Formulas and constants:

 $\begin{aligned} hc &= 12,400 \ eVA \ ; k_B = 1/11,600 \ eV/K \ ; ke^2 = 14.4eVA \ ; m_c c^2 = 0.511 \times 10^6 eV \ ; m_p/m_e = 1836 \\ \text{Relativistic energy - momentum relation} \quad E &= \sqrt{m^2 c^4 + p^2 c^2} \qquad ; \qquad c = 3 \times 10^8 \ m/s \\ \text{Photons:} \ E &= hf \quad ; p = E/c \ ; f = c/\lambda \\ \text{Lorentz force:} \ \vec{F} &= q\vec{E} + q\vec{v} \times \vec{B} \\ \text{Planck's law :} \quad u(\lambda) = n(\lambda) \bar{E}(\lambda) \quad ; \quad n(\lambda) = \frac{8\pi}{\lambda^4} \quad ; \quad \bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1} \\ \text{Energy in a mode/oscillator :} \quad E_f = nhf \quad ; \text{ probability } P(E) \propto e^{-E/k_B T} \\ \text{Stefan's law :} \ R &= \sigma T^4 \quad ; \sigma = 5.67 \times 10^{-8} W / m^2 K^4 \ ; R = cU/4 \ , U = \int_0^\infty u(\lambda) d\lambda \\ \text{Wien's displacement law :} \ \lambda_m T = hc/4.96k_B \\ \text{Photoelectric effect :} \quad eV_0 = (\frac{1}{2}mv^2)_{\text{max}} = hf - \phi \quad , \quad \phi = \text{work function} \\ \text{Compton scattering :} \qquad \lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta) \quad ; \quad \lambda_c = \frac{h}{m_e c} = 0.0243A \\ \text{Rutherford scattering:} \qquad b = \frac{kq_aQ}{m_av^2} \cot(\theta/2) \quad ; \qquad \Delta N \propto \frac{1}{\sin^4(\theta/2)} \\ \text{Electrostatics :} \ F = \frac{kq_1q_2}{r^2} \ (\text{force}) \quad ; \quad V = \frac{kq}{r} \ (\text{potential)} \quad ; \quad U = q_0V \ (\text{potential energy}) \end{aligned}$

Justify all your answers to all 3 problems

Problem 1 (10 pts)

A black body is at temperature 11,600 K.

(a) At what wavelength does it emit maximum radiation? Give your answer in A (Angstrom).

(b) What is the average energy per mode of oscillation for the wavelength found in (a)? Give your answer in eV.

(c) Find a range of wavelengths where the average energy per mode of oscillation is at least 0.9 eV. The range of wavelengths found should be at least 500 A wide. Justify your answer.

(d) Assume this blackbody, at temperature 11,600K, is a sphere of radius R. Another blackbody is a sphere of radius R/2 and emits the same total power as the first. What is its temperature?

Problem 2 (10 pts)

In a Compton scattering experiment, the outgoing photon has a wavelength 0.9 A and the outgoing electron has kinetic energy 188 eV.

(a) Find the wavelength of the incoming photon, in A.

(b) Find the angle at which the photon is scattered relative to the direction of the incident photon, in degrees.

(c) Find the angle at which the electron is scattered relative to the direction of the incident photon, in degrees.

Problem 3 (10 pts)

A beam of α particles is incident on an Aluminum foil (Z=13). The radius of the Al nucleus is R=3.6x10⁻⁵A.

(a) For what value of the α particles kinetic energy is the distance of closest approach to the nucleus equal to the radius of this nucleus? Give your answer in MeV.

(b) For a beam of α particles of kinetic energy of value found in (a) it is found that 100 α particles scatter at angle 90° per second. What is the value of the impact parameter for these α particles, expressed in terms of the nuclear radius R?

(c) For this beam of α particles, how many α particles scatter at angle 45° per second? What is their impact parameter expressed in terms of the nuclear radius R?

(d) In what range of kinetic energy of α particles will there be deviations from

Rutherford's scattering law for scattering with this Al foil? Justify your answer.

Justify all your answers to all problems