Approach

Jackson develops Maxwell's equations empirically starting from Coulomb's law and proceeding through the law of Biot and Savart, Faraday's law, and Maxwell's inclusion of the displacement current. Almost all of you have been through this development twice (in lower division and in upper division). Landau and Lifshitz start with the special theory of relativity and then develop relativistic dynamics and Maxwell's equations from Hamilton's principle. I plan to follow this latter approach. It provides a unified treatment of mechanics and electrodynamics; it places the important physics up front where it can be treated slowly and carefully; and it de-emphasizes the drudgery of mathematically complex boundary value problems. Pedagogically this approach fits the order of our curriculum now that we begin electrodynamics after a quarter of mechanics.

The main difficulty is that the *Classical Theory of Fields* (and the Landau and Lifshitz series in general) is a little too elegant and dense and is not always easy to learn from. I will try to compensate by proceeding slowly and carefully. This is a case where an instructor may be useful. The homework problems in the *Classical Theory of Fields* tend to be too hard and too few in number, so we will extract problems from Jackson's *Classical Electrodynamics*. We will also read relevant chapters in the two texts in parallel. When we get to boundary value problems (for electrostatics and waveguides) we will use *Classical Electrodynamics* as the primary text.

Outline

1. Special relativity
2. Relativistic mechanics from Hamilton's principle
3. Dynamics of charges in a given electromagnetic field (use of vector potential in mechanics problems, guiding center drift theory, and adiabatic invariants)
4. Transformation of the fields
5. The field equations from Hamilton's principle
6. Conservation theorems and the stress tensor (examples where momentum in field plays an important role)
7. Electrostatics and magnetostatics (solution of problems with symmetry, multipole expansions, circuit elements)
8. Electromechanical systems
9. Maxwell equations for continuous media (polarization vector **P** and magnetization vector **M**, forces on magnetized and polarized material)
10. Logical exposition of unit systems (Gaussian and rationalized MKS)

1/7/02 Skip(1-4)

Special relativity Chapter 1, L+L
Chapter 11, Jackson

reference system (or frame) consists of 3 orthogonal axis and a clock. (length and time are primitive units)

An event (e.g., a spark) is characterized by (t, x, y, z). It is sometimes called a worldpoint (4-space being world space)

The motion of a particle is characterized by a trajectory [i.e., $(t, x(t), y(t), z(t))$], in 4-space. The trajectory is sometimes called a world line (or curve).

Can define velocity, acceleration of the particle (i.e., $v_x = \frac{dx}{dt}$, etc.)

There exist reference frames such that space is homogeneous and isotropic and time is homogeneous. These are called inertial frames.

(Note that an isolated particle that is initially at rest in such a frame must remain at rest.)

1st. postulate { The Principle of Relativity states that the laws of physics are the same in all inertial frames.

2nd postulate { Speed of light (c) is independent of the source and is finite. ($c \simeq 2.998 \times 10^{10}$ cm/sec).

(Here, we should have substituted max signal velocity for speed of light; photons could have finite rest mass.)

Therefore speed of light is the same in all inertial frames.

An <u>interval</u> between two events is $s_{1,2}$ where

$$s_{12}^2 \equiv c^2(t_2-t_1)^2 - (x_2-x_1)^2 - (y_2-y_1)^2 - (z_2-z_1)^2$$

$$ds^2 \equiv (ds)^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

1/8/02

<u>Invariance of $s_{1,2}$</u> (i.e. $s_{1,2} = s'_{1,2}$)

in another inertial frame

First consider the case $s_{1,2} = 0$

Let event 1 be ~~emission~~ of pulse of light.

Let event 2 be reception of pulse of light.

$$s_{12}^2 = c^2t_{12}^2 - l_{12}^2 = 0$$

$$\therefore \; s_{12}'^2 = c^2t_{12}'^2 - l_{12}'^2 = 0$$

Since $ds^2 = 0$ implies that $ds'^2 = 0$
and since ds^2 and ds'^2 are
differentials of the same order

$$ds^2 = f(|\underset{\sim}{v}|)\, ds'^2 + \underset{\underset{0}{\shortparallel}}{d\lambda^2}$$

$\underset{\sim}{v}$ is relative velocity between
the two systems

$f(|\underset{\sim}{v}|)$ is independent of position
because space is homogeneous
$f(|\underset{\sim}{v}|)$ is independent of direction
of $\underset{\sim}{v}$ because space is isotropic

direct and inverse transformation

$$ds^2 = f(|\underset{\sim}{V}|)\, ds'^2 , \quad ds'^2 = f(|-\underset{\sim}{V}|)\, ds^2$$

$$\therefore ds^2 = f(|V|) \underbrace{f(|-\underset{\sim}{V}|)}_{f(|\underset{\sim}{V}|)} ds^2$$

$$\therefore f^2 = \pm 1 , \quad \therefore f = 1$$

By integration $ds = ds' \Rightarrow S_{12} = S'_{12}$

<u>Timelike intervals and spacelike intervals</u>

S_{12} is timelike if

$$S_{12}^2 = c^2 t_{12}^2 - l_{12}^2 > 0$$

Timelike intervals cannot be simultaneous in any frame

$$0 < S_{12}^2 = S_{12}'^2 \neq c^2 \underset{\substack{\parallel \\ 0}}{t_{12}'^2} - l_{12}'^2$$

S_{12} is <u>spacelike</u> if

$$S_{12}^2 = ct_{12}^2 - l_{12}^2 < 0$$

Events separated by spacelike intervals cannot occur at the same point in any reference frame

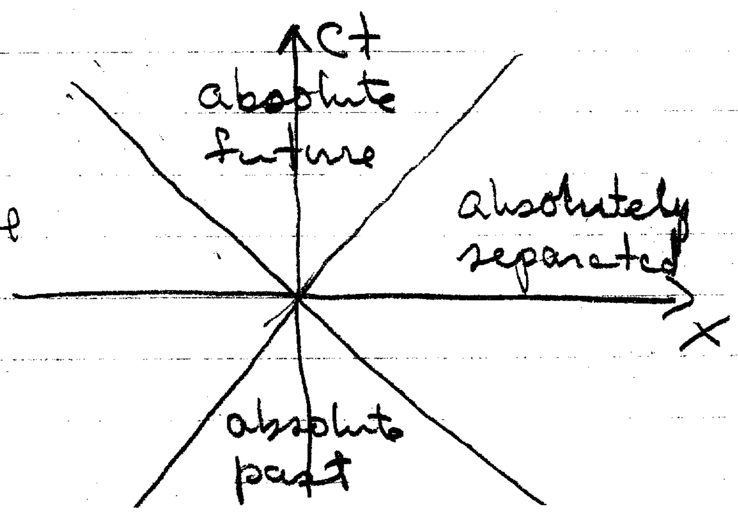
$$0 > S_{12}^2 = S_{12}'^2 \neq ct_{12}'^2 - \underbrace{l_{12}'^2}_{0}$$

Timelike and spacelike are not dependent on the reference system

Only events separated by timelike intervals can be causally related

<u>Light cone</u>

in 4-space



Proper time

A particle with an attached clock moves in an arbitrary manner

can be accelerating

ds = an interval defined by ticks on the clock

proper time $= \frac{ds}{c}$ ← independent of reference frame

Let an inertial system (x', y', z', t') be instantaneously coincident with the particle

$$dx' = dy' = dz' = 0$$

$$dt' = dt_{\substack{\text{particle} \\ \text{clock}}}$$

$$\therefore \; c^2 dt'^2 - 0 = ds'^2 = ds^2$$

$$\therefore \; dt_{\substack{\text{particle} \\ \text{clock}}} = \frac{ds}{c}$$

Let $[t, \underline{r}(t)]$ be world line (or curve) of the particle in some reference frame

$$c^2 dt^2_{\substack{\text{particle}\\\text{clock}}} = ds'^2 = ds^2 = c^2dt^2 - (d\underline{r})^2$$

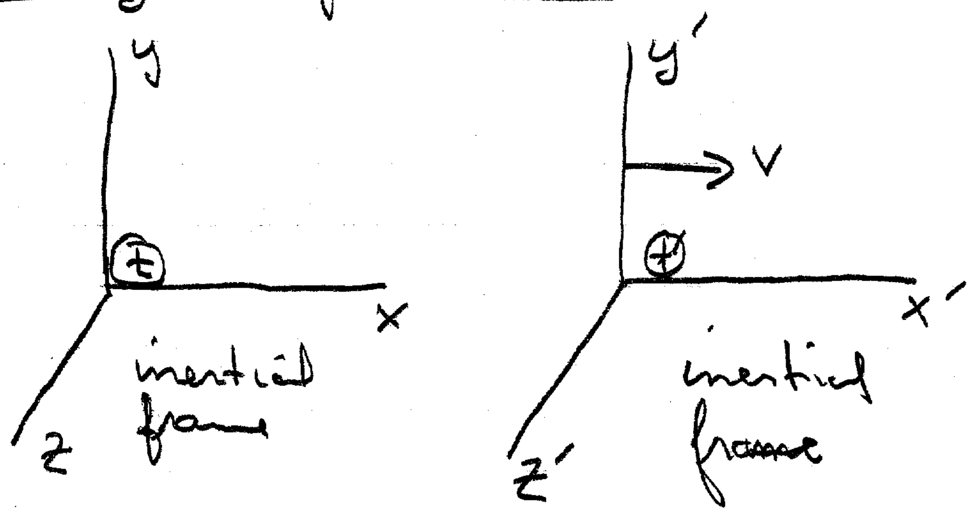
$$= c^2 dt^2 \left(1 - v^2/c^2\right)$$

$$(t_2 - t_1)_{\substack{\text{particle}\\\text{clock}}} = \int_{t_1}^{t_2} dt \left[1 - v^2/c^2\right]^{1/2}$$

Homework:

(1) At rest a μ meson has a life time of $\tau = 2.2 \times 10^{-6}$ sec. If it travels at speed $v = .988c$, how far will it go before it decays?

Lorentz transformation



For an event observed in the two frames, we want to determine the transformation

$$(t', x', y', z') \xrightarrow{T} (t, x, y, z)$$

T is linear since space and time are homogeneous

T preserves the length of intervals (i.e., $S_{12}^2 = S_{12}'^2$)

(in this space with strange metric)

∴ T is a rotation plus a displacement

only changes origin of space & time

A general rotation can be decomposed into rotations in the six planes

$$\underbrace{xy,\ xz,\ yz}_{\text{spatial rotations}},\ xt,\ yt,\ zt$$

this rotation corresponds to the diagram

Next require that $S_{12}^2 = S_{12}'^2$

Let event 1 occur at origin when origins are coincident

$$x_1', y_1', z_1', t_1' = x_1, y_1, z_1, t_1 = 0$$

Let event 2 be arbitrary event

$$(x', y', z', t') \qquad (x, y, z, t)$$

$y' = y$ and $z' = z$ ← for orientation in diagram

(The y and y' axis are coincident initially and are not changed by a rotation in the xt plane)

$$c^2t'^2 - x'^2 = c^2t^2 - x^2$$

try $\quad x = x'\cosh\psi + ct'\sinh\psi$

$$ct = x'\sinh\psi + ct'\cosh\psi$$

$$c^2t^2 - x^2 = x'^2\overbrace{[\sinh^2\psi - \cosh^2\psi]}^{-1}$$
$$+ c^2t'^2\underbrace{[\cosh^2\psi - \sinh^2\psi]}_{+1}$$

to evaluate ψ, consider $x' = 0$

$$vt = x = ct'\sinh\psi$$

$$ct = ct'\cosh\psi$$

$$\tanh\psi = \frac{v}{c}$$

$$\therefore \quad \cosh\psi = \frac{1}{\sqrt{1-\tanh^2\psi}} = \frac{1}{\sqrt{1-v^2/c^2}}, \quad \sinh\psi = \frac{v/c}{\sqrt{1-v^2/c^2}}$$

$$\therefore \quad x = \frac{x' + Vt'}{\sqrt{1 - V^2/c^2}} \qquad t = \frac{t' + x'V/c^2}{\sqrt{1 - V^2/c^2}}$$

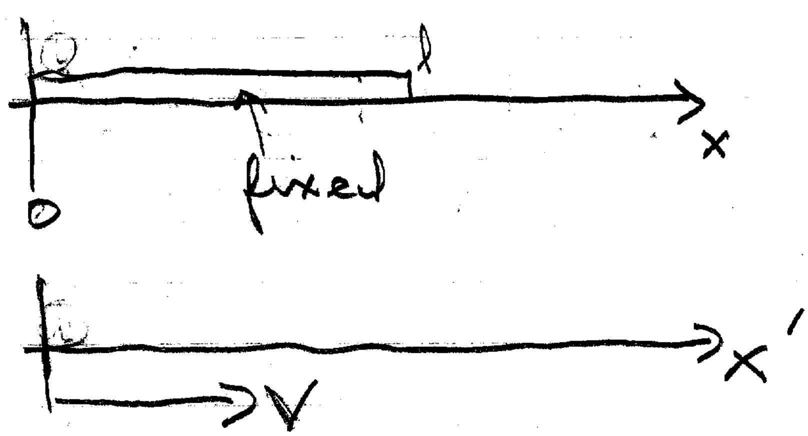
$$y' = y \quad , \quad z' = z$$

Comments

Obtain T^{-1} by letting $V \to -V$

Reduces to Galilean transformation in limit $V/c \ll 1$.

<u>length contraction</u>



events 1 and 2 occur at ends of rod simultaneously in primed frame

$$t_1' = t_2'$$

$$l = x_2 - x_1 = \frac{\overbrace{x_2' - x_1'}^{l'} + v(\overbrace{t_2' - t_1'}^{=0})}{\sqrt{1 - v^2/c^2}}$$

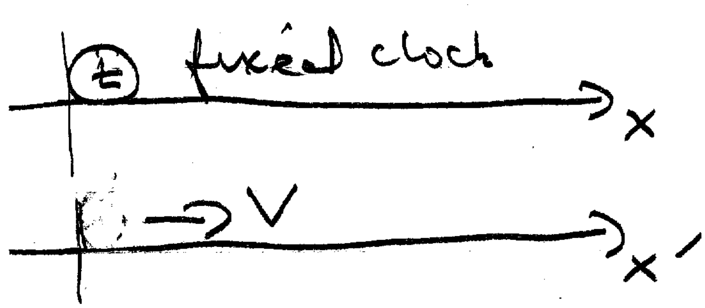
for any t_2, t_1

$$\therefore \quad l = \frac{l'}{\sqrt{1 - v^2/c^2}}$$

since $y' = y$, $z' = z$

$$V_{ol} = \frac{(V_{ol})'}{\sqrt{1 - v^2/c^2}}$$

time dilation



events 1 and 2 are ticks on unprimed clock $(x_1 = x_2)$

$$t' = \frac{t - xV/c^2}{\sqrt{1 - V^2/c^2}} \quad \text{(inverse tran.)}$$

$$t_2' - t_1' = \frac{t_2 - t_1 - \overbrace{(x_2 - x_1)}^{=0} V/c^2}{\sqrt{1 - V^2/c^2}}$$

$$\overbrace{t_2' - t_1'}^{\Delta t'} = \frac{\overbrace{t_2 - t_1}^{\Delta t}}{\sqrt{1 - V^2/c^2}}$$

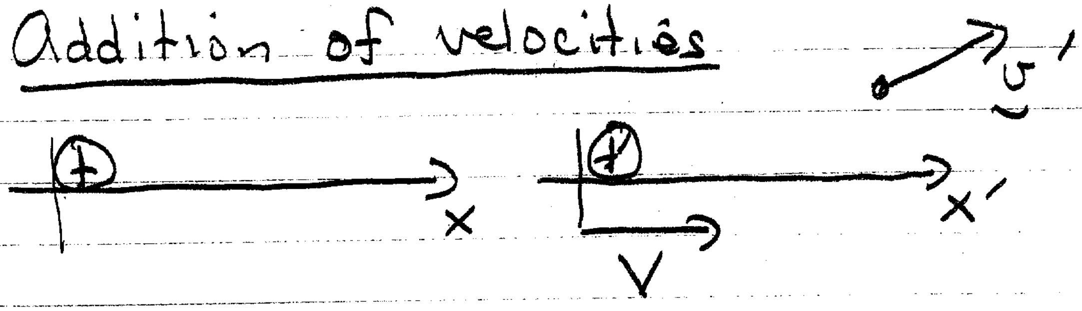
Note that

$$(\mathrm{Vol}')(\Delta t)' = \sqrt{1 - V^2/c^2}\,(\mathrm{Vol})\frac{\Delta t}{\sqrt{1 - V^2/c^2}} = \underbrace{\mathrm{Vol}\,\Delta t}$$

clock and rod both stationary

$\therefore \quad d^3x'\,dt' = d^3x\,dt$

Addition of velocities



$$dx = \frac{dx' + V\,dt'}{\sqrt{1 - V^2/c^2}}\,, \qquad dy = dy'\,, \quad dz = dz'$$

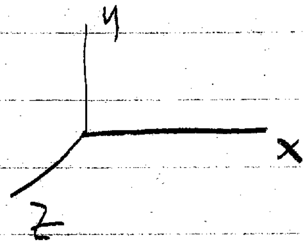
$$dt = \frac{dt' + \frac{V}{c^2}\,dx'}{\sqrt{1 - V^2/c^2}}$$

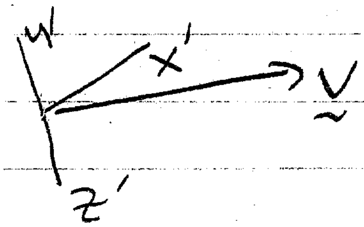
$$v_x = \frac{dx}{dt} = \frac{dx'/dt' + V}{1 + \frac{V}{c^2}\frac{dx'}{dt'}} = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}}$$

$$v_y = \frac{v_y' \sqrt{1 - V^2/c^2}}{1 + v_x' V/c^2}$$

$$v_z = \frac{v_z' \sqrt{1 - V^2/c^2}}{1 + v_x' V/c^2}$$

General Lorentz Transformation





$$\mathbf{r}_{\parallel}' = \frac{\mathbf{r}' \cdot \mathbf{v}\,\mathbf{v}}{v^2} \qquad \mathbf{r}_{\perp}' = \mathbf{r}' - \mathbf{r}_{\parallel}'$$

$$\mathbf{r}_{\perp} = \mathbf{r}_{\perp}' \qquad \mathbf{r}_{\parallel} = \frac{\mathbf{r}_{\parallel}' + \mathbf{v}t'}{\sqrt{1 - v^2/c^2}}$$

$$\mathbf{r}' = \mathbf{r}' + \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)\frac{\mathbf{r}' \cdot \mathbf{v}\,\mathbf{v}}{v^2} + \frac{\mathbf{v}t'}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{1}{\sqrt{1 - v^2/c^2}}\left(t' + \frac{\mathbf{r}' \cdot \mathbf{v}}{c^2}\right)$$

can now reduce to component representation

Homework

(2) Jackson 11.4

(3) J. 11.3 11.2 product

(4) J. 11.5 acceleration

(5) J. 11.6 twins

(6) Verify J Eq (11.29) ~~for Doppler shift~~ (Hint: phase is invariant)

phase is invariant

(7) Show that

$$1 = \frac{\partial(t', x', y', z')}{\partial(t, x, y, z)}$$

for a general Lorentz Transformation (Hint: decompose into product of rotations)