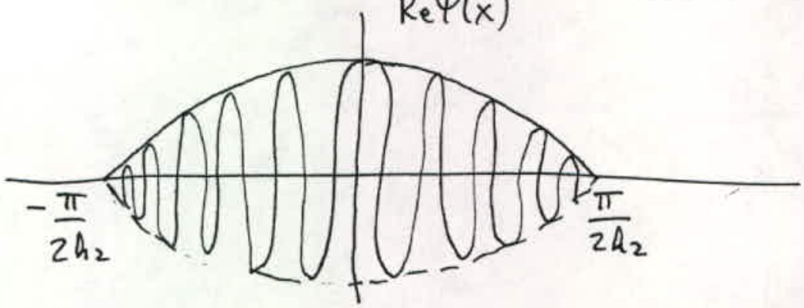
Physics 2D — Quiz 4 Solutions — Winter 2011

Problem 1

$$\Psi(x) = C\left(e^{i(k_1+k_2)x} + e^{i(k_1-k_2)x}\right) = Ce^{ik_1x}\left(e^{ik_2x} + e^{-ik_2x}\right) =$$

$$= 2Ce^{ik_1x}\cos(k_2x) \Rightarrow \boxed{\mathrm{Re}(\Psi(x)) = 2C\cos k_1x \cdot \cos k_2x}$$



(b) $\Delta x = 2 \cdot \frac{\pi}{2k_2} = \frac{\pi}{k_2} = \frac{\pi}{0.1\,\mathring{A}} = \boxed{31.4\,\mathring{A}^{-1}}$

(c) $p = \hbar k \Rightarrow \Delta p = \hbar \Delta k$ [redacted]

Δk is the spread in k values, here $\Delta k = k_1 + k_2 - (k_1 - k_2) = 2k_2$

$$\Rightarrow \Delta p = 2\hbar k_2 = \frac{2\hbar c k_2}{c} = \frac{2 \cdot 1973\,eV\mathring{A} \cdot 0.1\,\mathring{A}^{-1}}{c} = 395\frac{eV}{c}$$

$$\boxed{\Delta p = 395\,eV/c}$$

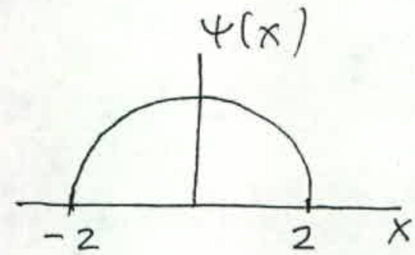
(d) $\Delta x \Delta p = \Delta x\, \Delta k \cdot \hbar = \frac{\pi}{k_2} \cdot 2k_2 \cdot \hbar = 2\pi\hbar$

Here $\boxed{\Delta x \Delta p = 2\pi\hbar}$, is consistent in order of magnitude with $\Delta x \Delta p \sim \hbar$

Problem 2

$$\psi(x) = C\sqrt{4-x^2} \quad ; \quad |\psi(x)|^2 = C^2(4-x^2)$$

(a) $$\frac{P(x=0)}{P(x=1)} = \frac{|\psi(x=0)|^2}{|\psi(x=1)|^2} = \frac{4}{4-1} = \frac{4}{3} = 1.333$$



So $$\boxed{\frac{P(x=0)}{P(x=1)} = 1.333}$$

(b) $$1 = \int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = \int_{-2}^{2} dx\, C^2(4-x^2) =$$

$$= C^2\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} = C^2\left(16 - 2\cdot\frac{8}{3}\right) = 16C^2\left(1-\frac{1}{3}\right) = 16C^2\cdot\frac{2}{3}$$

$$\Rightarrow C^2 = \frac{3}{32} \Rightarrow \boxed{C = \sqrt{\frac{3}{32}}}$$

(c) By symmetry, $\boxed{P(x \text{ between } 0 \text{ and } 2) = 0.5}$

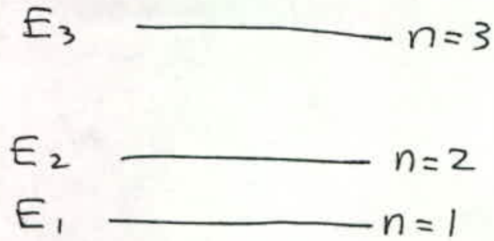
(d) $$P(-1 < x < 1) = \int_{-1}^{1} dx\, |\psi(x)|^2 =$$

$$= \frac{3}{32}\int_{-1}^{1} dx\,(4-x^2) = \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\Big|_{-1}^{1} =$$

$$= \frac{3}{32}\left(8 - \frac{2}{3}\right) = \frac{3}{32}\left(\frac{22}{3}\right) = \frac{22}{32} = \boxed{0.6875}$$

Problem 3

$$E_n = \frac{\hbar^2\pi^2}{2m_e L^2} n^2 = \frac{37.6\,\mathrm{eV}\mathring{A}^2}{L^2} n^2$$



Light with wavelength λ has photons of energy:

$$E = hf = \frac{hc}{\lambda} = \frac{12,400\,\mathrm{eV}\mathring{A}}{\lambda}$$

For $\lambda_1 = 1000\,\mathring{A}$, $\boxed{E_{\lambda_1} = 12.4\,\mathrm{eV}}$ For $\lambda_2 = 5000\,\mathring{A}$, $\boxed{E_{\lambda_2} = 2.48\,\mathrm{eV}}$

(a) We will get no absorption line if $E_2 - E_1 > 12.4\,\mathrm{eV}$

$$E_2 - E_1 = \frac{37.6\,\mathrm{eV}\mathring{A}^2}{L^2} \cdot 3 > 12.4\,\mathrm{eV} \Rightarrow \boxed{L < 3.02\,\mathring{A}}$$

So for example, for $L = 1\,\mathring{A}$ there is no absorption since

$$E_2 - E_1 = 37.6\,\mathrm{eV} \cdot 3 = 112.8\,\mathrm{eV} > E_{\lambda_1}, \text{ and } E_n - E_1 \geq E_2 - E_1$$

(b) As found above, to get any absorption we need $\boxed{L > 3.02\,\mathring{A}}$

(c) If L is too large we will get more than one absorption line.

To get 2 absorption lines we need

$$E_3 - E_1 < 12.4\,\mathrm{eV} \Rightarrow \frac{37.6\,\mathrm{eV}\mathring{A}^2}{L^2} \cdot 8 < 12.4\,\mathrm{eV} \Rightarrow$$

$\Rightarrow L > 4.93\,\mathring{A}$. Therefore, to get only one absorption line we need $\boxed{L < 4.93\,\mathring{A}}$. Note that for $L = 4.93\,\mathring{A}$,

$E_2 - E_1 = \frac{37.6}{4.93^2} \cdot 3\,\mathrm{eV} = 4.64\,\mathrm{eV}$ which is in the range of photon energies corresponding to $1000\,\mathring{A} < \lambda < 5000\,\mathring{A}$