PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) Consider a system described by the Hamiltonian

$$\hat{H} = -H \sum_{i=1}^{N} \sigma_i + \Delta \sum_{i=1}^{N} (1 - \sigma_i^2) \ ,$$

where each $\sigma_i \in \{-1, 0, +1\}$.

(a) Compute the ordinary canonical partition function $Z(T, N, H, \Delta)$ and the free energy $F(T, N, H, \Delta)$.

(b) Find the magnetization $M(T, N, H, \Delta)$.

(c) Show that $\frac{\partial M}{\partial \Delta} = -\frac{\partial N_0}{\partial H}$, where $N_0 = \sum_{i=1}^{N} \delta_{\sigma_i,0}$.

(2) Consider a three-dimensional gas of N identical particles of mass m, each of which has a magnetic dipole moment $\boldsymbol{m} = \mu_0 \hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is a three-dimensional unit vector. The Hamiltonian is

$$\hat{H} = \sum_{i=1}^{N} \left[\frac{\boldsymbol{p}_i^2}{2m} - \mu_0 \boldsymbol{H} \cdot \hat{\boldsymbol{n}}_i \right] .$$

(a) What is the grand potential $\Omega(T, V, \mu, \boldsymbol{H})$?

(b) Express $\boldsymbol{M}$ in terms of T, V, N, and $\boldsymbol{H}$, where N is the average number of particles.

(c) Find $M(T, V, N, \boldsymbol{H})$ to lowest order in the external field.