

7-29. (a) Every increment of charge follows a circular path of radius  $R$  and encloses an area

$\pi R^2$ , so the magnetic moment is the total current times this area. The entire

charge

$Q$  rotates with frequency  $f = \omega / 2\pi$ , so the current is

$$i = Qf = q\omega / 2\pi$$

$$\mu = iA = (Q\omega / 2\pi)(\pi R^2) = Q\omega R^2 / 2$$

$$L = I\omega = \frac{1}{2}MR^2\omega$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2 / 2}{QMR^2\omega / 2} = 2$$

(b) The entire charge is on the equatorial ring, which rotates with frequency  $f = \omega / 2\pi$ .

$$i = Qf = Q\omega / 2\pi$$

$$\mu = iA = (Q\omega / 2\pi)(\pi R^2) = Q\omega R^2 / 2$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2 / 2}{QMR^2\omega / 5} = 5/2 = 2.5$$

7-33. (a) There should be four lines corresponding to the four  $m_j$  values  $-3/2, -1/2, +1/2, +3/2$ .

(b) There should be three lines corresponding to the three  $m_\ell$  values  $-1, 0, +1$ .

7-35. For

$$\ell = 2, \quad L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{6}\hbar = 2.45\hbar, \quad j = \ell \pm 1/2 = 3/2, 5/2 \text{ and } J = \sqrt{j(j+1)}\hbar$$

$$\text{For } j = 3/2, \quad J = \sqrt{(3/2)(3/2+1)}\hbar = \sqrt{15/4}\hbar = 1.94\hbar$$

$$\text{For } j = 5/2, \quad J = \sqrt{(5/2)(5/2+1)}\hbar = \sqrt{35/4}\hbar = 2.96\hbar$$

7-39. (a)  $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$

$$\ell = (\ell_1 + \ell_2), (\ell_1 + \ell_2 - 1), \dots, |\ell_1 - \ell_2| = (1+1), (1+1-1), (1-1) = 2, 1, 0$$

(b)  $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

$$s = (s_1 + s_2), (s_1 + s_2 - 1), \dots, |s_1 - s_2| = (1/2 + 1/2), (1/2 - 1/2) = 1, 0$$

(c)  $\mathbf{J} = \mathbf{L} + \mathbf{S}$

$$j = (\ell + s), (\ell + s - 1), \dots, |\ell - s|$$

For  $\ell = 2$  and  $s = 1$ ,  $j = 3, 2, 1$

$\ell = 2$  and  $s = 0$ ,  $j = 2$

For  $\ell = 1$  and  $s = 1$ ,  $j = 2, 1, 0$

$\ell = 1$  and  $s = 0$ ,  $j = 1$

For  $\ell = 0$  and  $s = 1$ ,  $j = 1$

$\ell = 0$  and  $s = 0$ ,  $j = 0$

(d)  $\mathbf{J}_1 = \mathbf{L}_1 + \mathbf{S}_1 \quad j_1 = \ell_1 \pm 1/2 = 3/2, 1/2$

$\mathbf{J}_2 = \mathbf{L}_2 + \mathbf{S}_2 \quad j_2 = \ell_2 \pm 1/2 = 3/2, 1/2$

(e)  $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 \quad j = (j_1 + j_2), (j_1 + j_2 - 1), \dots, |j_1 - j_2|$

For  $j_1 = 3/2$  and  $j_2 = 3/2$ ,  $j = 3, 2, 1, 0$

$j_1 = 3/2$  and  $j_2 = 1/2$ ,  $j = 2, 1$

For  $j_1 = 1/2$  and  $j_2 = 3/2$ ,  $j = 2, 1$

$j_1 = 1/2$  and  $j_2 = 1/2$ ,  $j = 1, 0$

These are the same values as found in (c).

7-40. (a)  $E_{3/2} = \frac{hc}{\lambda}$  Using values from Figure 7-22,

$$E_{3/2} = \frac{1239.852 \text{ eV} \cdot \text{nm}}{588.99 \text{ nm}} = 2.10505 \text{ eV} \quad E_{1/2} = \frac{1239.852 \text{ eV} \cdot \text{nm}}{589.59 \text{ nm}} = 2.10291 \text{ eV}$$

(b)  $\Delta E = E_{3/2} - E_{1/2} = 2.10505 \text{ eV} - 2.10291 \text{ eV} = 2.14 \times 10^{-3} \text{ eV}$

(c)  $\Delta E = 2\mu_B B \rightarrow B = \frac{\Delta E}{2\mu_B} = \frac{2.14 \times 10^{-3} \text{ eV}}{2(5.79 \times 10^{-4} \text{ eV/T})} = 18.5 \text{ T}$

7-43. (a) For electrons: Including spin, two are in the  $n = 1$  state, two are in the  $n = 2$  state, and

one is in the  $n = 3$  state. The total energy is then:

$$E = 2E_1 + 2E_2 + E_3 \quad \text{where } E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2} \quad E = 2E_1 + 2(2^2 E_1) + (3^2 E_1) = 19E_1$$

where

$$E_1 = \frac{(hc)^2 \pi^2}{2m_e c^2 L^2} = \frac{(197.3)^2 \pi^2}{2(0.511 \times 10^6)(1.0)^2} = 0.376 eV \quad E = 19E_1 = 7.14 eV$$

(b) Pions are bosons and all five can be in the  $n = 1$  state, so the total energy is:

$$E = 5E_1 \quad \text{where } E_1 = \frac{0.376 eV}{264} = 0.00142 eV \quad E = 5E_1 = 0.00712 eV$$

7-44. (a) Carbon:  $Z = 6$ ;  $1s^2 2s^2 2p^2$

(b) Oxygen:  $Z = 8$ ;  $1s^2 2s^2 2p^4$

(c) Argon:  $Z = 18$ ;  $1s^2 2s^2 2p^6 3s^2 3p^6$

7-45. Using Figure 7-34:

Sn ( $Z = 50$ )

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^2$$

Nd ( $Z = 60$ )

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6 4f^4 6s^2$$

Yb ( $Z = 70$ )

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 6s^2$$

Comparison with Appendix C.

Sn: agrees

Nd:  $5p^6$  and  $4f^4$  are in reverse order

Yb: agrees

7-46. Both *Ga* and *In* have electron configurations  $(ns)^2 (np)$  outside of closed shells

$(n-1, s)^2 (n-1, p)^6 (n-1, d)^{10}$ . The last  $p$  electron is loosely bound and is more easily removed than one of the  $s$  electrons of the immediately preceding elements *Zn* and *Cd*.

7-48.  $E_n = -\frac{Z_{eff}^2 E_1}{n^2}$  (Equation 7-25)

$$Z_{eff} = n \sqrt{\frac{-E_n}{E_1}} = 3 \sqrt{\frac{5.14 eV}{13.6 eV}} = 1.84$$

7-49. (a) Fourteen electrons, so  $Z = 14$ . Element is silicon.

(b) Twenty electrons, so  $Z = 20$ . Element is calcium.

7-50. (a) For a  $d$  electron,  $\ell = 2$ , so  $L_z = -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar$

(b) For an  $f$  electron,  $\ell = 3$ , so  $L_z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-56. (a)  $E_1 = -13.6 eV (Z - 1)^2 = -13.6 eV (74 - 1)^2 = -7.25 \times 10^4 eV = -72.5 keV$

(b)  $E_1(\text{exp}) = -69.5 keV = -13.6 eV (Z - \sigma)^2 = -13.6 eV (74 - 1)^2$

$$74 - \sigma = \left(69.5 \times 10^3 eV / 13.6 eV\right)^{1/2} = 71.49$$

$$\sigma = 74 - 71.49 = 2.51$$

7-58. (a)  $\Delta E = hc / \lambda$

$$E(3P_{1/2}) - E(3S_{1/2}) = \frac{1240 eV \cdot nm}{589.59 nm} = 2.10 eV$$

$$E(3P_{1/2}) = E(3S_{1/2}) + 2.10 eV = -5.14 eV + 2.10 eV = -3.04 eV$$

$$E(3D) - E(3P_{1/2}) = \frac{1240 eV \cdot nm}{818.33 nm} = 1.52 eV$$

$$E(3D) = E(3P_{1/2}) + 1.52 eV = -3.04 eV + 1.52 eV = -1.52 eV$$

(b) For  $3P$ :  $Z_{eff} = 3 \sqrt{\frac{3.04 eV}{13.6 eV}} = 1.42$

For  $3D$ :  $Z_{eff} = 3 \sqrt{\frac{1.52 eV}{13.6 eV}} = 1.003$

(c) The Bohr formula gives the energy of the  $3D$  level quite well, but not the  $3P$  level.

- 7-59. (a)  $\Delta E = gm_j\mu_B B$  (Equation 7-72) where  $s = 1/2$ ,  $\ell = 0$  gives  $j = 1/2$  and (from Equation 7-73)  $g = 2$ .  $m_j = \pm 1/2$ .

$$\Delta E = (2)(\pm 1/2)(5.79 \times 10^{-5} \text{ eV/T})(0.55T) = \pm 3.18 \times 10^{-5} \text{ eV}$$

The total splitting between the  $m_j = \pm 1/2$  states is  $6.37 \times 10^{-5} \text{ eV}$ .

- (b) The  $m_j = 1/2$  (spin up) state has the higher energy.

- (c)  $\Delta E = hf \rightarrow f = \Delta E/h = 6.37 \times 10^{-5} \text{ eV} / 4.14 \times 10^{-15} \text{ eV}\cdot\text{s} = 1.54 \times 10^{10} \text{ Hz}$

This is in the microwave region of the spectrum.

7-61. (a)  $\Delta E = \frac{e\hbar}{2m} B = (5.79 \times 10^{-5} \text{ eV/T})(0.05T) = 2.90 \times 10^{-6} \text{ eV}$

(b)  $|\Delta\lambda| = \frac{\lambda^2}{hc} \Delta E = \frac{(579.07 \text{ nm})^2 (2.90 \times 10^{-6} \text{ eV})}{1240 \text{ eV}\cdot\text{nm}} = 7.83 \times 10^{-4} \text{ nm}$

- (c) The smallest measurable wavelength change is larger than this by the ratio  $0.01 \text{ nm} / 7.83 \times 10^{-4} \text{ nm} = 12.8$ . The magnetic field would need to be

increased by this

same factor because  $B \propto \Delta E \propto \Delta\lambda$ . The necessary field would be 0.638T.

7-66.  $\theta_{\min} = \cos^{-1} \left[ m_\ell \hbar / \sqrt{\ell(\ell+1)} \hbar \right]$  with  $m_\ell = \ell$ .

$$\cos \theta_{\min} = \ell / \sqrt{\ell(\ell+1)}. \text{ Thus, } \cos^2 \theta_{\min} = \ell^2 / [\ell(\ell+1)] = 1 - \sin^2 \theta_{\min}$$

$$\text{or, } \sin^2 \theta_{\min} = 1 - \frac{\ell^2}{\ell(\ell+1)} = \frac{\ell(\ell+1) - \ell^2}{\ell(\ell+1)} = \frac{\ell^2 + \ell - \ell^2}{\ell(\ell+1)}$$

$$\text{And, } \sin \theta_{\min} = \left( \frac{1}{\ell+1} \right)^{1/2} \text{ For large } \ell, \theta_{\min} \text{ is small.}$$

$$\text{Then } \sin \theta_{\min} \approx \theta_{\min} = \left( \frac{1}{\ell+1} \right)^{1/2} \approx \frac{1}{(\ell)^{1/2}}$$

