Physics 2D Quiz 5 Solutions Winter 2011

Problem 1

$$\Delta p = 500\,\text{eV}/c$$

Use $\Delta x \Delta p = \hbar/2 \Rightarrow \Delta x = \frac{\hbar}{2\Delta p} = \frac{\hbar c}{2 \cdot 500\,\text{eV}} = \frac{1973\,\text{eV}\,\text{Å}}{1000\,\text{eV}}$

$$\Rightarrow \boxed{\Delta x = 1.973\,\text{Å}}$$

Since $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$ and $\langle x \rangle = 0$ we have

$$\boxed{\langle x^2 \rangle = (1.973\,\text{Å})^2 = 3.893\,\text{Å}^2}$$

(b) $\langle K \rangle = \frac{\langle p^2 \rangle}{2m_e}$; since $(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2$ and $\langle p \rangle = 0$,

$$\langle K \rangle = \frac{(\Delta p)^2}{2m_e} = \frac{500^2}{2 \cdot m_e c^2}\,\text{eV}^2 = \frac{500^2}{2 \cdot 0.511 \times 10^6}\,\text{eV} = 0.245\,\text{eV}$$

So $\boxed{\langle K \rangle = 0.245\,\text{eV}}$

(c) Use $\langle x^2 \rangle = \frac{\hbar}{2m_e\omega} \Rightarrow \langle U \rangle = \frac{1}{2} m_e \omega^2 \langle x^2 \rangle = \frac{1}{2} m_e \omega^2 \frac{\hbar}{2m_e\omega} =$

$= \frac{\hbar\omega}{4}$. Now the ground state energy is $E_0 = \frac{\hbar\omega}{2}$, and

$$E_0 = \langle K \rangle + \langle U \rangle \Rightarrow \boxed{\langle K \rangle = \langle U \rangle = \frac{\hbar\omega}{4} = 0.245\,\text{eV}}$$

(d) $\hbar\omega = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\hbar\omega}$; we have $\hbar\omega = 4\langle K \rangle = 0.978\,\text{eV}$

$$\Rightarrow \lambda = \frac{12{,}400\,\text{eV}\,\text{Å}}{0.978\,\text{eV}} \Rightarrow \boxed{\lambda = 12{,}673\,\text{eV}}$$

Problem 2

(a) For electrons incident on step potential, reflection coef. is

$$R = \frac{(k_1 - k_2)^2}{(k_1+k_2)^2} \; ; \qquad k_1 = \sqrt{\frac{2m}{\hbar^2} E} \, , \text{ with } E = 10\,eV, \ \frac{\hbar^2}{2m_e} = 3.81\,eV\mathring{A}^2$$

$$\Rightarrow k_1 = \sqrt{\frac{10}{3.81}}\ \mathring{A}^{-1} \Rightarrow \boxed{k_1 = 1.62\,\mathring{A}^{-1}}$$

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(10-8)eV} = \sqrt{\frac{2}{3.81}}\ \mathring{A}^{-1} = 0.725\,\mathring{A}^{-1}$$

$$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} \Rightarrow R = 0.146 \Rightarrow \boxed{T = 1-R = 0.854}$$

So for 1,000 incident electrons, $\boxed{\text{854 electrons reach point A}}$

(b) For tunneling through barrier, $U = 13\,eV$, $E = 10\,eV$, $U - E = 3\,eV \Rightarrow$

$$T = e^{-2\sqrt{\frac{2m}{\hbar^2}(U-E)}\,\Delta x} = e^{-2\sqrt{\frac{3}{3.81}}\cdot 2} = e^{-3.55} = 0.0287$$

So $\boxed{T = 0.0287}$. For 854 electrons incident on the barrier, $T \times 854$ reach point B, i.e. $\boxed{\sim 25 \text{ electrons}}$

(c) According to classical mech, $\boxed{\text{1000 electrons reach A}}$, none is reflected since $E > U_{step}$

According to class mech, $\boxed{\text{0 electrons reach B}}$, all are reflected at barrier since $E < U_{barrier}$

Problem 3

$$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2 m_e L_1^2} \left(n_1^2 + n_2^2 + \frac{L_1^2}{L_3^2} n_3^2 \right)$$

Since $L_1 = 1\,\text{Å}$, $L_3 = 0.667\,\text{Å} \Rightarrow \frac{L_1^2}{L_3^2} = 2.25\,\text{Å}$; $\frac{\hbar^2\pi^2}{2 m_e L_1^2} = 37.6\,\text{eV} \Rightarrow$

$$\boxed{E_{n_1, n_2, n_3} = 37.6\,\text{eV} \left(n_1^2 + n_2^2 + 2.25\, n_3^2 \right)}$$

n_1, n_2, n_3	$n_1^2+n_2^2+2.25n_3^2$	E
1, 1, 1	4.25	160eV $\leftarrow \equiv E_1$
2, 1, 1	7.25	273eV $\leftarrow \equiv E_2$
1, 1, 2	11	414eV $\leftarrow \equiv E_4$
2, 2, 1	10.25	385eV $\leftarrow \equiv E_3$
3, 1, 1	12.25	461eV

E_4 ———	(1, 1, 2)	(1 state)	
E_3 ———	(2, 2, 1)	(1 state)	(b)
E_2 ———	(2,1,1) and (1,2,1)	(2 states)	
E_1 ———	(1, 1, 1)	(1 state)	

(c) $\psi = C \sin\frac{\pi n_1 x}{L_1} \sin\frac{\pi n_2 y}{L_2} \sin\frac{\pi n_3 z}{L_3}$; for $\psi(y = \frac{L_2}{2}) = 0$

we need $n_2 = 2$. The two lowest energy states with $n_2 = 2$ are (1, 2, 1) with energy E_2 and (2, 2, 1) with energy E_3.