PHYSICS 4E QUIZ2 SPRING QUARTER 2011
PROF. HIRSCH APRIL 25

Formulas and constants:

$hc = 12{,}400\ eV\,A$; $k_B = 1/11{,}600\ eV/K$; $ke^2 = 14.4 eVA$; $m_e c^2 = 0.511\times10^6 eV$; $m_p/m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3\times10^8 m/s$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator: $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67\times10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = hc/4.96 k_B$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton scattering : $\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1-\cos\theta)$; $\lambda_c \equiv \frac{h}{m_e c} = 0.0243A$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $V = \frac{kq}{r}$ (potential) ; $U = q_0 V$ (potential energy)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0\frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $E_0 = \frac{ke^2}{2a_0} = 13.6eV$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2Z}{r^2} = m\frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass : $\mu = \frac{mM}{m+M}$; X - ray spectra : $f^{1/2} = A_n(Z-b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973\ eV\,A$

Wave packets : $y(x,t) = \sum_j a_j\cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Justify all your answers to all 3 problems

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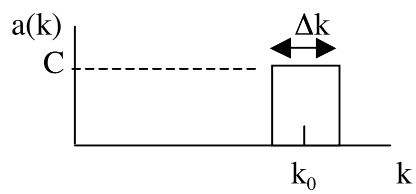
Problem 1 (10 pts)
Radiation with wavelengths in the range $240A \leq \lambda \leq 480A$ is incident on a gas of He^+ ions (Z=2) that is at room temperature.
(a) Find all the wavelengths that will be absorbed. Give your answers in A, to two decimal places.
(b) After the ions absorb radiation, they will emit radiation. Find all the wavelengths that will be emitted. Give your answers in A, no decimal places.
(c) For (a) and (b) you should have ignored the fact that the mass of the nucleus of He^+ is not infinite. Now take into account that the mass of the He^+ nucleus is finite and find a new value for the longest wavelength absorbed, in A. By how much does it differ from the result found in (a)?
Hint: He nucleus has 2 protons and 2 neutrons, you may assume that protons and neutrons have approximately the same mass.

Problem 2 (10 pts + 2 pts extra credit)
An electron in a hydrogen-like ion is in its ground state, its speed as given by the Bohr model is approximately 0.3c (c=speed of light). You may ignore relativistic corrections.
(a) What is the Z for this ion? What is the radius of the orbit, in A?
(b) For an electron confined to a spatial region of length given by the radius found in (a), estimate its kinetic energy by using the uncertainty principle. Give your answer in eV.
(c) For an electron at the distance given by the radius found in (a), find the potential energy in eV, and find the total energy in eV.
(d) For extra credit: assume now the radius of the orbit is half of what you found in (a) for this ion. Estimate the kinetic energy using the uncertainty principle, the potential energy and the total energy, and compare this total energy with the result found in (c).
Use: $\hbar^2/2m_e = 3.81\ eVA^2$

Problem 3 (10 pts)



An electron is described by the wavepacket $\psi(x,t) = \int dk a(k) e^{i(kx-\omega(k)t)}$, with k_0=10A^{-1} and Δk=1A^{-1} and a(k) shown in the graph above.
(a) Find an expression for $|\psi(x,t=0)|$ (|| means absolute value) and make a graph of it.
(b) Give the smallest positive and negative values of x for which $\psi(x,t=0)$=0. Call the distance between those points Δx and give the value of Δx in A.
(c) Decide whether this electron should be described using classical mechanics or relativistic mechanics. Explain your answer.
(d) Using what you decided in (c), find the phase velocity and the group velocity of this electron. Give your answers as v_p/c and v_g/c.

Justify all your answers to all problems