3-34. Equation 3-24: $\lambda_m = \frac{1.24\times10^3}{V} nm = \frac{1.24\times10^3}{80\times10^3 V} = 0.016 nm$

3-36. $\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) = \frac{(6.63\times10^{-34} J\cdot s)(1-\cos 110°)}{(9.11\times10^{-31} kg)(3.00\times10^8 m/s)} = 3.26\times10^{-12} m$

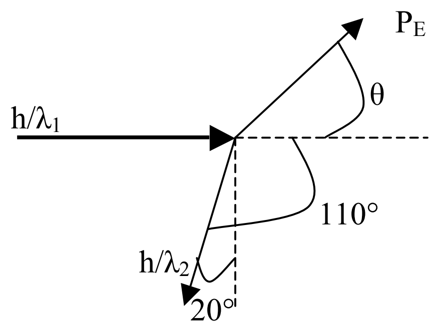
$$\lambda_1 = \frac{hc}{E_1} = \frac{(6.63\times10^{-34} J\cdot s)(3\times10^8 m/s)}{(0.511\times10^6 eV)(1.60\times10^{-19} J/eV)} = 2.43\times10^{-12} m$$

$$\lambda_2 = \lambda_1 + 3.26\times10^{-12} m = (2.43 + 3.26)\times10^{-12} m = 5.69\times10^{-12} m$$

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240 eV\cdot nm}{5.69\times10^{-3} nm} = 2.18\times10^5 eV = 0.218 MeV$$

Electron recoil energy $E_e = E_1 - E_2$ (Conservation of energy)

$E_e = 0.511 MeV - 0.218 MeV = 0.293 MeV$. The recoil electron momentum makes an angle θ with the direction of the initial photon.



$$\frac{h}{\lambda_2}\cos 20° = p_e \sin\theta = (1/c)\sqrt{E^2 - (mc^2)^2}\,\sin\theta \qquad \text{(Conservation of momentum)}$$

$$\sin\theta = \frac{(3.00\times10^8 m/s)(6.63\times10^{-34} J\cdot s)\cos 20°}{(5.69\times10^{-12} m)\left[(0.804 MeV)^2 - (0.511 MeV)^2\right]^{1/2}(1.60\times10^{-13} J/MeV)}$$

$= 0.330$ or $\theta = 19.3°$

3-37. $\Delta\lambda = \lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{mc}(1-\cos\theta) = 0.01\lambda_1$ Equation 3-25

$$\lambda_1 = (100)\frac{h}{mc}(1-\cos\theta) = (100)(0.00243nm)(1-\cos 90°) = 0.243nm$$

3-38. (a) $E_1 = \frac{hc}{\lambda_1} = \frac{1240eV\cdot nm}{0.0711nm} = 1.747\times10^4 eV$

(b) $\lambda_2 = \lambda_1 + \frac{h}{mc}(1-\cos\theta) = 0.0711 + (0.00243nm)(1-\cos 180°) = 0.0760nm$

(c) $E_2 = \frac{hc}{\lambda_2} = \frac{1240eV\cdot nm}{0.0760nm} = 1.634\times10^4 eV$

(d) $E_e = E_1 - E_2 = 1.128\times10^3 eV$

3-41. (a) Compton wavelength = $\frac{h}{mc}$

electron: $\frac{h}{mc} = \frac{6.63\times10^{-34} J\cdot s}{(9.11\times10^{-31}kg)(3.00\times10^8 m/s)} = 2.43\times10^{-12}m = 0.00243nm$

proton: $\frac{h}{mc} = \frac{6.63\times10^{-34} J\cdot s}{(1.67\times10^{-27}kg)(3.00\times10^8 m/s)} = 1.32\times10^{-15}m = 1.32 fm$

(b) $E = \frac{hc}{\lambda}$

(i) electron: $E = \frac{1240eV\cdot nm}{0.00243nm} = 5.10\times10^5 eV = 0.510MeV$

(ii) proton: $E = \frac{1240eV\cdot nm}{1.32\times10^{-6}nm} = 9.39\times10^8 eV = 939MeV$

3-50. (a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \quad \therefore \quad T = \frac{2.898\times10^{-3} m\cdot K}{82.8\times10^{-9}m} = 3.50\times10^4 K$

(b) Equation 3-18: $\frac{u(70nm)}{u(82.8nm)} = \frac{(70nm)^{-5}/\left(e^{hc/(70nm)kT}-1\right)}{(82.8nm)^{-5}/\left(e^{hc/(82.8nm)kT}-1\right)}$

where $\frac{hc}{(70nm)kT} = \frac{(6.63\times10^{-34} J\cdot s)(3.00\times10^{8} m/s)}{(70\times10^{-9} m)(1.38\times10^{-23} J/K)(3.5\times10^{4} K)} = 5.88$ and

$\frac{hc}{(82.8nm)kT} = 4.97$ $\qquad$ $\frac{u(70nm)}{u(82.8nm)} = \frac{(70nm)^{-5}/(e^{5.88}-1)}{(82.8nm)^{-5}/(e^{4.97}-1)} = 0.929$

Similarly, $\frac{u(100nm)}{u(82.8nm)} = \frac{(100nm)^{-5}/(e^{4.12}-1)}{(82.8nm)^{-5}/(e^{4.97}-1)} = 0.924$

3-53. (a) Equation 3-18: $u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$ Letting $C = 8\pi hc$ and $a = hc/kT$

gives $u(\lambda) = \frac{C\lambda^{-5}}{e^{a/\lambda}-1}$

(b)

$$\frac{du}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C\lambda^{-5}}{e^{a/\lambda}-1}\right] = C\left[\frac{\lambda^{-5}(-1)e^{a/\lambda}(-a\lambda^{-2})}{(e^{a/\lambda}-1)^2} - \frac{5\lambda^{-6}}{e^{a/\lambda}-1}\right]$$

$$= \frac{C\lambda^{-6}}{(e^{a/\lambda}-1)^2}\left[\frac{a}{\lambda}e^{a/\lambda} - 5(e^{a/\lambda}-1)\right] = \frac{C\lambda^{-6}e^{a/\lambda}}{(e^{a/\lambda}-1)^2}\left[\frac{a}{\lambda} - 5(1-e^{a/\lambda})\right] = 0$$

The maximum corresponds to the vanishing of the quantity in brackets.
Thus, $5\lambda(1-e^{-a/\lambda}) = a$

(c) This equation is most efficiently solved by trial and error; i.e., guess at a value for a/λ in the expression $5\lambda(1-e^{-a/\lambda}) = a$, solve for a better value of a/λ; substitute the new value to get an even better value, and so on. Repeat the process until the calculated value no longer changes. One succession of values is 5, 4.966310, 4.965156, 4.965116, 4.965114, 4.965114. Further iterations repeat the same value (to seven digits), so we have $\frac{a}{\lambda_m} = 4.965114 = \frac{hc}{\lambda_m kT}$

(d) $\lambda_m T = \frac{hc}{(4.965114)k} = \frac{(6.63\times10^{-34} J\cdot s)(3.00\times10^{8} m/s)}{(4.965114)(1.38\times10^{-23} J/K)}$

Therefore, $\lambda_m T = 2.898\times10^{-3} m\cdot K$ Equation 3-5

4-1. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $R = 1.097 \times 10^7 m^{-1}$ (Equation 4-2)

The Lyman series ends on m =1, the Balmer series on m =2, and the Paschen series on

m =3. The series limits all have $n = \infty$, so $\frac{1}{n} = 0$.

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2}\right) = 1.097 \times 10^7 m^{-1}$$

$$\lambda_L(\text{limit}) = 1.097 \times 10^7 m^{-1} = 91.16 \times 10^{-9} m = 91.16nm$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2}\right) = 1.097 \times 10^7 m^{-1}/4$$

$$\lambda_B(\text{limit}) = 4/1.097 \times 10^7 m^{-1} = 3.646 \times 10^{-7} m = 364.6nm$$

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2}\right) = 1.097 \times 10^7 m^{-1}/9$$

$$\lambda_P(\text{limit}) = 9/1.097 \times 10^7 m^{-1} = 8.204 \times 10^{-7} m = 820.4nm$$

4-2. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 2$ for Balmer series (Equation 4-2)

$$\frac{1}{379.1nm} = \frac{1.097 \times 10^7 m^{-1}}{10^9 nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^9 nm/m}{379.1nm\left(1.097 \times 10^7 m^{-1}\right)} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095} \quad \rightarrow \quad n = (1/0.0095)^{1/2} = 10.3 \quad \rightarrow \quad n = 10$$

$$n = 10 \rightarrow n = 2$$

4-3. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 1$ for Lyman series (Equation 4-2)

$$\frac{1}{164.1nm} = \frac{1.097\times10^{7}m^{-1}}{10^{9}nm/m}\left(1-\frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{10^{9}nm/m}{164.1nm\left(1.097\times10^{7}m^{-1}\right)} = 1 - 0.5555 = 0.4445$$

$$n = \left(1/0.4445\right)^{1/2} = 1.5$$

No, this is not a hydrogen Lyman series transition because n is not an integer.

4-4. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ (Equation 4-2)

For the Brackett series $m = 4$ and the first four (i.e., longest wavelength lines have $n = 5$,

6, 7, and 8.

$$\frac{1}{\lambda_{45}} = 1.097\times10^{7}m^{-1}\left(\frac{1}{4^2}-\frac{1}{5^2}\right) = 2.468\times10^{5}m^{-1}$$

$\lambda_{45} = \frac{1}{2.68\times10^{5}m^{-1}} = 4.052\times10^{-6}m = 4052nm$. Similarly,

$$\lambda_{46} = \frac{1}{3.809\times10^{5}m^{-1}} = 2.625\times10^{-6}m = 2625nm$$

$$\lambda_{47} = \frac{1}{4.617\times10^{5}m^{-1}} = 2.166\times10^{-6}m = 2166nm$$

$$\lambda_{48} = \frac{1}{5.142\times10^{5}m^{-1}} = 1.945\times10^{-6}m = 1945nm$$

These lines are all in the infrared.

4-7. $\Delta N \propto \frac{1}{\sin^4(\theta/2)} = \frac{A}{\sin^4(\theta/2)}$ (From Equation 4-6), where A is the product of the two

quantities in parentheses in Equation 4-6.

(a) $\frac{\Delta N(10°)}{\Delta N(1°)} = \frac{A/\sin^4(10°/2)}{A/\sin^4(1°/2)} = \frac{\sin^4(0.5°)}{\sin^4(5°)} = 1.01\times10^{-4}$

(b) $\dfrac{\Delta N(30^\circ)}{\Delta N(1^\circ)} = \dfrac{\sin^4(0.5^\circ)}{\sin^4(15^\circ)} = 1.29\times10^{-6}$

4-9. $r_d = \dfrac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \dfrac{ke^2 \cdot 2 \cdot 79}{E_{k\alpha}}$ (Equation 4-11)

For $E_{k\alpha} = 5.0MeV$: $r_d = \dfrac{(1.44MeV \cdot fm)(2)(79)}{5.0MeV} = 45.5\,fm$

For $E_{k\alpha} = 7.7MeV$: $r_d = 29.5\,fm$

For $E_{k\alpha} = 12MeV$: $r_d = 19.0\,fm$

4-10. $r_d = \dfrac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \dfrac{ke^2 \cdot 2 \cdot 79}{E_{k\alpha}}$ (Equation 4-11)

$E_{k\alpha} = \dfrac{(1.44MeV \cdot fm)(2)(13)}{4\,fm} = 9.4MeV$

4-40. Those scattered at $\theta = 180^\circ$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\frac{1}{2}m_\alpha v^2 = 7.7MeV = \dfrac{k(2e)(79e)}{r}$ where r = upper limit of the nuclear radius.

$r = \dfrac{k(2)(79)e^2}{7.7MeV} = \dfrac{2(79)(1.440MeV \cdot fm)}{7.7MeV} = 29.5\,fm$