PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) A box of volume V contains N_1 identical atoms of mass m_1 and N_2 identical atoms of mass m_2.

(a) Compute the density of states $D(E, V, N_1, N_2)$.

(b) Let $x_1 \equiv N_1/N$ be the fraction of particles of species #1. Compute the statistical entropy $S(E, V, N, x_1)$.

(c) Under what conditions does increasing the fraction x_1 result in an increase in statistical entropy of the system? Why?

(2) Two chambers containing Argon gas at $p = 1.0\,\text{atm}$ and $T = 300\,\text{K}$ are connected via a narrow tube. One chamber has volume $V_1 = 1.0\,\text{L}$ and the other has volume $V_2 = r\,V_1$.

(a) Compute the RMS energy fluctuations of the particles in the smaller chamber when the volume ration is $r = 2$.

(b) Compute the RMS energy fluctuations of the particles in the smaller chamber when the volume ration is $r = \infty$.

(3) Consider a system of N identical but distinguishable particles, each of which has a nondegenerate ground state with energy zero, and a g-fold degenerate excited state with energy $\varepsilon > 0$.

(a) Let the total energy of the system be fixed at $E = M\varepsilon$, where M is the number of particles in an excited state. What is the total number of states $\Omega(E, N)$?

(b) What is the entropy $S(E, N)$? Assume the system is thermodynamically large. You may find it convenient to define $\nu \equiv M/N$, which is the fraction of particles in an excited state.

(c) Find the temperature $T(\nu)$. Invert this relation to find $\nu(T)$.

(d) Show that there is a region where the temperature is negative.

(e) What happens when a system at negative temperature is placed in thermal contact with a heat bath at positive temperature?

(4) Solve for the model in problem 3 using the ordinary canonical ensemble. The Hamiltonian is

$$\hat{H} = \varepsilon \sum_{i=1}^{N} \left(1 - \delta_{\sigma_i,1}\right) ,$$

where $\sigma_i \in \{1, \ldots, g+1\}$.

(a) Find the partition function $Z(T, N)$ and the Helmholtz free energy $F(T, N)$.

(b) Show that $\hat{M} = \frac{\partial \hat{H}}{\partial \varepsilon}$ counts the number of particles in an excited state. Evaluate the thermodynamic average $\nu(T) = \langle \hat{M} \rangle / N$.

(c) Show that the entropy $S = -\left(\frac{\partial F}{\partial T}\right)_N$ agrees with your result from problem 3.