PHYSICS 4E QUIZ 3 SPRING QUARTER 2011
PROF. HIRSCH MAY 9th

Formulas and constants:

$hc = 12{,}400\ eV\,A$; $k_B = 1/11{,}600\ eV/K$; $ke^2 = 14.4 eVA$; $m_e c^2 = 0.511\times10^6 eV$; $m_p/m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3\times10^8 m/s$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67\times10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Planck : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$; Wien : $\lambda_m T = hc/4.96k_B$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton : $\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1-\cos\theta)$; $\lambda_c \equiv \frac{h}{m_e c} = 0.0243A$; Rutherford : $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $V = \frac{kq}{r}$ (potential) ; $U = q_0V$ (potential energy)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0\frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $E_0 = \frac{ke^2}{2a_0} = 13.6eV$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2Z}{r^2} = m\frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass : $\mu = \frac{mM}{m+M}$; X - ray spectra : $f^{1/2} = A_n(Z-b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973\ eV\,A$

Wave packets : $y(x,t) = \sum_j a_j\cos(k_jx - \omega_jt)$, or $y(x,t) = \int dk\ a(k)\ e^{i(kx-\omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\ dx = |\Psi(x,t)|^2\ dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $<A> = \int_{-\infty}^{\infty} dx\psi^* A_{op}\psi$

Eigenvalues and eigenfunctions : $A_{op}\Psi = a\Psi$ (a is a constant) ; uncertainty : $\Delta A = \sqrt{<A^2> - <A>^2}$

Harmonic oscillator : $\Psi_n(x) = C_nH_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2$; $\Delta n = \pm1$

Step potential : $R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E-V)}$

PHYSICS 4E QUIZ 3 SPRING QUARTER 2011
PROF. HIRSCH MAY 9th

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

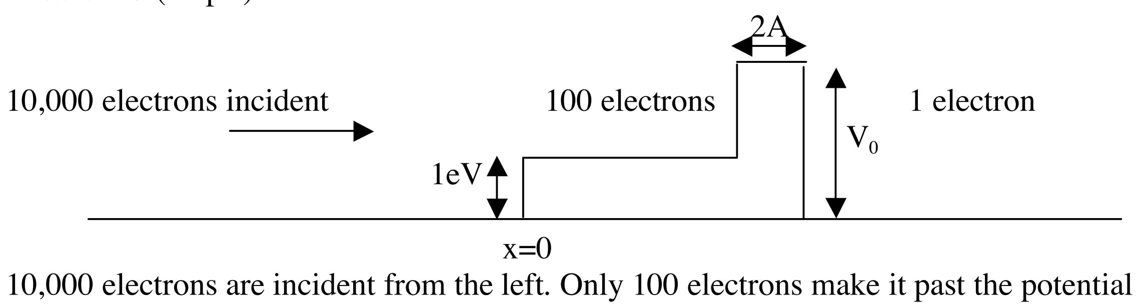
Justify all your answers to all problems

Problem 1 (10 pts)
An electron in an infinite one-dimensional square well has energy 5 eV.
(a) What is the smallest possible width of this well? Give your answer in A. What are two other possible value for the width of this well?

Assuming the width of the well is the smallest found in (a),
(b) What is the uncertainty in the momentum of this electron, Δp? Give your answer in units eV/c. Do not use Heisenberg's uncertainty principle to find this answer, use the definition of uncertainty given in the formula sheet.
(c) To check whether your answer in (b) makes sense, do the following: if L is the width of the well, you can assume that $\Delta x \sim L$, so by the uncertainty principle ΔpL should equal a dimensionless number (of order unity) times $\hbar$. Find that dimensionless number.
(d) How much more likely is it to find this electron in a small region around x=L/2 than in a small region of the same width around x=L/6?
Use: $\hbar^2/2m_e = 3.81\ eVA^2$

Problem 2 (10 pts)
The wavefunction for an electron in a harmonic oscillator potential is
$\psi(x) = Ce^{-x^2/a^2}$ with a=1A.
(a) Find C, in units $A^{-1/2}$.
(b) Find the uncertainty in the position, Δx, give its value in A.
(c) Find the energy of this electron, in eV.
Use $\int_{-\infty}^{\infty} dx\, e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$, $\int_{-\infty}^{\infty} dx\, x^2 e^{-\lambda x^2} = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$

Problem 3 (10 pts)



10,000 electrons are incident from the left. Only 100 electrons make it past the potential step of height 1 eV at x=0, and only 1 electron makes it past the barrier of height V_0 and width 2A.
(a) Find the kinetic energy of the incident electrons, in eV.
(b) Find the value of V_0 in eV.

Justify all your answers to all problems