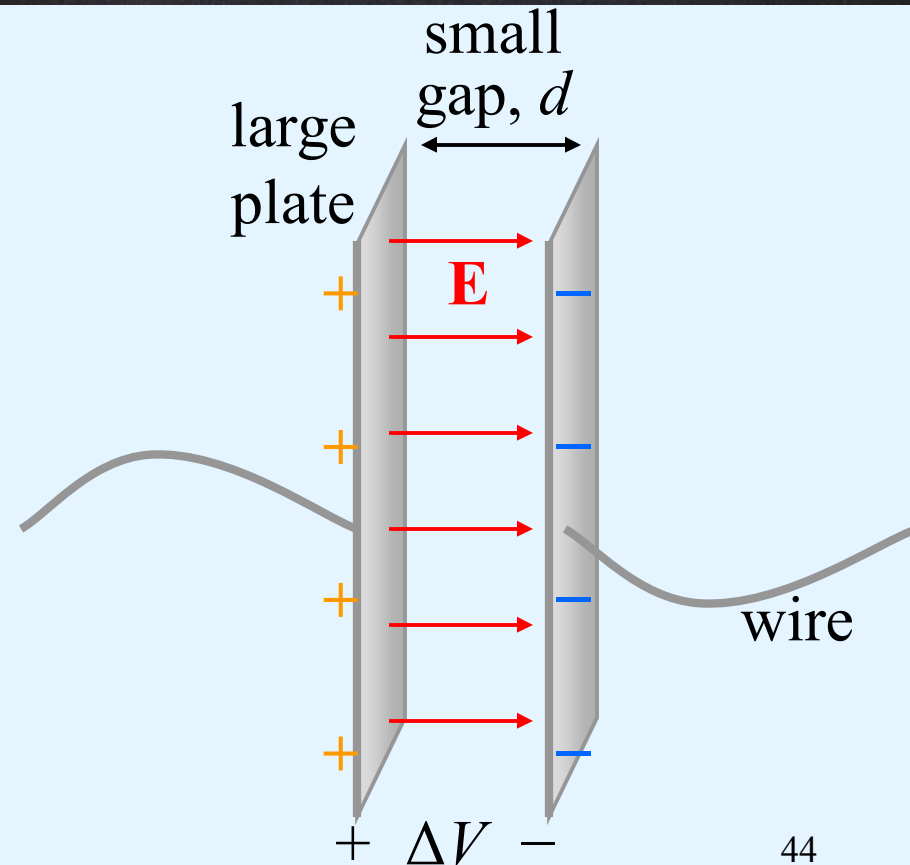
Relationship between E-field and V-field

- **E** points between equipotentials
- **E** is perpendicular to equipotentials
 - Tighter equipotential surfaces = stronger **E** (more volts/meter)
 - Wider equipotential surfaces = weaker **E** (less volts/meter)



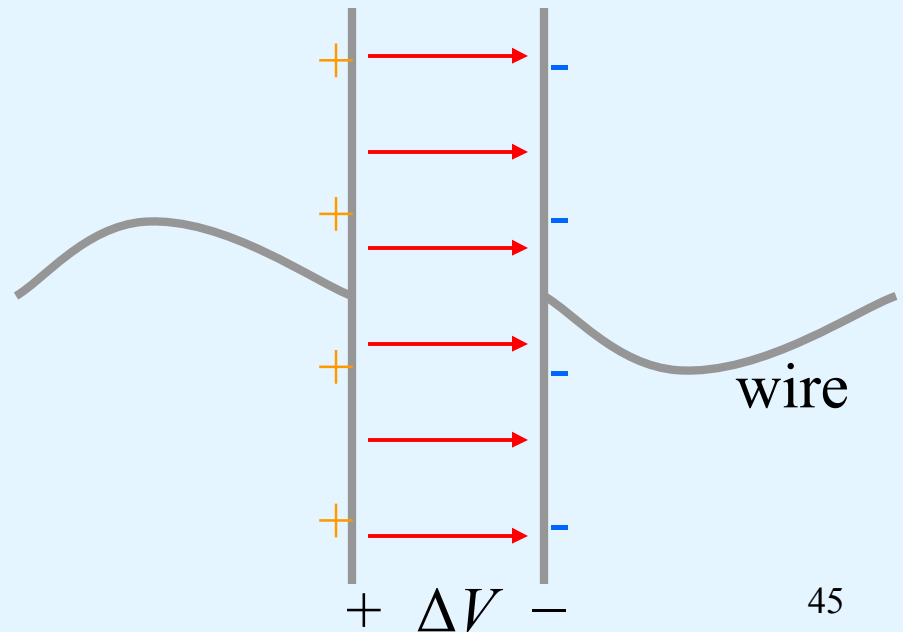
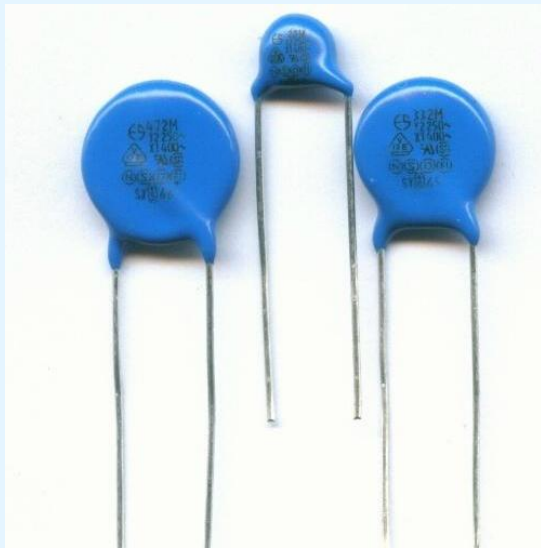
Capacitor

- Two parallel thin plates
 - Small gap => neglect edges
- Charges symmetrically
 - By induction!
- Stores charge
- Q is proportional to ΔV
- Defines **capacitance** C :
 - $Q = C\Delta V$
 - Measured in $C/V \equiv F$, Farads



What is the E-field outside the plates of a capacitor?

- A left
- B zero
- C right
- D depends on which plate has more charge
- E not enough information



Energy in a capacitor

- Given a capacitor charged to voltage ΔV , how much work does it take to increase the charge by a small amount, dq ?

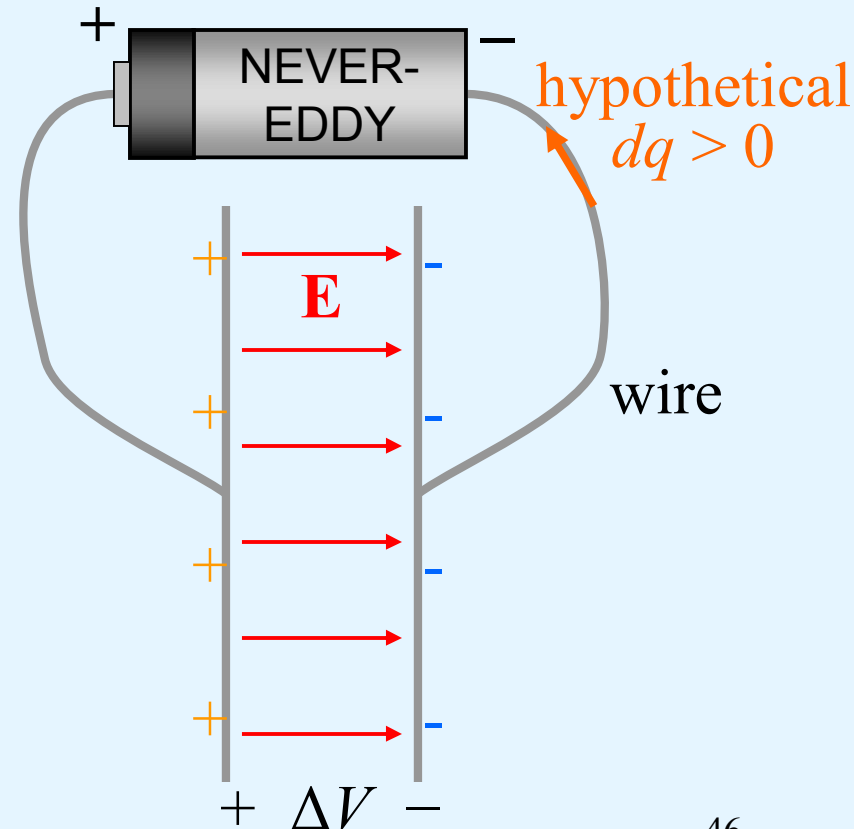
$$dU = dW_{outside} = \Delta V dq = \frac{q}{C} dq$$

- How much work to charge the whole thing from 0 to ΔV ?

$$\Delta U = \int_0^W dW_{outside} = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \frac{q^2}{2} \Big|_0^Q$$

final charge

$$= \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C (\Delta V)^2$$



STOP

Sign convention for a capacitor

- Choose reference plate: defines polarities
 - Eliminates minus sign

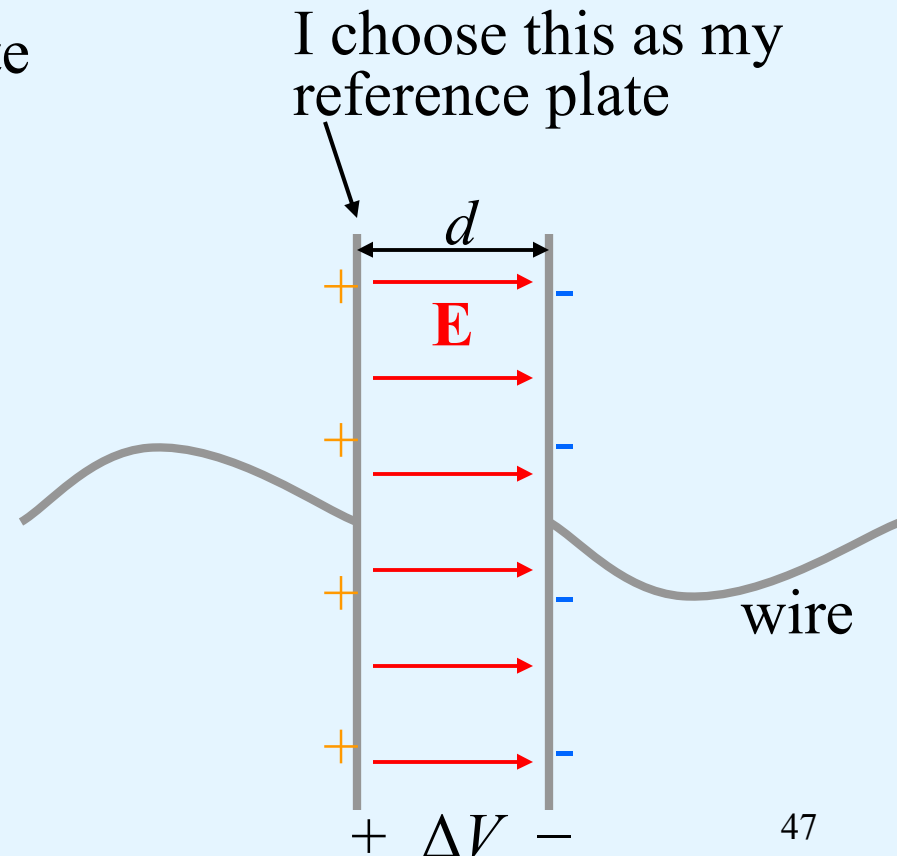


$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \longleftarrow \text{charge on reference plate}$$

$$\Delta V = E d = Q \frac{d}{A\epsilon_0}$$

$$\Rightarrow Q = \frac{A\epsilon_0}{d} \Delta V \equiv C \Delta V$$

capacitance \propto area,
inversely \propto separation



What is the total *net* charge in a capacitor charged to voltage ΔV ?

A $Q = C \Delta V$

B $Q = (1/2)C \Delta V$

C $Q = 0$

D $Q = C(\Delta V)^2$

E $Q = (1/2)C(\Delta V)^2$

If I charge a capacitor so that $Q < 0$, what describes its energy?

A $U > 0$

B $U = 0$

C $U < 0$

D depends on ΔV

How to solve any problem: Example 1

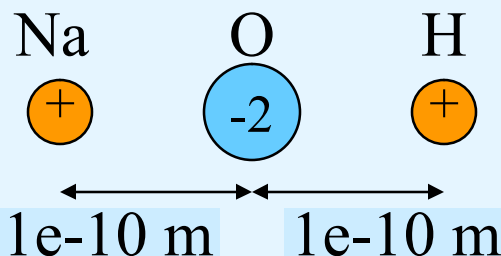
- A bound NaOH molecule is modeled as Na⁺ bound to O⁽⁻²⁾ bound to H⁺, in a straight line in that order. The Na⁺ ion is 1.0e-10 m from the O⁽⁻²⁾, which is 1.0e-10 m from the H⁺. How much energy does it take to remove the Na⁺ ion from the molecule (i.e., take it to far away), in J?

- What does it look like?
- What does it want? Energy
- What do we know about what it wants? $\Delta U = q\Delta V$
- What is given? q_1, q_2, q_3, r_1, r_2
- What can we find from what is given? $V(\mathbf{r}) = k_e \frac{q_s}{r}$
- What principles/formulas relate what is given or what we can find to what it wants?

$$V_{total}(\mathbf{r}) = V_1(\mathbf{r}) + V_2(\mathbf{r})$$

$$\Delta U = q_0 \Delta V_{total} = 0 - q_0 k_e \left(\frac{-2q_p}{1 \times 10^{-10}} + \frac{q_p}{2 \times 10^{-10}} \right)$$

$V_{total}(\infty)$



if
needed

How to solve any problem: Example 2

- An electron in a picture tube starts at rest. It is accelerated by 25,000 V across a distance of 0.50 m to the screen. How long does it take to travel to the screen, in s?

- What does it look like?

- What does it want? Energy

- What do we know about what it wants? $\Delta x = \frac{1}{2}at^2$, $\Delta x = v_{avg}t$

- What is given? $d, \Delta V$

- What can we find from what is given? $W_e = F_e d = qEd = q\Delta V \Rightarrow E = \frac{\Delta V}{d}$

- What principles/formulas relate what is given or what we can find to what it wants?

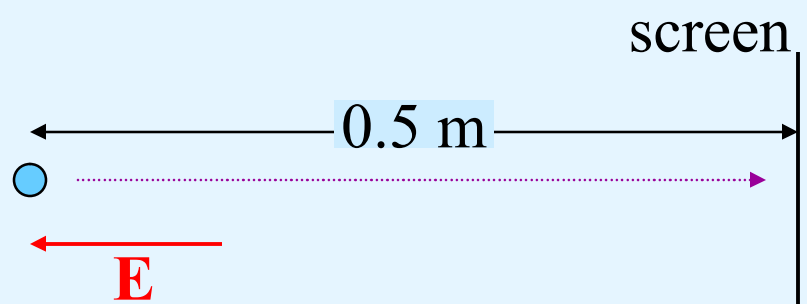
$$a = \frac{F}{m} = \frac{qE}{m}, \quad t = \sqrt{\frac{2\Delta x}{a}}$$

$$E = (25,000 \text{ V}) / 0.5 \text{ m} = 50,000 \text{ V/m}$$

$$a = \frac{(1.6 \times 10^{-19} \text{ C})(50,000 \text{ V/m})}{9.1 \times 10^{-31} \text{ kg}} = 8.8 \times 10^{15} \text{ m/s}^2$$

$$t = \sqrt{\frac{2(0.5 \text{ m})}{8.8 \times 10^{15}}} = 1.1 \times 10^{-8} \text{ s}$$

if needed



When mobile charges move, what do they do to the applied E-field?

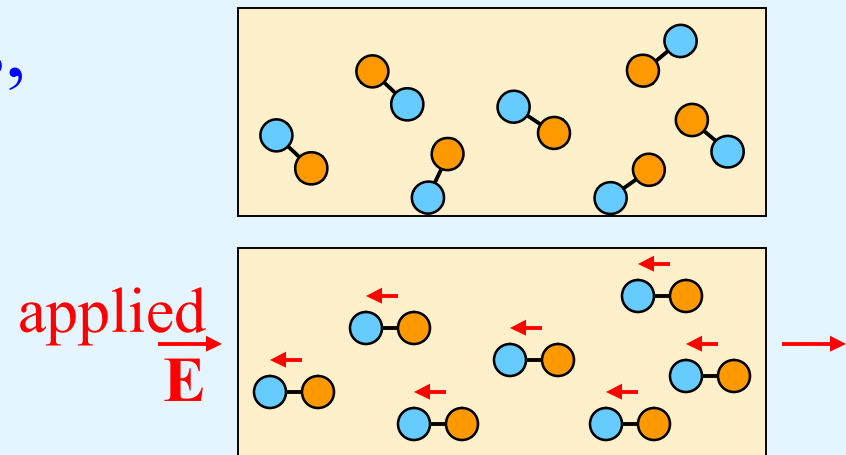
- A they reduce it
- B nothing
- C they increase it

Dielectrics (1)

- Have rotating **dipoles**
- For same charge on plates, moving charges **reduce** E-field

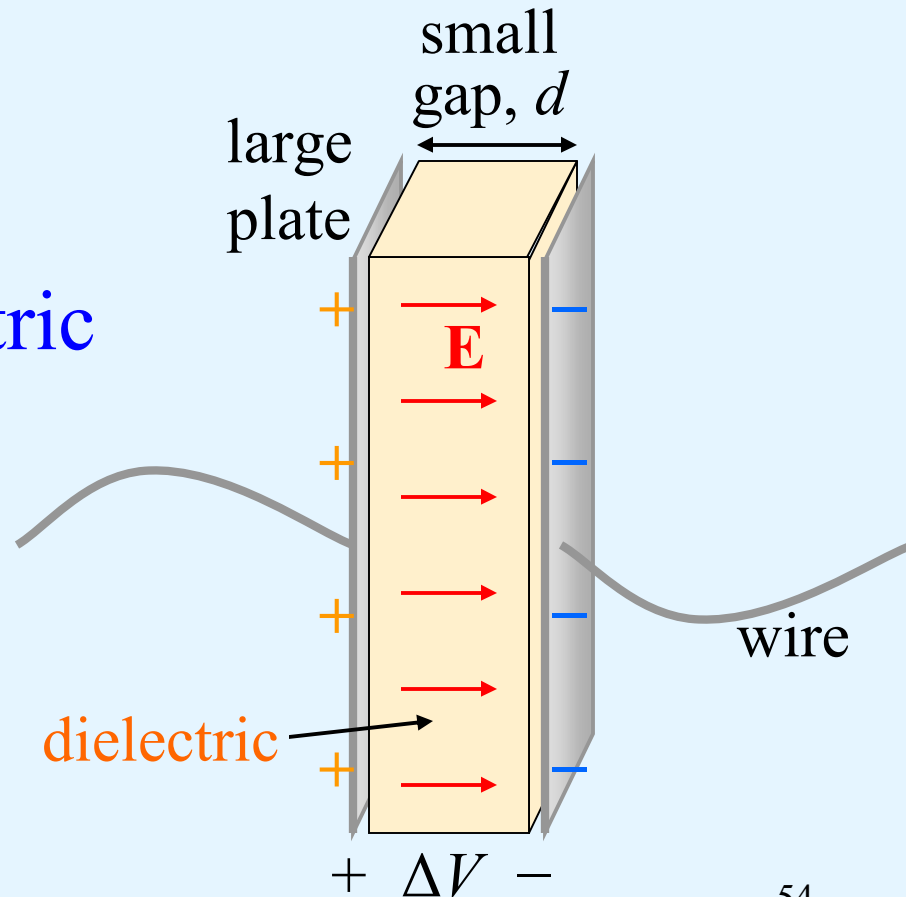
$$E_{net} = \frac{E_0}{\kappa} \quad \Rightarrow \quad \Delta V_{net} = \frac{\Delta V}{\kappa}$$

- Dielectrics are *not* conductors
 - Charges move a little, but are *bound* to a small region



When I insert a dielectric between the plates of a capacitor, what happens to the capacitance?

- A It decreases
- B nothing
- C it increases
- D depends on the dielectric



Dielectrics (2)

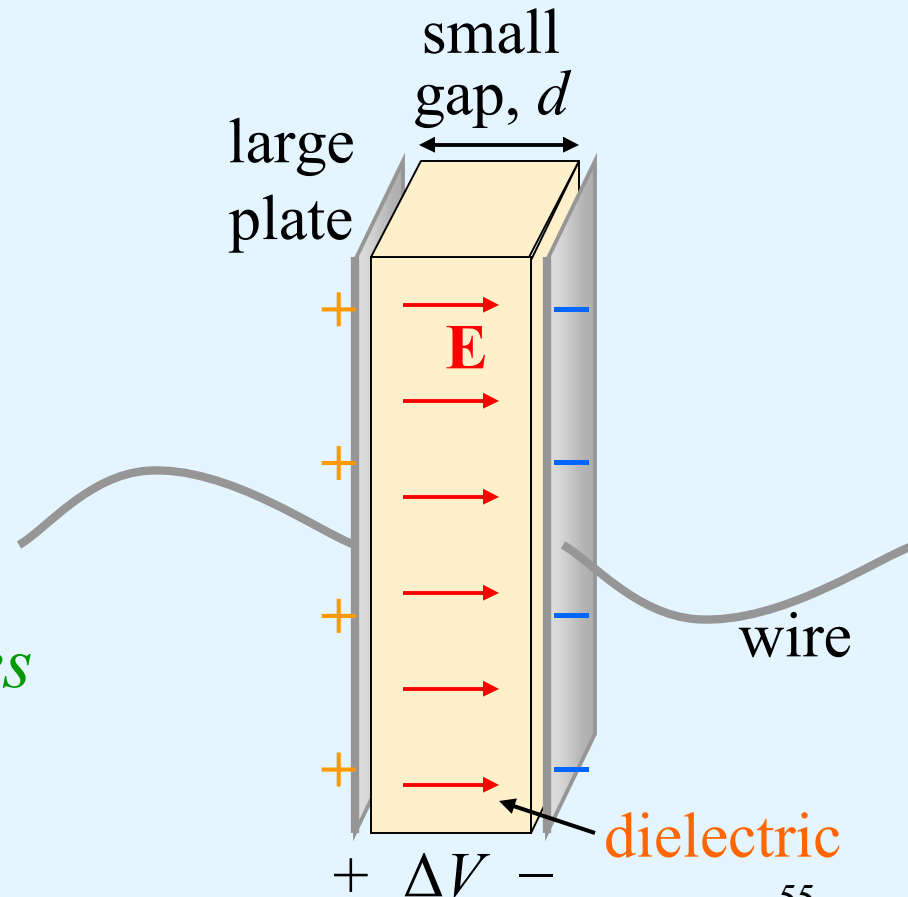
- Have rotating **dipoles**
- For same charge on plates, moving charges **reduce E-field**

$$E_{net} = \frac{E_0}{\kappa} \Rightarrow \Delta V_{net} = \frac{\Delta V}{\kappa}$$

↙ **dielectric constant**

- Coulombs per volt increases
 - => **capacitance increases**

$$C = \frac{\kappa \epsilon_0 A}{d}$$



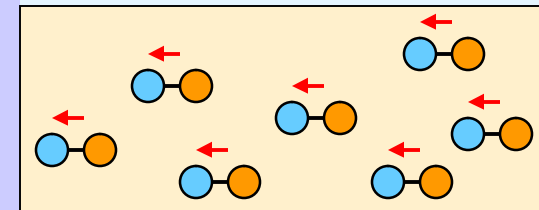
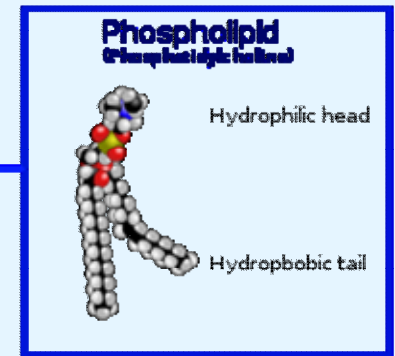
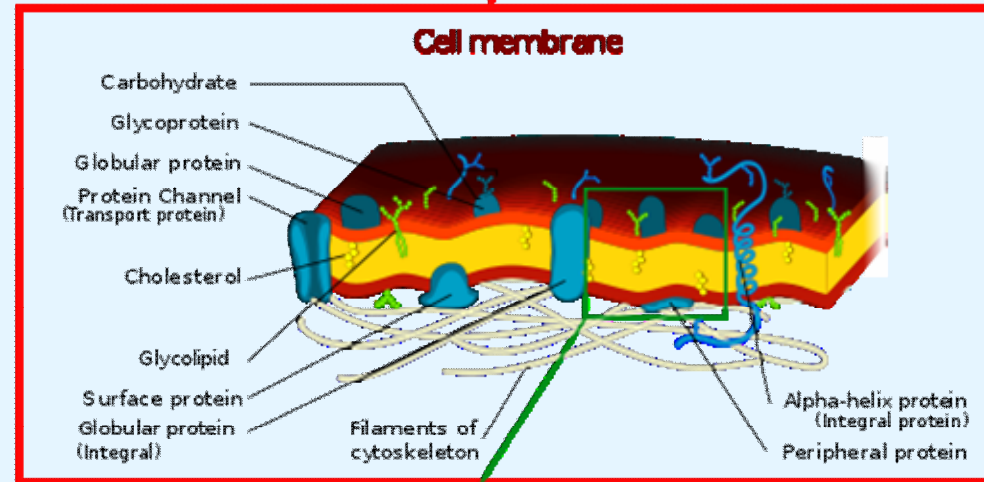
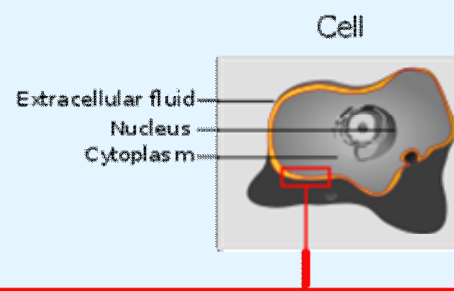
Cellular technology

“... local dielectric properties can play crucial roles in membrane functions.”

-- Local Dielectric Properties Around Polar Region of Lipid Bilayer Membranes, *J. Membrane Biol.* 85,225-231 (1985)

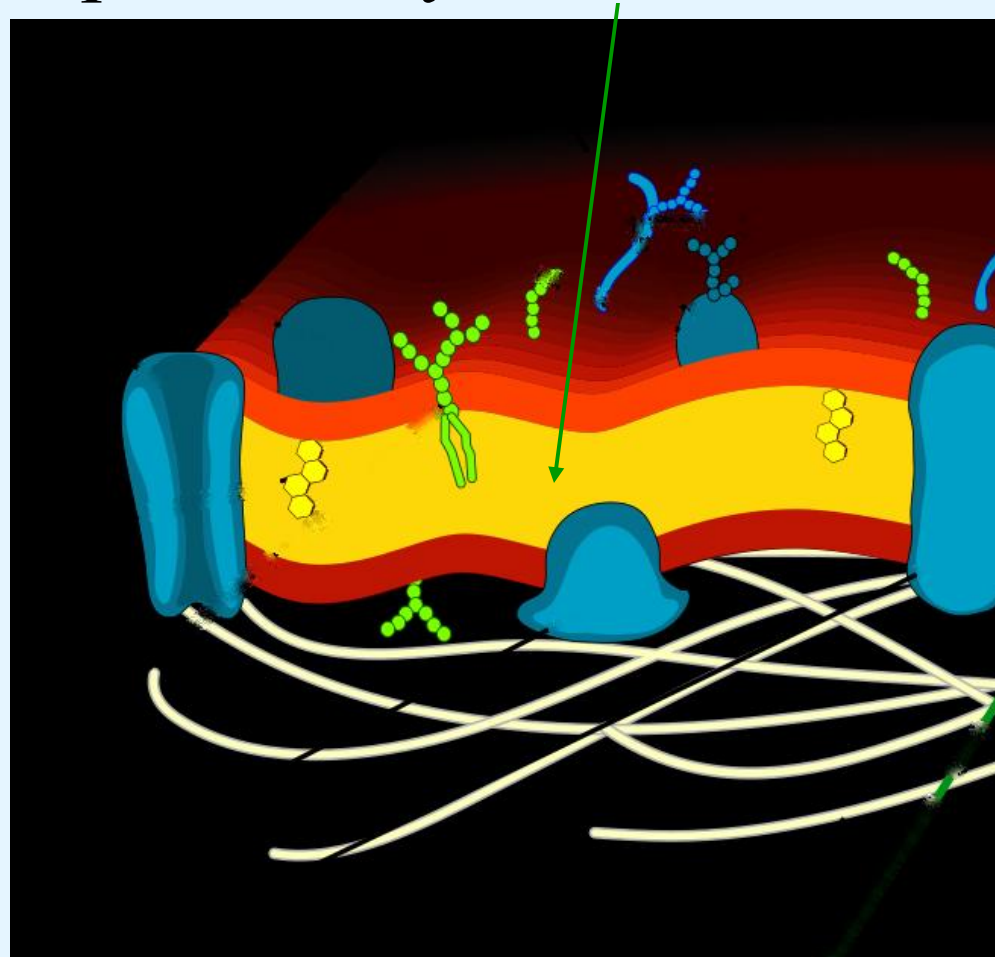
“An important physical effect of cholesterol is to increase the membrane's internal **electrical dipole potential**, which is one of the major mechanisms by which it modulates ion permeability. The dipole potential ... arises because of the alignment of dipolar residues ... in the .. interior of the membrane. ... its magnitude can vary from **~100 to >400 mV**.... Recent investigations have suggested that it affects numerous different biological membrane processes.”

-- Cholesterol Effect on the Dipole Potential of Lipid Membranes, *Biophys J.* 2006 June 1; 90(11).



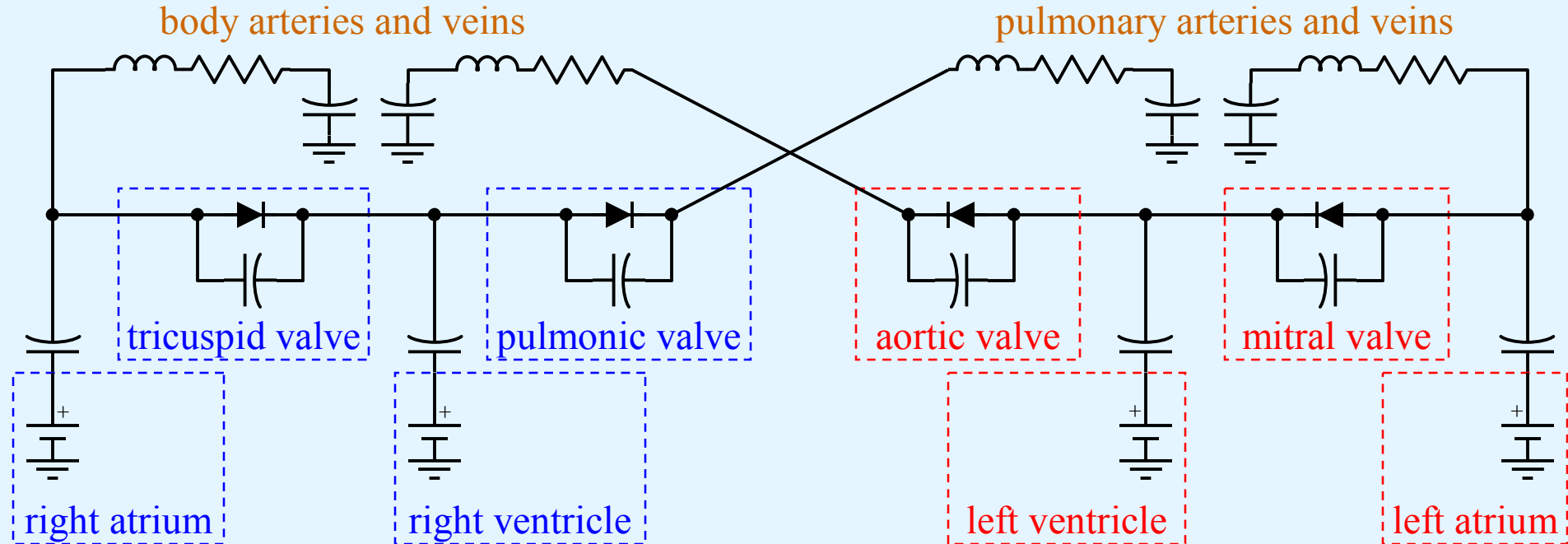
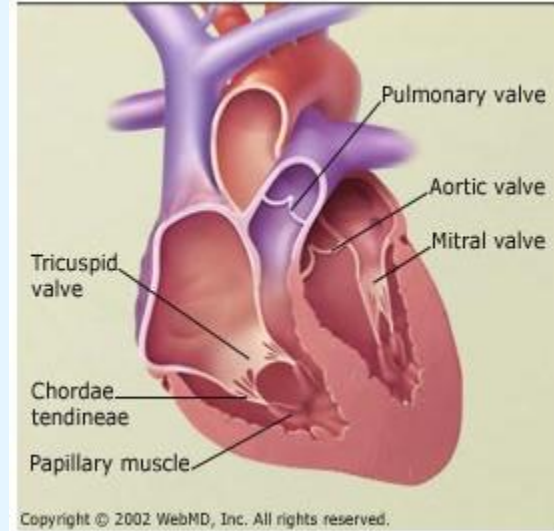
I'm a cell, and ions are expensive. I need to create a transmembrane potential difference.
What should I choose to perfuse my membrane?

- A Cholesterol, $\kappa = 2$
- B Water, $\kappa = 80$
- C Air, $\kappa = 1$
- D It doesn't matter



Electricity, with all my heart

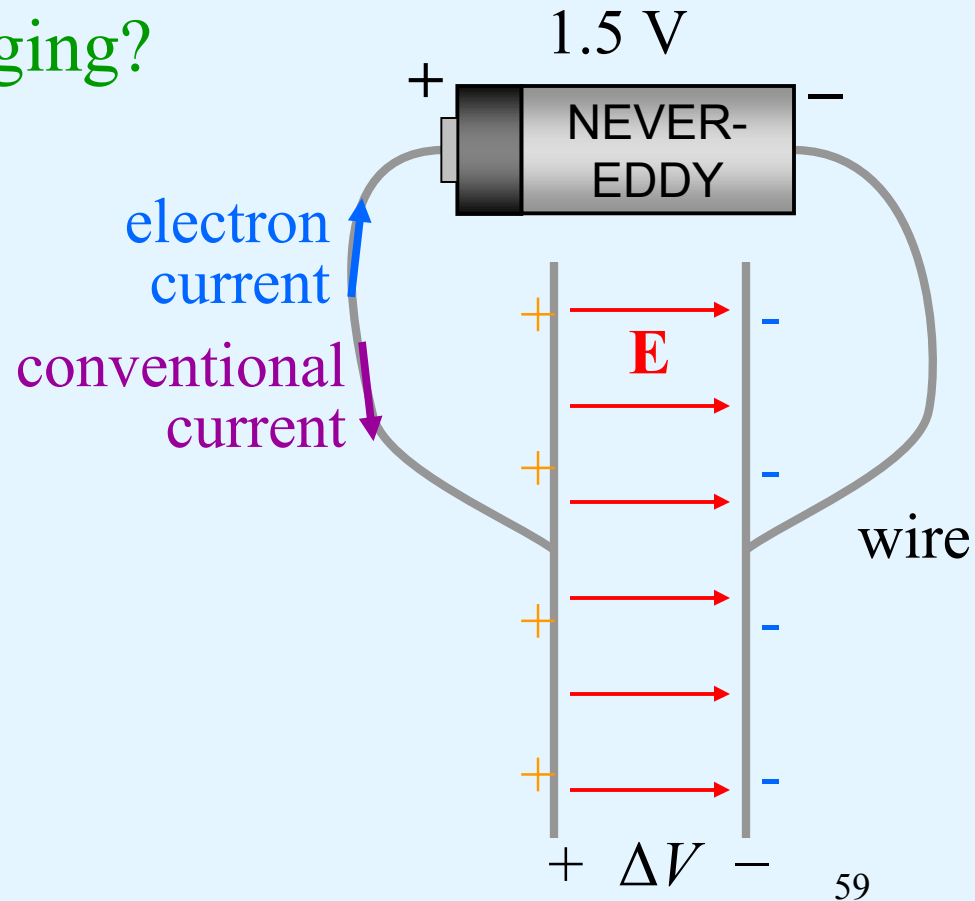
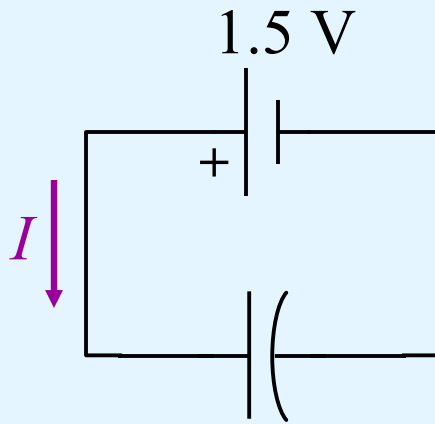
- Electrical model allows localizing performance of individual muscles or small groups from multiple measurements on the skin



Jose Bohorquez, *An Integrated-Circuit Switched-Capacitor Model and Implementation of the Heart*, Proceedings of the First International Symposium on Applied Sciences on Biomedical and Communication Technologies, 25-28 October 2008. DOI 10.1109/ISABEL.2008.4712624

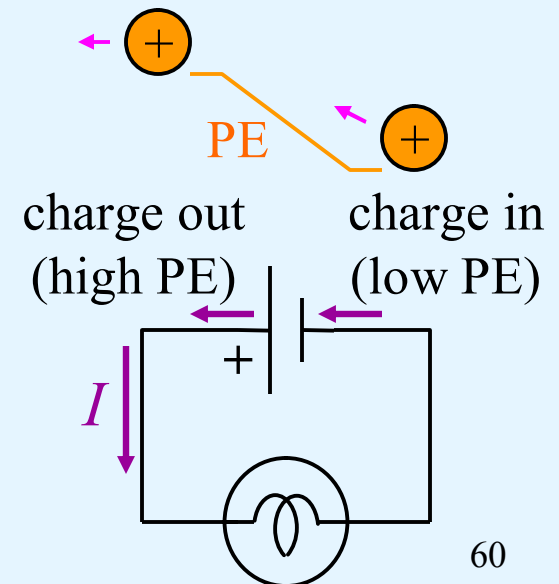
Charging a capacitor: some things just take time

- To what voltage (ΔV) does the capacitor charge?
 - Why does it stop charging?



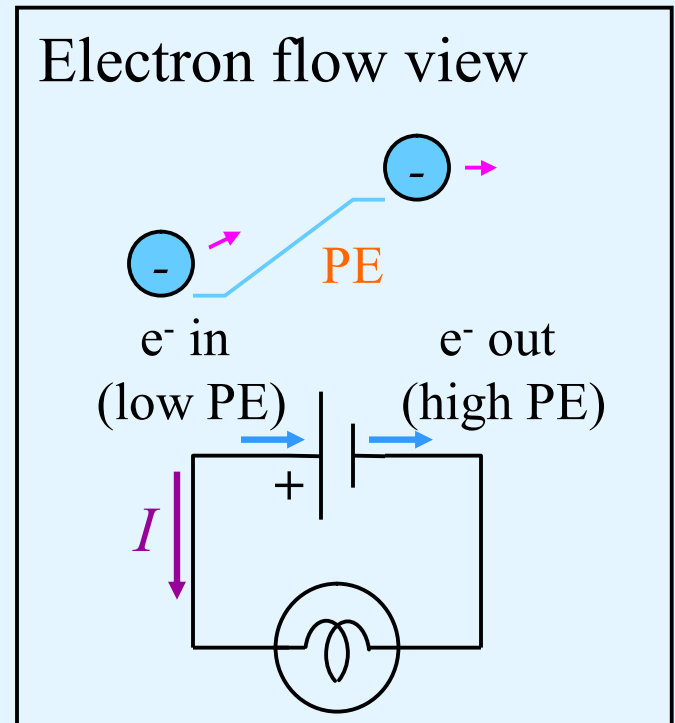
What is a voltage source?

- An **ideal voltage source** maintains a constant voltage across its terminals under all conditions
 - A battery is a voltage source. So is ...
 - ... a generator
 - Nothing is really ideal, but it's a useful model
- When charges move to reduce the voltage across the terminals ...
 - The voltage source pumps out more charge to keep the voltage up
 - Pulls in charge at low potential energy, pumps out a charge at high PE
 - This may or may not result in a continuous cycle of charge (current)
- An ideal voltage source can ...
 - Supply or absorb an arbitrarily large charge
 - Supply or absorb an arbitrary large current (arbitrarily large charge in arbitrarily short time)
 - Supply or absorb arbitrarily large energy



Ideal voltage source (2)

- Usually, voltage sources *supply* energy to the circuit

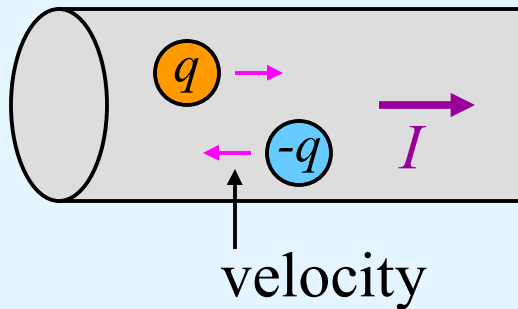


- But they can **just as well absorb** energy from the circuit



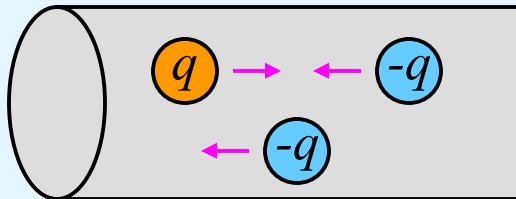
Electric current

- Conductors (good or poor) have mobile charges
- If immersed in an electric field, mobile charges move
- While they are moving, we have an electric **current**
 - **current** is the motion of charge



What describes the electric current below?

- A It is to the left
- B It is about zero
- C It is to the right
- D Depends on the conductor



Continuous current

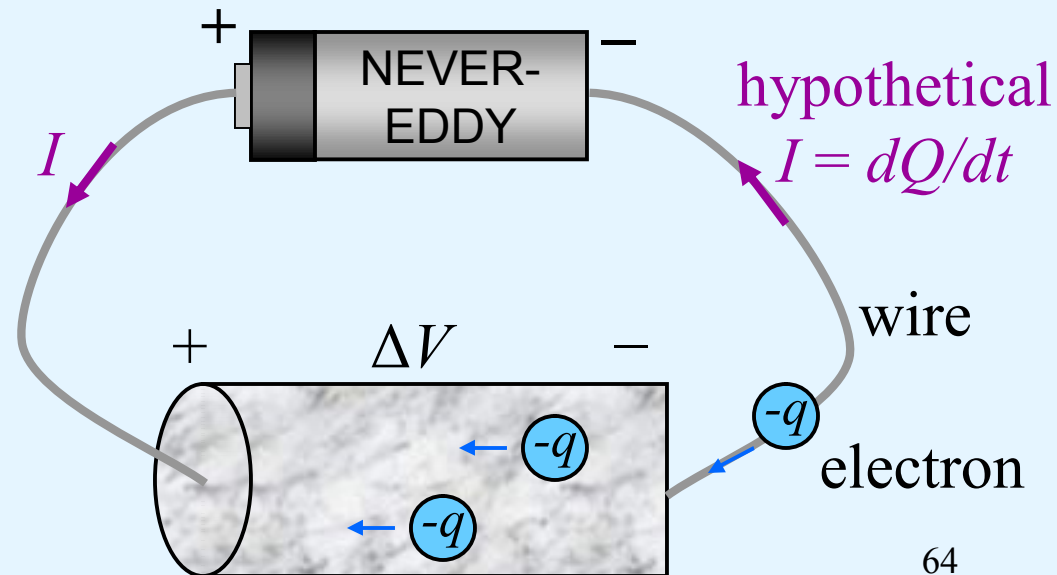
- If mobile charges are continuously removed from one side, and resupplied on the other, we have a **continuous current**
- **Conventional current** is the flow of hypothetical **positive** charges, in C/s \equiv **amperes**

(A)

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

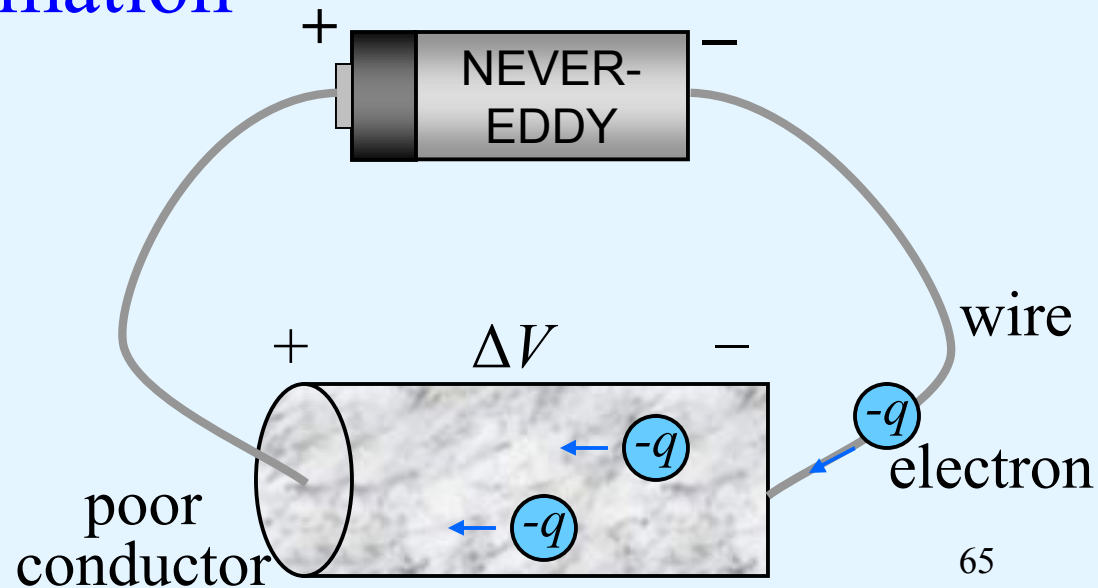
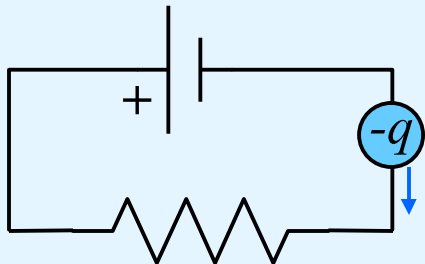
$$I(t) \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Most references use 'i' and 'q' for time-dependent values



How does the PE of the e^- going into the poor conductor compare to the PE of the e^- coming out?

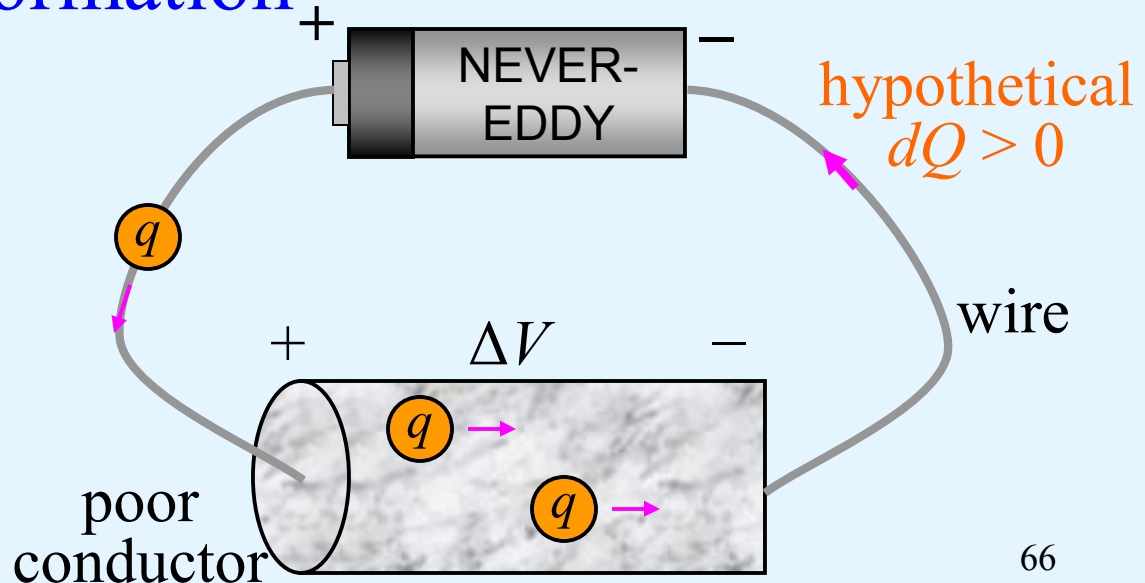
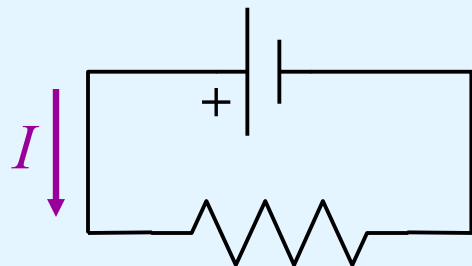
- A It's less
- B It's equal
- C It's more
- D not enough information



How does the PE of the hypothetical (+) charge going into the poor conductor compare to the PE of it coming out?

- A It's less
- B It's equal
- C It's more
- D not enough information

From now on, we *always* use conventional current when discussing circuits.



A wire is a good conductor. How much work does it take to move a charge through it?

A exactly zero

B essentially zero

C a lot

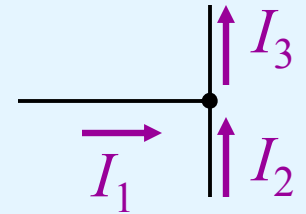
D it's impossible to move a charge through it

Kirchoff's current law (junction rule)

- Conservation of charge

- Sum of currents into a node = sum of currents out of a node

$$I_1 + I_2 = I_3$$

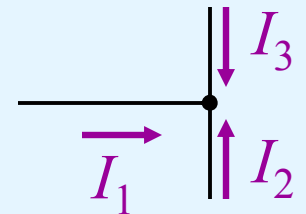


- “Node” is aka “junction”

- Sum of currents into a node = 0

- This is the book's version

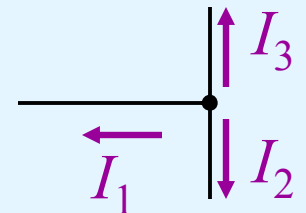
$$I_1 + I_2 + I_3 = 0$$



- Sum of currents out of a node = 0

- Not used much

$$I_1 + I_2 + I_3 = 0$$



Kirchoff's voltage law (loop rule)

- Conservation of energy

- PE of charge depends on its location, but not how it got there

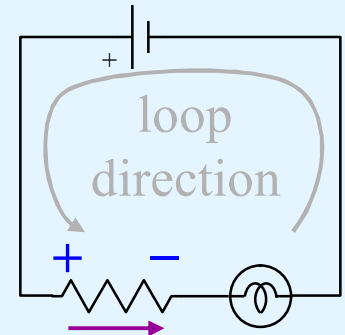
- Consider PE change of +1 C of test charge, at each node around a loop

- If possible, choose loop direction out from battery +
- I find it easiest to start from battery -

- Final PE must equal starting PE

- PE is proportional to charge (voltage)

- Must also be true of voltages

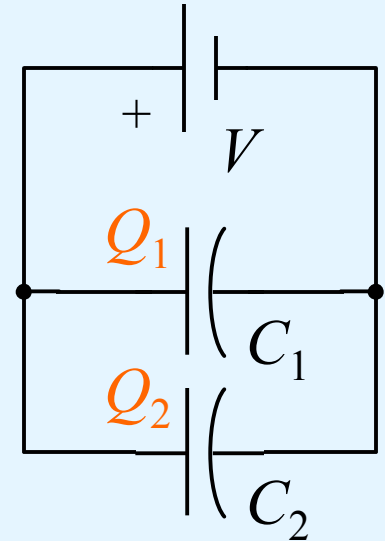


Capacitor combinations

Recall: $C \equiv \frac{Q}{\Delta V}$

- Capacitors in parallel

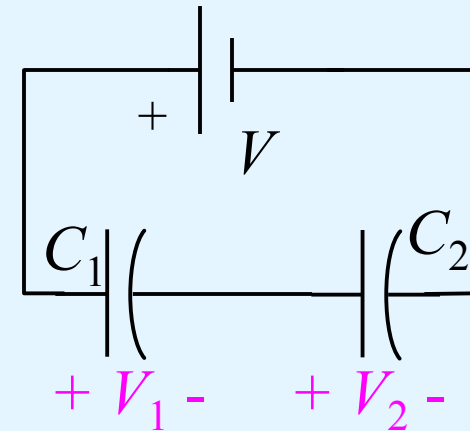
- When charged, how does V_1 compare to V_2 , and to V ?
- How about charges Q_1 and Q_2 ?



$$Q_{tot} = Q_1 + Q_2 \quad \Rightarrow \quad C_{tot} = \frac{Q_{tot}}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$$

- Capacitors in series

- When charged, how does V_1 compare to V_2 , and to V ?
- How about charges Q_1 and Q_2 ?



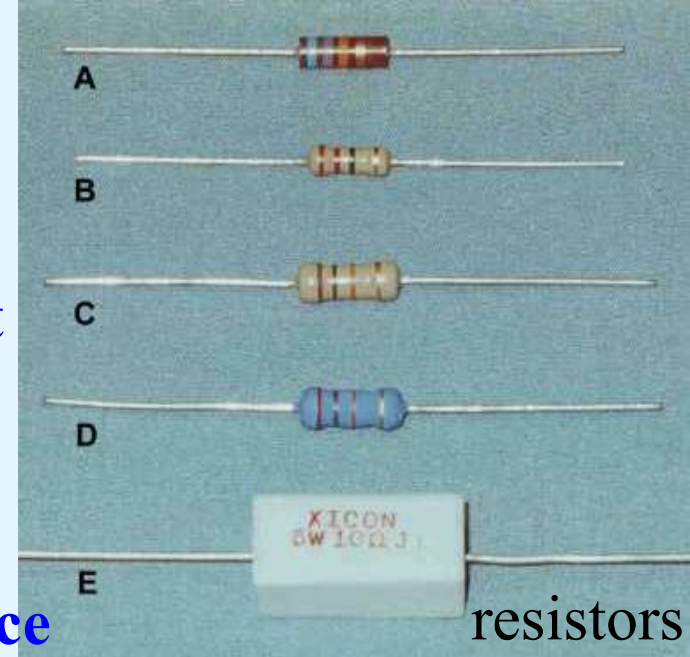
$$V_{tot} = V_1 + V_2 \quad \Rightarrow \quad C_{tot} = \frac{Q_{tot}}{V_{tot}} = \frac{Q}{Q/C_1 + Q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$

Ohm's Law

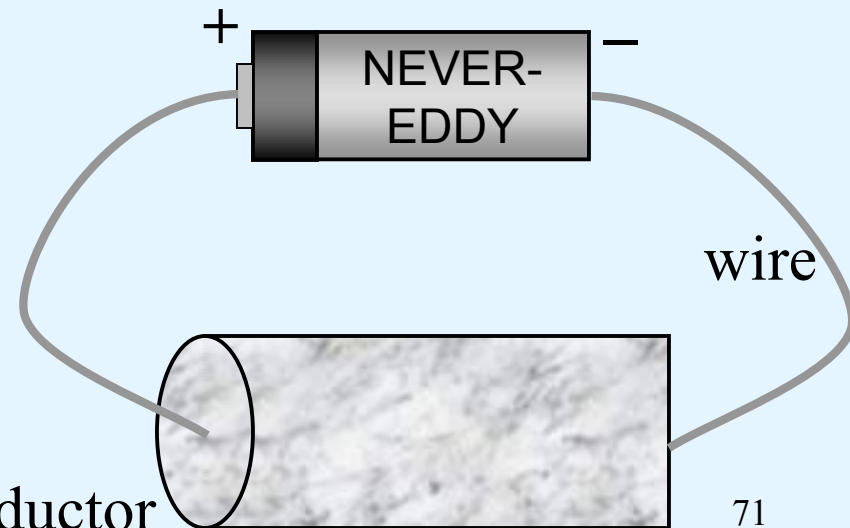
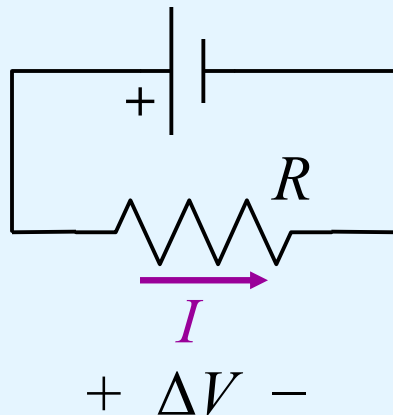
- For a large class of conductors (at constant temperature), the current through it is proportional to the voltage across it:

$$\Delta V = IR$$

- The constant of proportionality is **resistance**
 - Requires *consistency* in reference directions (sign conventions)
 - measured in **ohms** (Ω)
- Some things don't follow Ohm's Law
 - Light bulbs ?



resistors



Resistor combinations



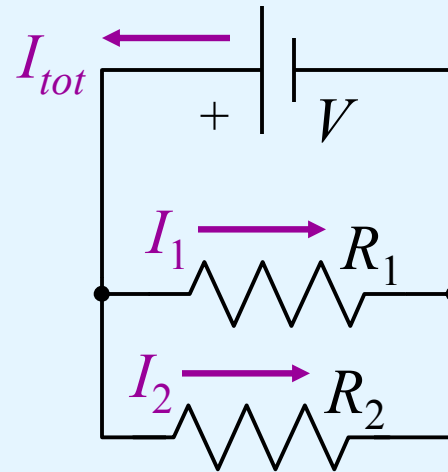
- Resistors in parallel

Recall: $\Delta V = IR$

- How does V_1 compare to V_2 , and to V ?
- How about currents I_1 and I_2 ?

$$I_{tot} = I_1 + I_2 \Rightarrow$$

$$R_{tot} = \frac{V}{I_{tot}} = \frac{V}{V/R_1 + V/R_2} = \frac{1}{1/R_1 + 1/R_2}$$

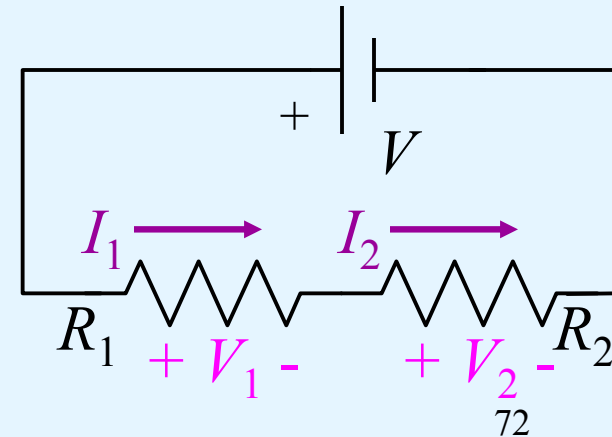


- Resistors in series

- How does V_1 compare to V_2 , and to V ?
- How about currents I_1 and I_2 ?

$$V_{tot} = V_1 + V_2 \Rightarrow$$

$$R_{tot} = \frac{V_{tot}}{I} = \frac{V_1 + V_2}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

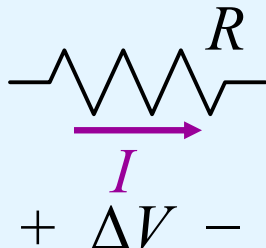


What we need is “More power”

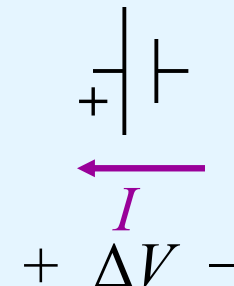
- Two terminal devices (R, C, diodes, batteries, etc.) supply or absorb electrical energy
 - How much? $P = (\Delta V)I$
 $\text{J/s} = \text{J/C} \cdot \text{C/s}$
 - Sign convention for sources is opposite that of all other devices
 - Resistors convert electrical energy into heat
 - Capacitors sometimes absorb & sometimes supply



Positive power **absorbs**
energy from circuit

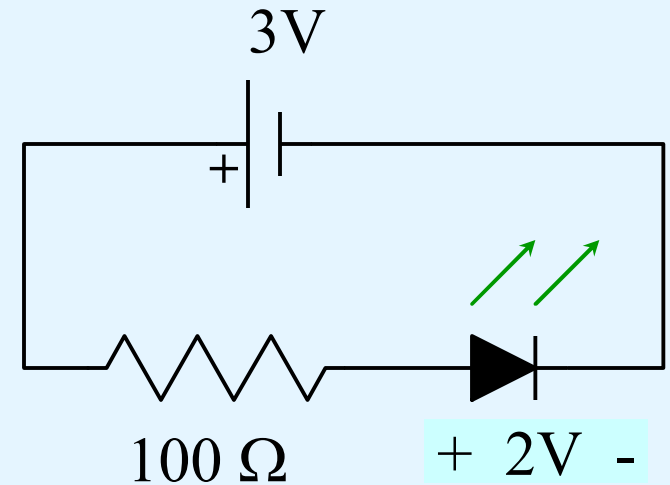


Positive power **supplies**
energy to circuit



What is the power dissipated in the diode, in W?

- A 0.10
- B 0.06
- C 0.04
- D 0.02
- E not enough information



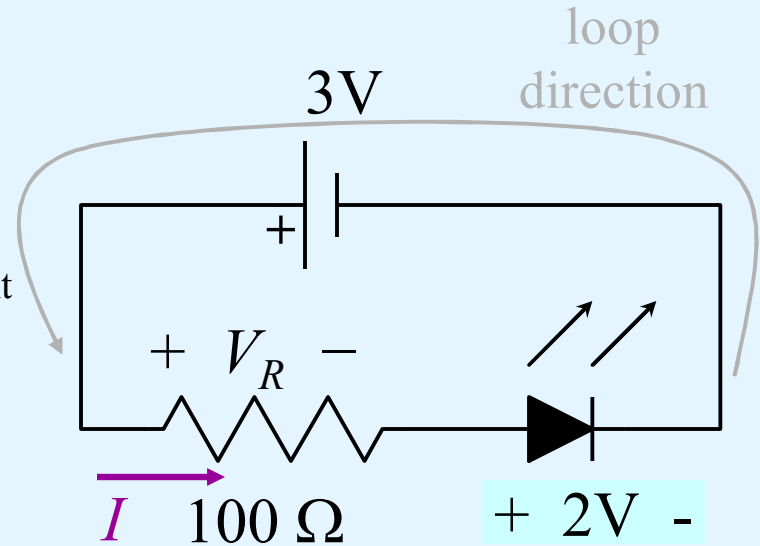
Solution

From the loop rule: $3 - V_R - 2 = 0 \Rightarrow V_R = 1 \text{ volt}$

Note the polarities on the loop rule

From Ohm's law: $I = V_R / 100 = 0.01 \text{ amperes}$

Power law on diode: $P = (2)(0.01) = 0.02 \text{ watts}$



Without using any formulas, what is the total resistance?

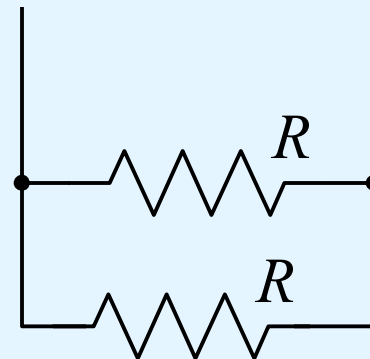
A $R/2$

B R

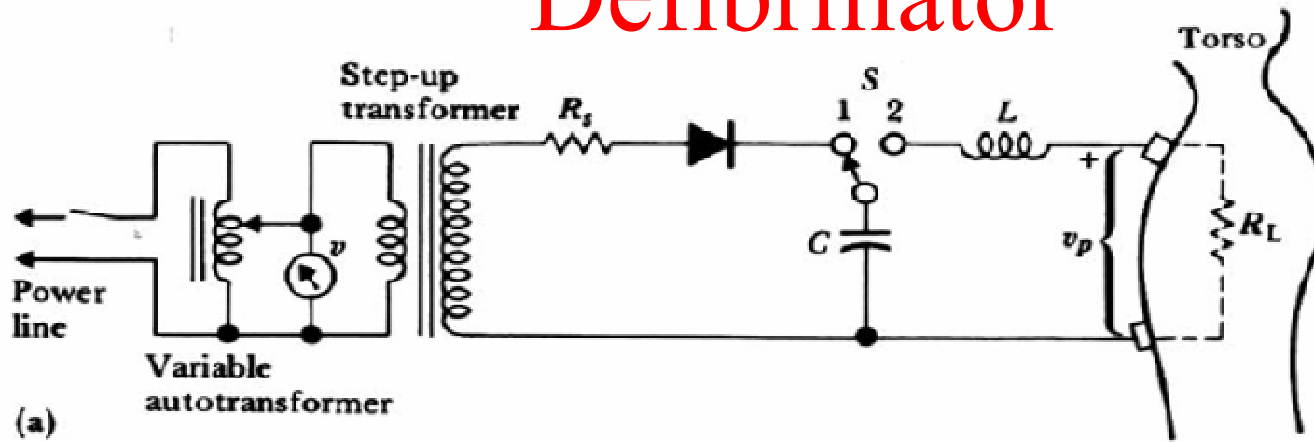
C $2R$

D Depends on R

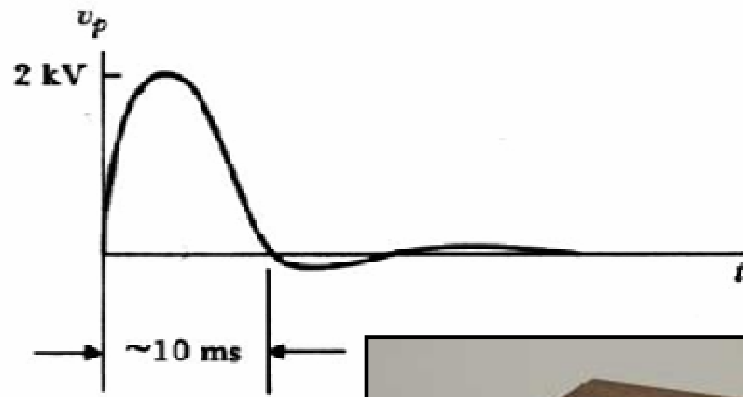
E 10 ohms



Defibrillator



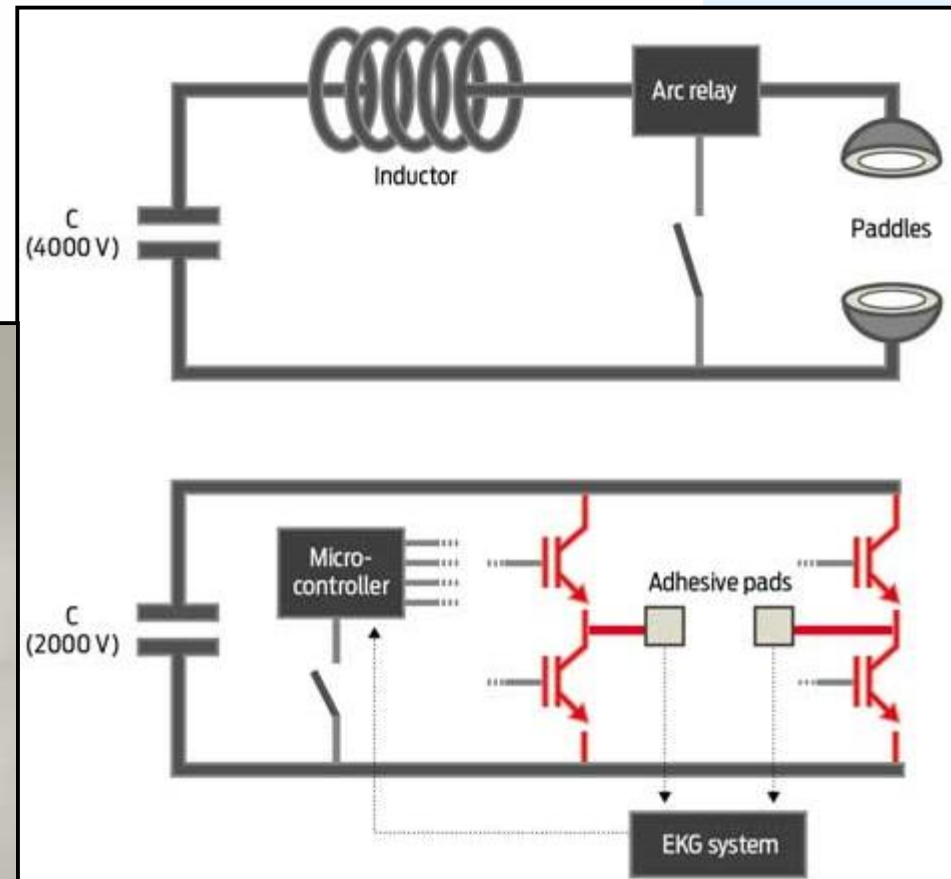
(a)



(b)

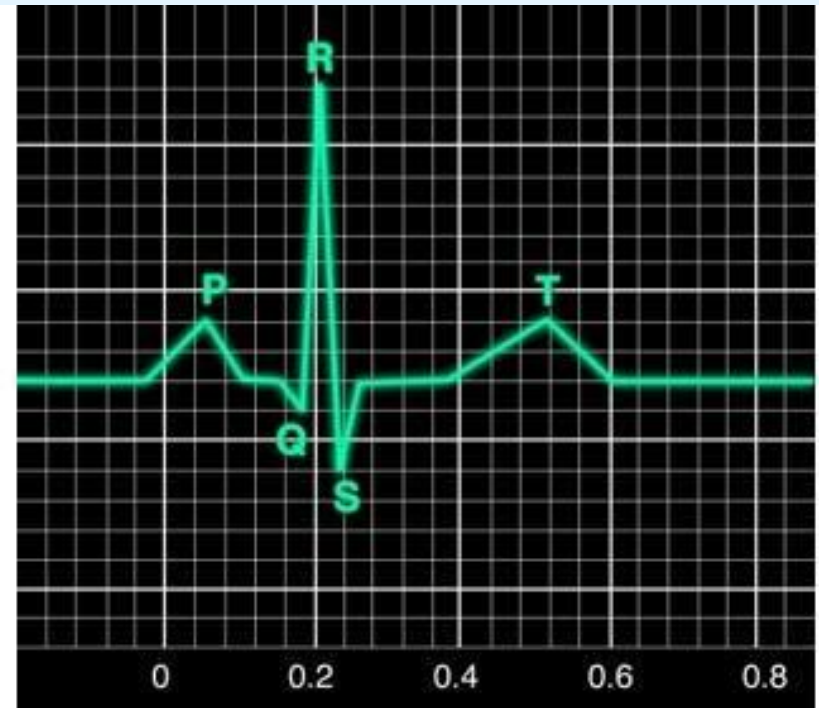


spoons →

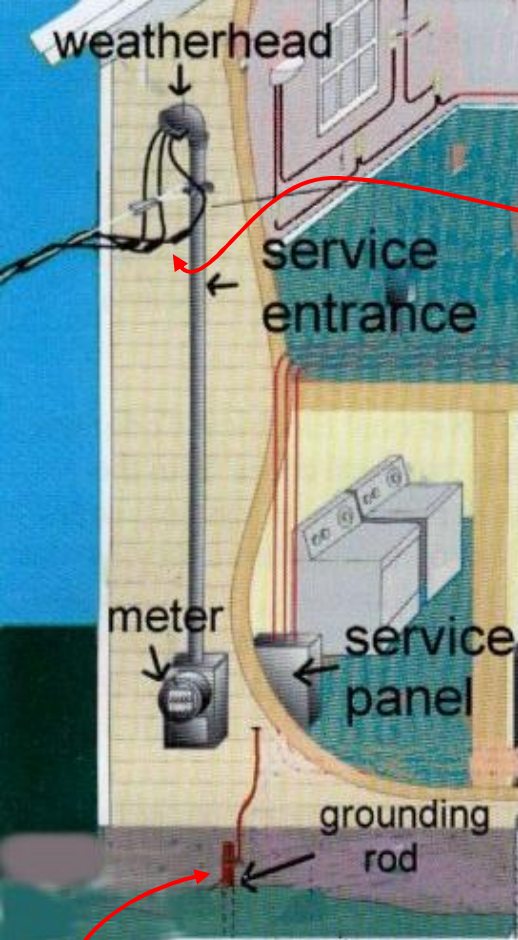


EKG

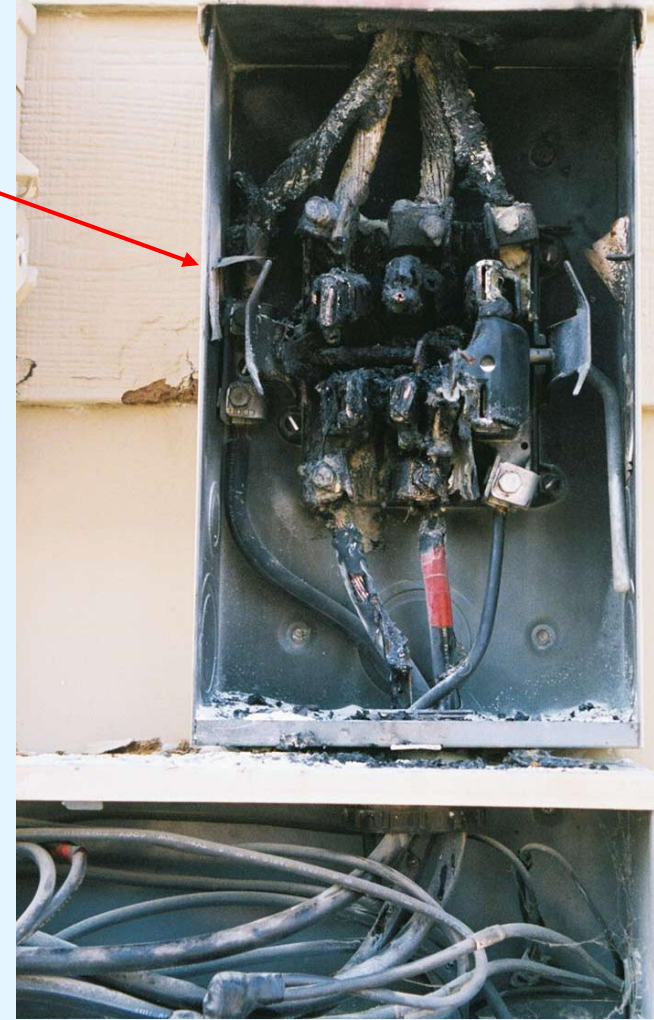
- Voltages measured between pairs of leads
- Sample of *one* voltage below right
- Different pairs measure different regions of the heart



Home electrical service



drip loops
prevent this



- Drip loops keep water out
- Grounding rod prevents arcing
- Find your grounding rod at home