Chapter 26

Image Formation by Mirrors and Lenses

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q26.1 With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.

Q26.2 With a convex spherical mirror, all images of real objects are upright, virtual, and smaller than the object. As seen in Question 26.1, you only get a change of orientation when you pass the focal point—but the focal point of a convex mirror is on the non-reflecting side!

Q26.3 The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$ we have $\frac{1}{p}=-\frac{1}{q}$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-\frac{q}{p}=\frac{p}{p}=1$. The image is right side up and actual size.

Q26.4 Stones at the bottom of a clear stream always appear closer to the surface because light is refracted away from the normal at the surface. Example 26.5 in the textbook shows that its apparent depth is three quarters of its actual depth.

Q26.5 For definiteness, we consider real objects ($p>0$).

(a) For $M=-\frac{q}{p}$ to be negative, q must be positive. This will happen in $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}$ if $p>f$, if the object is farther than the focal point.

(b) For $M=-\frac{q}{p}$ to be positive, q must be negative.

From $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}$ we need $p<f$.

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(c) For a real image, q must be positive.

As in part (a), it is sufficient for p to be larger than f.

(d) For $q<0$ we need $p<f$.

(e) For $|M|>1$, we consider separately $M<-1$ and $M>1$.

If $M=-\frac{q}{p}<-1$, we need $\frac{q}{p}>1$ or $q>p$

or $\frac{1}{q}<\frac{1}{p}$.

From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, $\frac{1}{p}+\frac{1}{p}>\frac{1}{f}$ or $\frac{2}{p}>\frac{1}{f}$

or $\frac{p}{2}<f$ or $p<2f$.

Now if $-\frac{q}{p}>1$ or $-q>p$ or $q<-p$

we may require $q<0$, since then $\frac{1}{p}=\frac{1}{f}-\frac{1}{q}$ with $\frac{1}{f}>0$

gives $\frac{1}{p}>-\frac{1}{q}$ as required or $-p>q$.

For $q<0$ in $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}$ we need $p<f$.

Thus the overall condition for an enlarged image is simply $p<2f$.

(f) For $|M|<1$, we have the reverse of part (e), requiring $p>2f$.

Q26.6 Using the same analysis as in Question 26.5 except $f<0$.

(a) Never.

(b) Always.

(c) Never, for light rays passing through the lens will always diverge.

(d) Always.

(e) Never.

(f) Always.

Q26.7 We assume the lens has a refractive index higher than its surroundings. For the biconvex lens in Figure 26.20(a), $R_1 > 0$ and $R_2 < 0$. Then all terms in $(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ are positive and $f > 0$. For the other two lenses in part (a) of the figure, R_1 and R_2 are both positive but R_1 is less than R_2. Then $\frac{1}{R_1} > \frac{1}{R_2}$ and the focal length is again positive.

For the biconcave lens and the plano-concave lens in Figure 26.20(b), $R_1 < 0$ and $R_2 > 0$. Then both terms are negative in $\frac{1}{R_1} - \frac{1}{R_2}$ and the focal length is negative. For the middle lens in part (b) of the figure, R_1 and R_2 are both positive but R_1 is greater than R_2. Then $\frac{1}{R_1} < \frac{1}{R_2}$ and the focal length is again negative.

Q26.8 Both words are inverted. However OXIDE has up-down symmetry whereas LEAD does not.

Q26.9 In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.

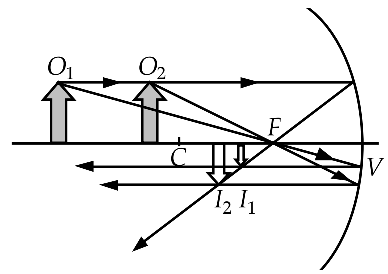


FIG. Q26.9

Q26.10 As in the diagram, let the center of curvature *C* of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex *V*. A ray from the top of the fish that reaches the bowl surface along a radial line through *C* has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to *V* is refracted to bend away from the normal. Its extension back inside the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.

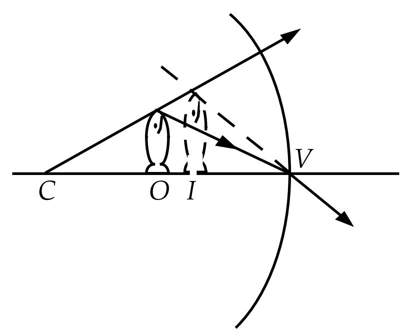


FIG. Q26.10

Q26.11 Because when you look at the ƎƆИA⅃UᙠMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

Q26.12 With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

Q26.13 An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays. All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure Q26.13.

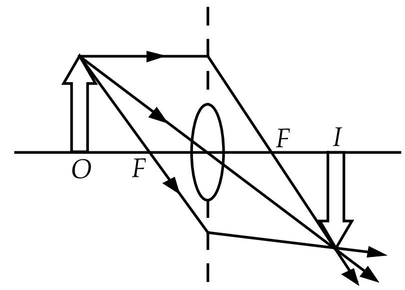


FIG. Q26.13

Q26.14 Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.

Q26.15 The focal point is defined as the location of the image formed by rays originally parallel to the axis. An object at a large but finite distance will radiate rays nearly but not exactly parallel. Infinite object distance describes the definite limiting case in which these rays become parallel. To measure the focal length of a converging lens, set it up to form an image of the farthest object you can see outside a window. The image distance will be equal to the focal length within one percent or better if the object distance is a hundred times larger or more.

Q26.16 Use a converging lens as the projection lens in a slide projector. Place the brightly illuminated slide slightly farther than its focal length away from it, so that the lens will produce a real, inverted, enlarged image on the screen.

Q26.17 Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

Q26.18 The artist's statements are accurate, perceptive, and eloquent. The image you see is "almost one's whole surroundings," including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher's index and middle fingers or their reflections in the globe.

The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose. Outside the globe, you are at the center of your observable universe. If you wink at the ball, the center of the looking-glass world hops over to the location of the image of your open eye.

Q26.19 The three mirrors, two of which are shown as M and N in the figure to the right, reflect any incident ray back parallel to its original direction. When you look into the corner you see image I_3 of yourself.

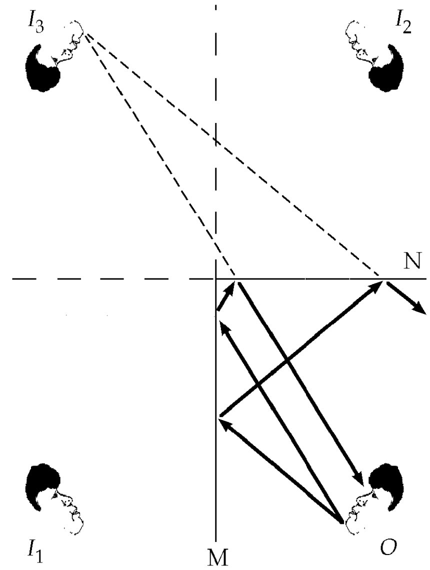


FIG. Q26.19

SOLUTIONS TO PROBLEMS

Section 26.1 Images Formed by Flat Mirrors

P26.1 I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8\ \text{m}}{3\times 10^{8}\ \text{m/s}}\boxed{\sim 10^{-9}\ \text{s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

P26.2 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800\ \text{m}+5.30\ \text{m}=6.10\ \text{m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600\ \text{m}}=\frac{6.10\ \text{m}}{0.800\ \text{m}}$$

or $$h'=(0.600\ \text{m})\left(\frac{6.10\ \text{m}}{0.800\ \text{m}}\right)=\boxed{4.58\ \text{m}}.$$

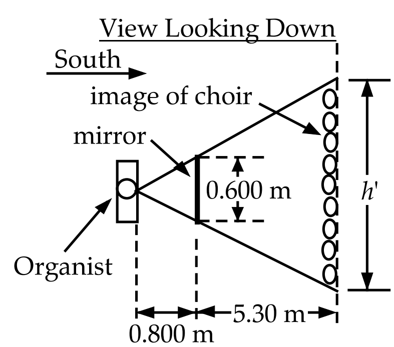


FIG. P26.2

P26.3 The flatness of the mirror is described

by $R = \infty$, $f = \infty$

and $\frac{1}{f} = 0$.

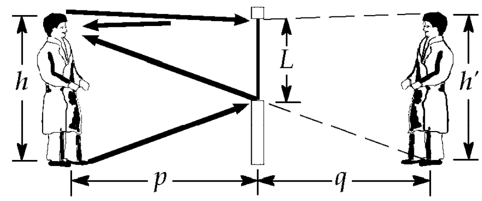


FIG. P26.3

By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$$

or $q = -p$.

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ inches.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}.$$

Thus, the mirror must be $\boxed{\text{at least 35.0 inches high}}$.

P26.4 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O,

and (O, I_3, I_4)

and (I_2, I_1, I_5)

forming the vertices of equilateral triangles.

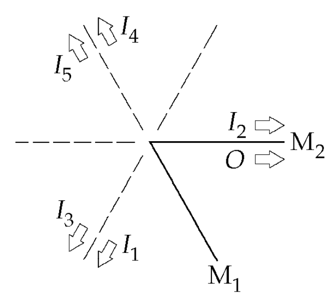


FIG. P26.4

P26.5 (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft}}$ from the person.

P26.6 (a) The flat mirrors have

$$R \to \infty$$

and $f \to \infty$.

The upper mirror M_1 produces a virtual, actual sized image I_1 according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{\infty} = 0$$
$$q_1 = -p_1$$

with $M_1 = -\frac{q_1}{p_1} = +1$.

As shown, this image is above the upper mirror. It is the object for mirror M_2, at object distance

$$p_2 = p_1 + h.$$

The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2} + \frac{1}{q_2} = 0$$
$$q_2 = -p_2 = -(p_1 + h)$$

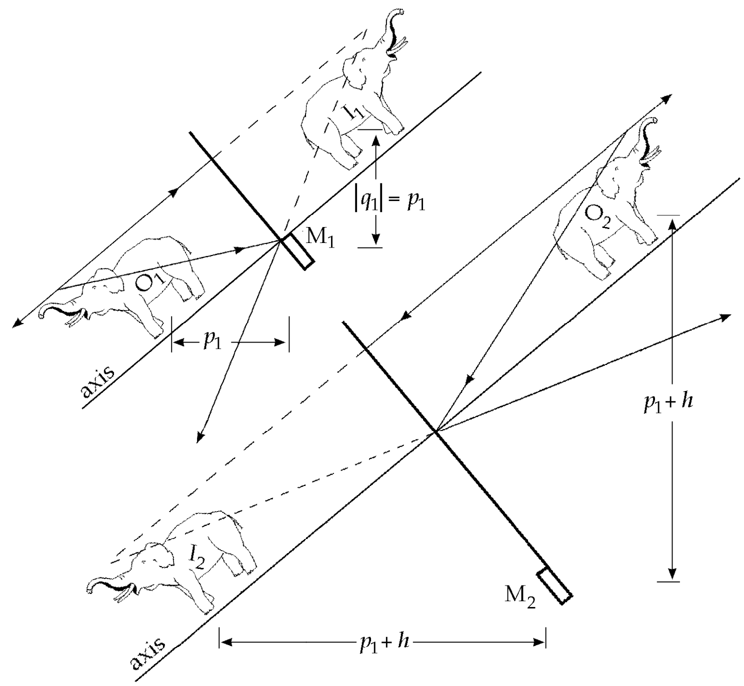


FIG. P26.6

with $M_2 = -\frac{q_2}{p_2} = +1$ and $M_{\text{overall}} = M_1 M_2 = 1$.

Thus the final image is at distance $\boxed{p_1 + h}$ behind the lower mirror.

(b) It is $\boxed{\text{virtual}}$.

(c) $\boxed{\text{Upright}}$

(d) With magnification $\boxed{+1}$.

(e) It $\boxed{\text{does not appear to be reversed}}$ left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

Section 26.2 Images Formed by Spherical Mirrors

P26.7 For a concave mirror, both R and f are positive.

We also know that $$f=\frac{R}{2}=10.0\text{ cm}.$$

(a) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{10.0\text{ cm}}-\frac{1}{40.0\text{ cm}}=\frac{3}{40.0\text{ cm}}$$

and $$\boxed{q=13.3\text{ cm}}$$

$$M=\frac{q}{p}=-\frac{13.3\text{ cm}}{40.0\text{ cm}}=\boxed{-0.333}.$$

The image is 13.3 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(b) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{10.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=\frac{1}{20.0\text{ cm}}$$

and $$\boxed{q=20.0\text{ cm}}$$

$$M=\frac{q}{p}=-\frac{20.0\text{ cm}}{20.0\text{ cm}}=\boxed{-1.00}.$$

The image is 20.0 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(c) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{10.0\text{ cm}}-\frac{1}{10.0\text{ cm}}=0$$

Thus, q = infinity.

$\boxed{\text{No image is formed}}$. The rays are reflected parallel to each other.

***P26.8** $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{1}{0.275\text{ m}}-\frac{1}{10.0\text{ m}}$$ gives $\boxed{q=-0.267\text{ m}}$.

Thus, the image is $\boxed{\text{virtual}}$.

$$M=\frac{-q}{p}=-\frac{-0.267}{10.0\text{ m}}=\boxed{0.0267}$$

Thus, the image is $\boxed{\text{upright}}\,(+M)$ and $\boxed{\text{diminished}}\,(|M|<1)$.

P26.9 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ gives $\frac{1}{30.0\text{ cm}}+\frac{1}{q}=\frac{2}{(-40.0\text{ cm})}$

$\frac{1}{q}=-\frac{2}{40.0\text{ cm}}-\frac{1}{30.0\text{ cm}}=-0.083\,3\text{ cm}^{-1}$ so $q=\boxed{-12.0\text{ cm}}$

$M=\frac{-q}{p}=-\frac{(-12.0\text{ cm})}{30.0\text{ cm}}=\boxed{0.400}$.

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ gives $\frac{1}{60.0\text{ cm}}+\frac{1}{q}=\frac{2}{(-40.0\text{ cm})}$

$\frac{1}{q}=-\frac{2}{40.0\text{ cm}}-\frac{1}{60.0\text{ cm}}=-0.066\,6\text{ cm}^{-1}$ so $q=\boxed{-15.0\text{ cm}}$

$M=\frac{-q}{p}=-\frac{(-15.0\text{ cm})}{60.0\text{ cm}}=\boxed{0.250}$.

(c) Since $M>0$, the images are $\boxed{\text{upright}}$.

P26.10 With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f=\frac{R}{2}=1.25\text{ m}.$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad\text{so}\qquad\frac{1}{2.00\text{ m}}+\frac{1}{q}=\frac{1}{1.25\text{ m}}$$

$$q=\boxed{3.33\text{ m}}.$$

P26.11 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{90.0\text{ cm}}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=\boxed{-0.500}$.

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{20.0\text{ cm}}$

$q=\boxed{-60.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$.

(c) The image (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are shown.

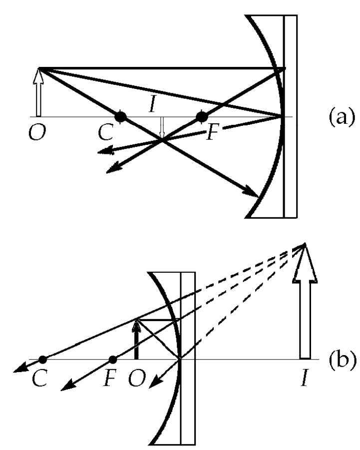


FIG. P26.11

***P26.12** (a) Since the object is in front of the mirror, $p > 0$. With the image behind the mirror, $q < 0$. The mirror equation gives the radius of curvature as $\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.00\text{ cm}} - \frac{1}{10.0\text{ cm}} = \frac{10-1}{10.0\text{ cm}}$, or

$$R = 2\left(\frac{10.0\text{ cm}}{9}\right) = \boxed{+2.22\text{ cm}}.$$

(b) The magnification is $M = -\frac{q}{p} = -\frac{(-10.0\text{ cm})}{1.00\text{ cm}} = \boxed{+10.0}$.

***P26.13** The ball is a convex mirror with $R = -4.25$ cm and $f = \frac{R}{2} = -2.125$ cm. We have

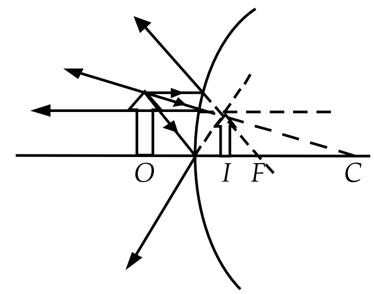


FIG. P26.13

$$M = \frac{3}{4} = -\frac{q}{p}$$

$$q = -\frac{3}{4}p$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125\text{ cm}}$$

$$\frac{3}{3p} - \frac{4}{3p} = \frac{1}{-2.125\text{ cm}}$$

$$3p = 2.125\text{ cm}$$

$\boxed{p = 0.708\text{ cm}}$ in front of the sphere.

The image is upright, virtual, and diminished.

P26.14 (a) $M = -4 = -\frac{q}{p}$ $\quad q = 4p$

$q - p = 0.60\text{ m} = 4p - p$ $\quad p = 0.2\text{ m}$ $\quad q = 0.8\text{ m}$

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2\text{ m}} + \frac{1}{0.8\text{ m}}$ $\quad f = \boxed{160\text{ mm}}$

(b) $M = +\frac{1}{2} = -\frac{q}{p}$ $\quad p = -2q$

$|q| + p = 0.20\text{ m} = -q + p = -q - 2q$

$q = -66.7\text{ mm}$ $\quad p = 133\text{ mm}$

$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{0.133\text{ m}} + \frac{1}{-0.066\,7\text{ m}}$ $\quad R = \boxed{-267\text{ mm}}$

P26.15 $M=-\dfrac{q}{p}$

$$q=-Mp=-0.013(30\text{ cm})=-0.39\text{ cm}$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{30\text{ cm}}+\frac{1}{-0.39\text{ cm}}=\frac{2}{R}$$

$$R=\frac{2}{-2.53\text{ m}^{-1}}=-0.790\text{ cm}$$

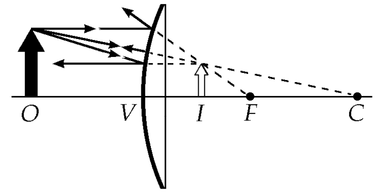


FIG. P26.15

The cornea is convex, with radius of curvature $\boxed{0.790\text{ cm}}$.

***P26.16** With

$$M=\frac{h'}{h}=\frac{+4.00\text{ cm}}{10.0\text{ cm}}=+0.400=-\frac{q}{p}$$

$$q=-0.400p$$

the image must be virtual.

(a) It is a $\boxed{\text{convex}}$ mirror that produces a diminished upright virtual image.

(b) We must have

$$p+|q|=42.0\text{ cm}=p-q$$

$$p=42.0\text{ cm}+q$$

$$p=42.0\text{ cm}-0.400p$$

$$p=\frac{42.0\text{ cm}}{1.40}=30.0\text{ cm}$$

The mirror is $\boxed{\text{at the 30.0 cm mark}}$.

(c) $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}=\dfrac{1}{30\text{ cm}}+\dfrac{1}{-0.4(30\text{ cm})}=\dfrac{1}{f}=-0.050\,0/\text{cm}$ $\boxed{f=-20.0\text{ cm}}$

The ray diagram looks like Figure 26.12(c) in the text.

P26.17 (a) $q=(p+5.00\text{ m})$ and, since the image must be real,

$$M=-\frac{q}{p}=-5 \qquad \text{or} \qquad q=5p.$$

Therefore, $p+5.00\text{ m}=5p$

or $p=1.25\text{ m}$ and $q=6.25\text{ m}$.

From $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{2}{R}$, $R=\dfrac{2pq}{p+q}=\dfrac{2(1.25)(6.25)}{1.25+6.25}$

$$=\boxed{2.08\text{ m(concave)}}$$

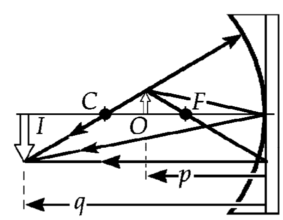


FIG. P26.17

(b) From part (a), $p=1.25\text{ m}$; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

P26.18 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ($q = -10.0$ cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R = |R|$, $q = -30.0$ cm)

$$\frac{1}{p} - \frac{1}{30.0} = \frac{2}{|R|}$$

or
$$\frac{2}{|R|} = \frac{30.0\text{ cm} - p}{(30.0\text{ cm})p} \qquad (1)$$

(convex side: $R = -|R|$, $q = -10.0$ cm)

$$\frac{1}{p} - \frac{1}{10.0} = -\frac{2}{|R|}$$

or
$$\frac{2}{|R|} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p}. \qquad (2)$$

(a) Equating Equations (1) and (2) gives:

$$\frac{30.0\text{ cm} - p}{3.00} = p - 10.0\text{ cm}$$

or
$$p = 15.0\text{ cm}.$$

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ($p = 15.0$ cm) in Equation (1) gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})}$$

or
$$\frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and
$$|R| = 60.0\text{ cm}.$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

P26.19 (a) The flat mirror produces an image according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \qquad \frac{1}{24\text{ cm}} + \frac{1}{q} = \frac{1}{\infty} = 0 \qquad q = -24.0\text{ m}.$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55\text{ m} + 24.0\text{ m} = \boxed{25.6\text{ m}}.$$

(b) The image is the same size as the object, so $\theta = \frac{h}{d} = \frac{1.50\text{ m}}{25.6\text{ m}} = \boxed{0.058\,7\text{ rad}}$.

continued on next page

(c) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ $\quad \frac{1}{24\text{ m}}+\frac{1}{q}=\frac{2}{(-2\text{ m})}$ $\quad q=\frac{1}{-(1/1\text{ m})-(1/24\text{ m})}=-0.960\text{ m}$

This image is distant from your eyes by $1.55\text{ m}+0.960\text{ m}=\boxed{2.51\text{ m}}$.

(d) The image size is given by $M=\frac{h'}{h}=-\frac{q}{p}$ $\quad h'=-h\frac{q}{p}=-1.50\text{ m}\left(\frac{-0.960\text{ m}}{24\text{ m}}\right)=0.060\,0\text{ m}$.

So its angular size at your eye is $\theta'=\frac{h'}{d}=\frac{0.06\text{ m}}{2.51\text{ m}}=\boxed{0.023\,9\text{ rad}}$.

(e) Your brain assumes that the car is 1.50 m high and calculate its distance as

$d'=\frac{h}{\theta'}=\frac{1.50\text{ m}}{0.023\,9}=\boxed{62.8\text{ m}}$.

***P26.20** (a) The image starts from a point whose height above the mirror vertex is given by

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ $\quad \frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}}$. $\quad$ Therefore, $\quad q=0.600\text{ m}$.

As the ball falls, p decreases and q increases. Ball and image pass when $q_1=p_1$. When this is true,

$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1}$ $\quad$ or $\quad p_1=1.00\text{ m}$.

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2=q_2=0$.

(b) The falling ball passes its real image when it has fallen

$3.00\text{ m}-1.00\text{ m}=2.00\text{ m}=\frac{1}{2}gt^2$, or when $t=\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.639\text{ s}}$.

The ball reaches its virtual image when it has traversed

$3.00\text{ m}-0=3.00\text{ m}=\frac{1}{2}gt^2$, or at $t=\sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.782\text{ s}}$.

Section 26.3 Images Formed by Refraction

P26.21 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}=0$ and $R\to\infty$

$q=-\frac{n_2}{n_1}p=-\frac{1}{1.309}(50.0\text{ cm})=-38.2\text{ cm}$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

P26.22 When $R \to \infty$, the equation describing image formation at a single refracting surface becomes $q = -p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1} = -\left(\frac{1.33}{1.66}\right)(8.00\text{ cm}) = -6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41\text{ cm}) = -13.84\text{ cm} \qquad \text{or} \qquad 13.84\text{ cm below the water surface.}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0\text{ cm}) = -9.02\text{ cm} \qquad \text{or} \qquad 9.02\text{ cm below the water surface.}$$

Therefore, the apparent thickness of the glass is $\Delta t = 13.84\text{ cm} - 9.02\text{ cm} = \boxed{4.82\text{ cm}}$.

P26.23 For refraction at a curved surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.

Solve for q to find $q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}$.

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0\text{ cm}$

and $p = 10.0\text{ cm}$.

So $q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57\text{ cm}$.

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$.

P26.24 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ so $\frac{1.00}{\infty} + \frac{1.40}{21.0\text{ mm}} = \frac{1.40 - 1.00}{6.00\text{ mm}}$

and $0.066\,7 = 0.066\,7$.

They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

P26.25 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $\frac{1.00}{p}+\frac{1.50}{q}=\frac{1.50-1.00}{6.00\text{ cm}}=\frac{1}{12.0\text{ cm}}$

(a) $\frac{1.00}{20.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/20.0\text{ cm})\right]}=\boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/10.0\text{ cm})\right]}=\boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/3.0\text{ cm})\right]}=\boxed{-6.00\text{ cm}}$

***P26.26** For a plane surface, $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $q=-\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right|=\frac{n_2}{n_1}\left|\frac{dp}{dt}\right|.$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}}=\left|\frac{dq}{dt}\right|=\frac{1.00}{1.33}(2.00\text{ cm/s})=\boxed{1.50\text{ cm/s}}.$$

Section 26.4 Thin Lenses

P26.27 (a) $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(0.440)\left[\frac{1}{12.0\text{ cm}}-\frac{1}{(-18.0\text{ cm})}\right]$

$f=\boxed{16.4\text{ cm}}$

(b) $\frac{1}{f}=(0.440)\left[\frac{1}{18.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$

$f=\boxed{16.4\text{ cm}}$

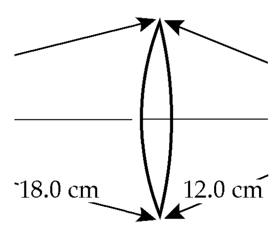


FIG. P26.27

P26.28 Let $R_1=$ outer radius and $R_2=$ inner radius

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{2.00\text{ m}}-\frac{1}{2.50\text{ cm}}\right]=0.050\,0\text{ cm}^{-1}$$

so $f=\boxed{20.0\text{ cm}}$.

P26.29 (a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{26.0\text{ cm}}$ $\boxed{q=650\text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{24.0\text{ cm}}$ $\boxed{q=-600\text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

P26.30 For a converging lens, f is positive. We use $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$.

(a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{40.0\text{ cm}}=\frac{1}{40.0\text{ cm}}$ $\boxed{q=40.0\text{ cm}}$

$$M=-\frac{q}{p}=-\frac{40.0}{40.0}=\boxed{-1.00}$$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=0$ $\boxed{q=\text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{10.0\text{ cm}}=-\frac{1}{20.0\text{ cm}}$ $\boxed{q=-20.0\text{ cm}}$

$$M=-\frac{q}{p}=-\frac{(-20.0)}{10.0}=\boxed{2.00}$$

The image is $\boxed{\text{upright, virtual}}$ and 20.0 cm in front of the lens.

P26.31 We are looking at an enlarged, upright, virtual image:

$M=\frac{h'}{h}=2=-\frac{q}{p}$ so $p=-\frac{q}{2}=-\frac{(-2.84\text{ cm})}{2}=+1.42\text{ cm}$

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ gives $\frac{1}{1.42\text{ cm}}+\frac{1}{(-2.84\text{ cm})}=\frac{1}{f}$

$\boxed{f=2.84\text{ cm}}$.

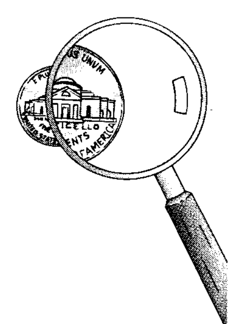


FIG. P26.31

P26.32 (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\frac{1}{32.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{1}{f}$

so $\boxed{f=6.40\text{ cm}}$

(b) $M=-\frac{q}{p}=-\frac{8.00\text{ cm}}{32.0\text{ cm}}=\boxed{-0.250}$

(c) Since $f>0$, the lens is $\boxed{\text{converging}}$.

P26.33 $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $p^{-1}+q^{-1}=\text{constant}$

We may differentiate through with respect to *p*: $-1p^{-2}-1q^{-2}\frac{dq}{dp}=0$

$$\frac{dq}{dp}=-\frac{q^2}{p^2}=-M^2.$$

P26.34 The image is inverted: $M=\frac{h'}{h}=\frac{-1.80\text{ m}}{0.024\,0\text{ m}}=-75.0=\frac{-q}{p}$ $q=75.0p$.

(b) $q+p=3.00\text{ m}=75.0p+p$ $p=\boxed{39.5\text{ mm}}$

(a) $q=2.96\text{ m}$ $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.039\,5\text{ m}}+\frac{1}{2.96\text{ m}}$ $f=\boxed{39.0\text{ mm}}$

P26.35 (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{(-32.0\text{ cm})}$

so $q=-\left(\frac{1}{20.0}+\frac{1}{32.0}\right)^{-1}=\boxed{-12.3\text{ cm}}$

The image is 12.3 cm to the left of the lens.

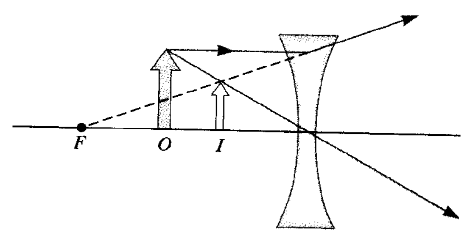


FIG. P26.35

(b) $M=-\frac{q}{p}=-\frac{(-12.3\text{ cm})}{20.0\text{ cm}}=\boxed{0.615}$

(c) See the ray diagram to the right.

***P26.36** Comparing $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ with $\frac{1}{p}+\frac{1}{-3.5p}=\frac{1}{7.5\text{ cm}}$ we see $q=-3.5p$ and $f=7.50$ cm for a converging lens.

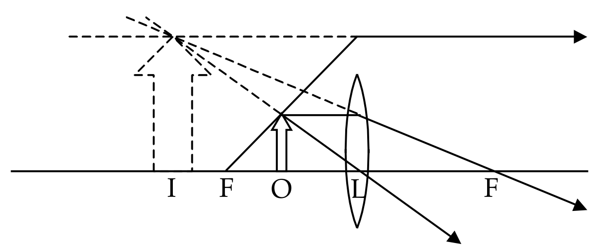
(a) To solve, we add the fractions:

$$\frac{-3.5+1}{-3.5p}=\frac{1}{7.5\text{ cm}}$$
$$\frac{3.5p}{2.5}=7.5\text{ cm}$$
$$p=\boxed{5.36\text{ cm}}$$

(b) $q=-3.5(5.36\text{ cm})=\boxed{-18.8\text{ cm}}$

$$M=-\frac{q}{p}=-\frac{-18.8\text{ cm}}{5.36\text{ cm}}=+3.50$$

(c)



P26.36(c)

The image is $\boxed{\text{enlarged, upright, and virtual.}}$

(d) The lens is being used as a magnifying glass. Statement: A magnifying glass with focal length 7.50 cm is used to form an image of a stamp, enlarged 3.50 times. Find the object distance. Locate and describe the image.

***P26.37** In

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$p^{-1}+q^{-1}=\text{constant,}$$

we differentiate with respect to time

$$-1\left(p^{-2}\right)\frac{dp}{dt}-1\left(q^{-2}\right)\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}.$$

continued on next page

We must find the momentary image location q:

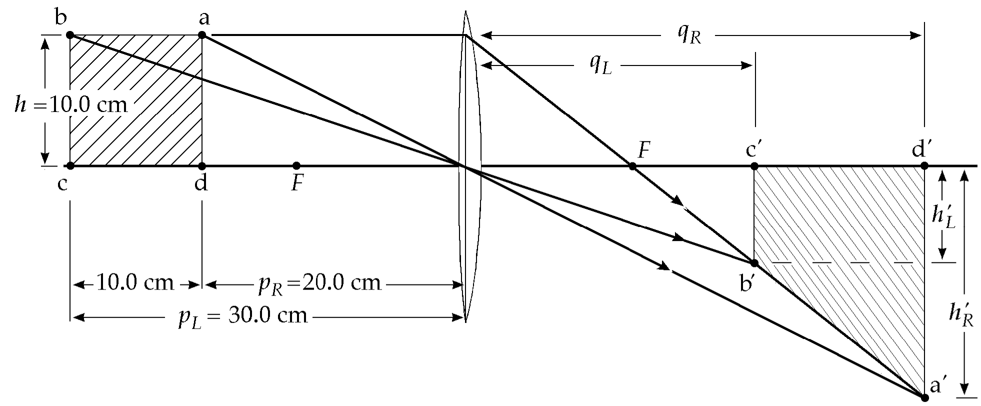
$$\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$$

$q=0.305\text{ m}$.

Now $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.001\,16\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}$.

P26.38 (a) $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{15.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$ or $\boxed{f=13.3\text{ cm}}$

(b)



The square is imaged as a trapezoid.

FIG. P26.38(b)

(c) To find the area, first find q_R and q_L, along with the heights h'_R and h'_L, using the thin lens equation.

$\frac{1}{p_R}+\frac{1}{q_R}=\frac{1}{f}$ becomes $\frac{1}{20.0\text{ cm}}+\frac{1}{q_R}=\frac{1}{13.3\text{ cm}}$ or $q_R=40.0\text{ cm}$

$$h'_R=hM_R=h\left(\frac{-q_R}{p_R}\right)=(10.0\text{ cm})(-2.00)=-20.0\text{ cm}$$

$\frac{1}{30.0\text{ cm}}+\frac{1}{q_L}=\frac{1}{13.3\text{ cm}}$ or $q_L=24.0\text{ cm}$

$$h'_L=hM_L=(10.0\text{ cm})(-0.800)=-8.00\text{ cm}$$

Thus, the area of the image is: Area $=|q_R-q_L||h'_L|+\frac{1}{2}|q_R-q_L||h'_R-h'_L|=\boxed{224\text{ cm}^2}$.

P26.39 (a) The image distance is: $q = d - p$.

Thus, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{p}+\frac{1}{d-p}=\frac{1}{f}$.

This reduces to a quadratic equation: $p^2+(-d)p+fd=0$

which yields: $\boxed{p=\frac{d\pm\sqrt{d^2-4fd}}{2}=\frac{d}{2}\pm\sqrt{\frac{d^2}{4}-fd}}$.

Since $f<\frac{d}{4}$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$.

The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$.

P26.40 To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f}$$

becomes $$\frac{1}{2\,000\text{ mm}}+\frac{1}{q_2}=\frac{1}{65.0\text{ mm}}$$

and $$q_2=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right).$$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D=q_2-q_1=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right)-65.0\text{ mm}=\boxed{2.18\text{ mm}}.$$

Section 26.5 Context Connection—Medical Fiberscopes

P26.41 The image will be inverted. With $h = 6$ cm, we require $h' = -1$ mm.

(a) $M=\frac{h'}{h}=-\frac{q}{p}$ $\qquad q=-p\frac{h'}{h}=-50\text{ mm}\frac{(-1\text{ mm})}{60\text{ mm}}=\boxed{0.833\text{ mm}}$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{50\text{ mm}}+\frac{1}{0.833\text{ mm}}$ $\qquad f=\boxed{0.820\text{ mm}}$

P26.42 (a) Light leaving the sphere refracts away from the normal, so light that travels toward the upper right comes from the $\boxed{\text{bottom half}}$ of the sphere.

(b) $\sin\theta_1 = \dfrac{1\text{ mm}}{R}$ $\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.50\left(\dfrac{1\text{ mm}}{R}\right) = 1\sin 90°$ $\qquad R = \boxed{1.50\text{ mm}}$

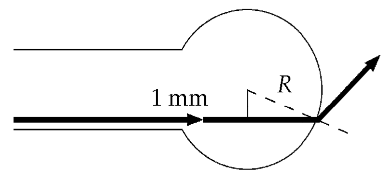


FIG. P26.42(b)

(c) $\delta = \left|\theta_2 - \theta_1\right| = \left|90° - \sin^{-1}\left(\dfrac{1}{1.5}\right)\right| = \boxed{48.2°}$

(d) No ray can have an angle of refraction larger than 90°, so the ray considered in parts (b) and (c) has the largest possible angle of refraction and then the largest possible deviation.

All other rays, at distances from the axis of less than 1 mm, will leave the sphere at smaller angles with the axis than 48.2°. The angular diameter of the cone of diverging light is $2 \times 48.2° = \boxed{96.4°}$.

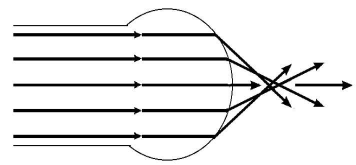


FIG. P26.42(d)

(e) Light can be absorbed by a coating on the sphere and reradiated in any direction. The sphere deviates from a perfectly spherical shape, probably macroscopically and surely at the scale of the wavelength of light. Light rays enter the sphere along directions not parallel to the axis of the fiber. Inhomogeneities within the sphere scatter light. Light reflects from the interior surface of the sphere.

Additional Problems

P26.43 If $M < 1$, the lens is diverging and the image is virtual. $\qquad d = p - |q| = p + q$

$M = -\dfrac{q}{p}$ $\qquad$ so $\qquad q = -Mp$ $\qquad$ and $\qquad d = p - Mp$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{(-Mp)} = \frac{(-M+1)}{-Mp} = \frac{(1-M)^2}{-Md} \qquad \boxed{f = \frac{-Md}{(1-M)^2}}.$$

If $M > 1$, the lens is converging and the image is still virtual.

Now $\qquad d = -q - p$.

We obtain in this case $\qquad \boxed{f = \dfrac{Md}{(M-1)^2}}.$

P26.44 Start with the first pass through the lens.

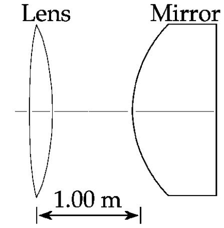


FIG. P26.44

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0 \text{ cm}} - \frac{1}{100 \text{ cm}} \qquad q_1 = 400 \text{ cm to right of lens}$$

For the mirror, $\quad p_2 = -300 \text{ cm}$

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-50.0 \text{ cm})} - \frac{1}{(-300 \text{ cm})} \qquad q_2 = -60.0 \text{ cm}$$

For the second pass through the lens, $\quad p_3 = 160 \text{ cm}$

$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0 \text{ cm}} - \frac{1}{160 \text{ cm}} \qquad q_3 = \boxed{160 \text{ cm to the left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400 \text{ cm}}{100 \text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{(-60.0 \text{ cm})}{(-300 \text{ cm})} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160 \text{ cm}}{160 \text{ cm}} = -1 \qquad M = M_1 M_2 M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

P26.45 The real image formed by the concave mirror serves as a real object for the convex mirror with $p = 50$ cm and $q = -10$ cm. Therefore,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad \frac{1}{f} = \frac{1}{50 \text{ cm}} + \frac{1}{(-10 \text{ cm})}$$

gives $f = -12.5$ cm and $R = 2f = \boxed{-25.0 \text{ cm}}$.

P26.46 With $n_1 = 1$ and $n_2 = n$ for the lens material, the first refraction is described by

$$\frac{1}{\infty} + \frac{1}{q_1} = \frac{n-1}{R_1}$$
$$q_1 = \frac{R_1}{n-1}$$

The real image formed by the first surface is a virtual object seen by the second, at $p_2 = -q_1$. Then for the second refraction

$$\frac{1}{-q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$
$$\frac{1-n}{R_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$
$$\frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This image distance for an object at infinity is by definition the focal length.

***P26.47** (a) $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{5\text{ cm}} - \frac{1}{7.5\text{ cm}} \qquad \therefore q_1 = 15\text{ cm}$

$$M_1 = -\frac{q_1}{p_1} = -\frac{15\text{ cm}}{7.5\text{ cm}} = -2$$

$$M = M_1 M_2 \qquad \therefore 1 = (-2)M_2$$

$$\therefore M_2 = -\frac{1}{2} = -\frac{q_2}{p_2} \qquad \therefore p_2 = 2q_2$$

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \qquad \therefore \frac{1}{2q_2} + \frac{1}{q_2} = \frac{1}{10\text{ cm}} \qquad \therefore q_2 = 15\text{ cm},\ p_2 = 30\text{ cm}$$

$$p_1 + q_1 + p_2 + q_2 = 7.5\text{ cm} + 15\text{ cm} + 30\text{ cm} + 15\text{ cm} = \boxed{67.5\text{ cm}}$$

(b) $\frac{1}{p_1'} + \frac{1}{q_1'} = \frac{1}{f_1} = \frac{1}{5\text{ cm}}$

Solve for q_1' in terms of p_1': $\qquad q_1' = \frac{5p_1'}{p_1' - 5}$ (1)

$$M_1' = -\frac{q_1'}{p_1'} = -\frac{5}{p_1' - 5}\text{, using (1).}$$

$$M' = M_1' M_2' \qquad \therefore M_2' = \frac{M'}{M_1'} = -\frac{3}{5}(p_1' - 5) = -\frac{q_2'}{p_2'}$$

$$\therefore q_2' = \frac{3}{5} p_2' (p_1' - 5) \tag{2}$$

Substitute (2) into the lens equation $\frac{1}{p_2'} + \frac{1}{q_2'} = \frac{1}{f_2} = \frac{1}{10\text{ cm}}$ and obtain p_2' in terms of p_1':

$$p_2' = \frac{10(3p_1' - 10)}{3(p_1' - 5)}. \tag{3}$$

Substituting (3) in (2), obtain q_2' in terms of p_1':

$$q_2' = 2(3p_1' - 10). \tag{4}$$

Now, $p_1' + q_1' + p_2' + q_2' =$ a constant.

Using (1), (3) and (4), and the value obtained in (a):

$$p_1' + \frac{5p_1'}{p_1' - 5} + \frac{10(3p_1' - 10)}{3(p' - 5)} + 2(3p_1' - 10) = 67.5.$$

This reduces to the quadratic equation

$$21p_1'^2 - 322.5p_1' + 1\,212.5 = 0,$$

which has solutions $p_1' = 8.784$ cm and 6.573 cm.

continued on next page

Case 1: $p_1' = 8.784 \text{ cm}$

$\therefore p_1' - p_1 = 8.784 \text{ cm} - 7.5 \text{ cm} = 1.28 \text{ cm}.$

From (4): $q_2' = 32.7 \text{ cm}$

$\therefore q_2' - q_2 = 32.7 \text{ cm} - 15 \text{ cm} = 17.7 \text{ cm}.$

Case 2: $p_1' = 6.573 \text{ cm}$

$\therefore p_1' - p_1 = 6.573 \text{ cm} - 7.5 \text{ cm} = -0.927 \text{ cm}.$

From (4): $q_2' = 19.44 \text{ cm}$

$\therefore q_2' = q_2 = 19.44 \text{ cm} - 15 \text{ cm} = 4.44 \text{ cm}.$

From these results it is concluded that:

The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object.

P26.48 $$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}}$$

so $q_1 = 50.0 \text{ cm}$ (to left of mirror).

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-16.7 \text{ cm})} - \frac{1}{(-25.0 \text{ cm})} \text{ and } q_2 = -50.3 \text{ cm},$$

meaning 50.3 cm to the right of the lens. Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0 \text{ cm}}{12.5 \text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

P26.49 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm.

The incident rays are parallel, so $p = \infty$.

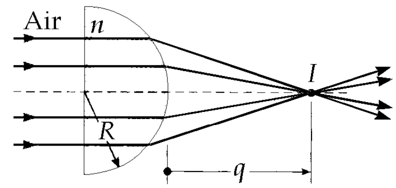


FIG. P26.49

Then, $$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes $$0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}}$$

and $\boxed{q = 10.7 \text{ cm}}$.

P26.50 (a) $$I = \frac{\mathscr{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi\left(1.60 \times 10^{-2} \text{ m}\right)^2} = \boxed{1.40 \text{ kW/m}^2}$$

(b) $$I = \frac{\mathscr{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(7.20 \text{ m})^2} = \boxed{6.91 \text{ mW/m}^2}$$

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $$\frac{1}{7.20 \text{ m}} + \frac{1}{q} = \frac{1}{0.350 \text{ m}}$$

so $q = 0.368$ m

and $$M = \frac{h'}{3.20 \text{ cm}} = -\frac{q}{p} = -\frac{0.368 \text{ m}}{7.20 \text{ m}}$$

$$h' = \boxed{0.164 \text{ cm}}$$

(d) The lens intercepts power given by $$\mathscr{P} = IA = \left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150 \text{ m})^2\right]$$

and puts it all onto the image where $$I = \frac{\mathscr{P}}{A} = \frac{\left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\pi(15.0 \text{ cm})^2/4\right]}{\pi(0.164 \text{ cm})^2/4}$$

$$I = \boxed{58.1 \text{ W/m}^2}.$$

P26.51 From the thin lens equation, $$q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00 \text{ cm})(12.0 \text{ cm})}{12.0 \text{ cm} - (-6.00 \text{ cm})} = -4.00 \text{ cm}.$$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$.

In this case, $p_2 = d - (-4.00 \text{ cm})$.

Therefore, $d + 4.00 \text{ cm} = f_2 = 12.0 \text{ cm}$ and $d = \boxed{8.00 \text{ cm}}$.

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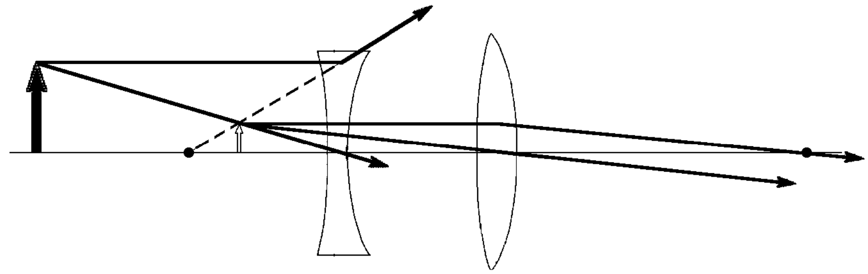

FIG. P26.51

P26.52 The inverted real image is formed by the lens operating on light directly from the object, on light that has not reflected from the mirror.

For this we have $M = -1.50 = -\dfrac{q}{p}$ $\qquad q = 1.50p$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{p}+\frac{1}{1.50p}=\frac{1}{10\text{ cm}}=\frac{2.50}{1.50p} \qquad p = 10\text{ cm}\left(\frac{2.5}{1.5}\right)=16.7\text{ cm}$$

Then the object is distant from the mirror by $40.0\text{ cm} - 16.7\text{ cm} = 23.3\text{ cm}$.

The second image seen by the person is formed by light that first reflects from the mirror and then goes through the lens. For it to be in the same position as the inverted image, the lens must be receiving light from an image formed by the mirror at the same location as the physical object. The formation of this image is described by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{23.3\text{ cm}}+\frac{1}{23.3\text{ cm}}=\frac{1}{f} \qquad f=\boxed{11.7\text{ cm}}\,.$$

P26.53 For the mirror, $f = \dfrac{R}{2} = +1.50\text{ m}$. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}$, then gives $\quad q = f = \boxed{1.50\text{ m}}$.

Now, in $\quad M = -\dfrac{q}{p} = \dfrac{h'}{h}$

the magnification is nearly zero, but we can be more precise: $\dfrac{h}{p}$ is the angular diameter of the object.

Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi\text{ rad}}{180°}\right)(1.50\text{ m}) = -0.0140\text{ m} = \boxed{-1.40\text{ cm}}\,.$$

P26.54 (a) For the light the mirror intercepts,

$$\mathscr{P} = I_0 A = I_0 \pi R_a^2$$

$$350\text{ W} = \left(1\,000\text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334\text{ m or larger}}$.

(b) In $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}=\dfrac{2}{R}$

we have $p \to \infty$

so $q = \dfrac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q\left(\dfrac{h}{p}\right) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where $\dfrac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \dfrac{\mathscr{P}}{\pi h'^2/4} = \dfrac{4I_0\pi R_a^2}{\pi h'^2} = \dfrac{4I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$

$$120\times10^3\text{ W/m}^2 = \frac{16\left(1\,000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$$

so $\boxed{\dfrac{R_a}{R} = 0.025\,5\text{ or larger}}$.

P26.55 In the original situation, $p_1 + q_1 = 1.50\text{ m}$.

In the final situation, $p_2 = p_1 + 0.900\text{ m}$

and $q_2 = q_1 - 0.900\text{ m} = 0.600\text{ m} - p_1$.

Our lens equation is $\dfrac{1}{p_1}+\dfrac{1}{q_1}=\dfrac{1}{f}=\dfrac{1}{p_2}+\dfrac{1}{q_2}$.

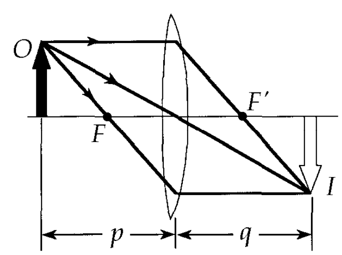

FIG. P26.55

Substituting, we have $\dfrac{1}{p_1}+\dfrac{1}{1.50\text{ m}-p_1}=\dfrac{1}{p_1+0.900}+\dfrac{1}{0.600-p_1}$.

Adding the fractions, $\dfrac{1.50\text{ m}-p_1+p_1}{p_1(1.50\text{ m}-p_1)}=\dfrac{0.600-p_1+p_1+0.900}{(p_1+0.900)(0.600-p_1)}$.

Simplified, this becomes $p_1(1.50\text{ m}-p_1)=(p_1+0.900)(0.600-p_1)$.

continued on next page

(a) Thus, $p_1 = \frac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$ $\quad p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$

(b) $\frac{1}{f} = \frac{1}{0.300\text{ m}} + \frac{1}{1.50\text{ m} - 0.300\text{ m}}$ and $f = \boxed{0.240\text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$

with $M = -\frac{q_2}{p_2} = \boxed{-0.250}$.

P26.56 A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:

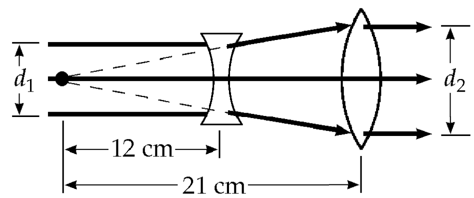

FIG. P26.56

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\qquad \frac{1}{\infty} + \frac{1}{q} = \frac{1}{-12\text{ cm}}$

$q = -12\text{ cm}$.

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p = 21\text{ cm}$. Then

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\qquad \frac{1}{21\text{ cm}} + \frac{1}{q} = \frac{1}{21\text{ cm}}$

$q = \infty$.

The exiting rays will be parallel. The lenses must be $21.0\text{ cm} - 12.0\text{ cm} = 9.00\text{ cm}$ apart.

By similar triangles, $\frac{d_2}{d_1} = \frac{21\text{ cm}}{12\text{ cm}} = \boxed{1.75\text{ times}}$.

***P26.57** (a) For the lens in air,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{79\text{ cm}} = (1.55-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the same lens in water

$$\frac{1}{f'} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f'} = \left(\frac{1.55}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

continued on next page

By division,

$$\frac{\frac{1}{79\text{ cm}}}{\frac{1}{f'}} = \frac{0.55}{0.165} = \frac{f'}{79\text{ cm}}$$

$$f' = 79\text{ cm}(3.32) = \boxed{263\text{ cm}}$$

(b) The path of a reflected ray does not depend on the refractive index of the medium which the reflecting surface bounds. Therefore the focal length of a mirror does not change when it is put into a different medium: $f' = \frac{R}{2} = f = \boxed{79.0\text{ cm}}$.

P26.58 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

becomes: $\frac{1}{5.00\text{ cm}} = (n-1)\left[\frac{1}{9.00\text{ cm}} - \frac{1}{(-11.0\text{ cm})}\right]$ giving $n = \boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

becomes: $\frac{1}{8.00\text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00\text{ cm}}$

or $q_1 = 13.3\text{ cm}$, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3\text{ cm}}{8.00\text{ cm}} = -1.67$.

This image becomes the object for the concave mirror with:

$$p_m = 20.0\text{ cm} - q_1 = 20.0\text{ cm} - 13.3\text{ cm} = 6.67\text{ cm}$$

and $f = \frac{R}{2} = +4.00\text{ cm}$.

The mirror equation becomes: $\frac{1}{6.67\text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00\text{ cm}}$

giving $q_m = 10.0\text{ cm}$

and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0\text{ cm}}{6.67\text{ cm}} = -1.50$.

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0\text{ cm} - q_m = +10.0\text{ cm}.$$

The thin lens equation yields: $\frac{1}{10.0\text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00\text{ cm}}$

continued on next page

or $$q_3 = 10.0\text{ cm}$$

and $$M_3 = -\frac{q_3}{p_3} = -\frac{10.0\text{ cm}}{10.0\text{ cm}} = -1.00\,.$$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{\text{total}} = M_1 M_2 M_3 = \boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

P26.59 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

For the upper mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \qquad \frac{1}{7.50\text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50\text{ cm}} \qquad q_1 = \infty.$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50\text{ cm}} \qquad q_2 = 7.50\text{ cm}.$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

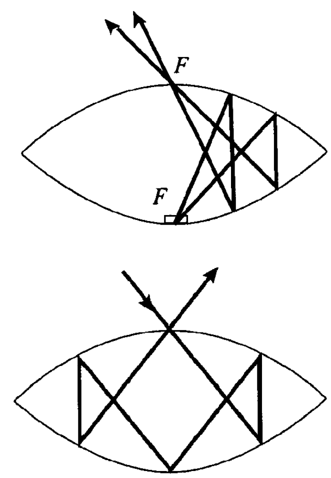

FIG. P26.59

P26.60 (a) For lens one, as shown in the first figure,

$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}$$

$$q_1=120\text{ cm}$$

$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

This real image $I_1=O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}:$$

$$q_2=\boxed{20.0\text{ cm}}$$

$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is $\boxed{\text{inverted}}$.

(c) If lens two is a converging lens (third figure):

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2=\boxed{6.67\text{ cm}}$$

$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

Again, $M_{\text{overall}}<0$ and the final image is $\boxed{\text{inverted}}$.

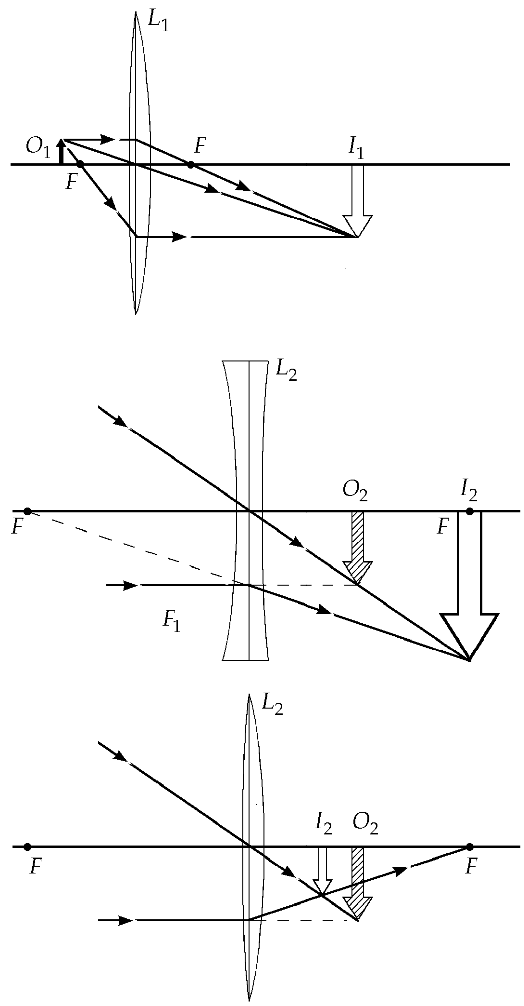

FIG. P26.60

ANSWERS TO EVEN PROBLEMS

P26.2 4.58 m

P26.4 see the solution

P26.6 (a) $-(p_1+h)$; (b) virtual; (c) upright; (d) 1.00; (e) no

P26.8 at $q=-0.267$ m virtual upright diminished with $M=0.0267$

P26.10 behind the worshiper, 3.33 m from the deepest point in the niche

P26.12 (a) 2.22 cm; (b) 10.0

P26.14 (a) 160 mm; (b) –267 mm

P26.16 (a) convex; (b) at the 30.0 cm mark; (c) –20.0 cm

P26.18 (a) 15.0 cm; (b) 60.0 cm

P26.20 (a) see the solution; (b) at 0.639 s and 0.782 s

P26.22 4.82 cm

P26.24 see the solution, the image is real, inverted, and diminished

P26.26 1.50 cm/s

P26.28 20.0 cm

P26.30 (a) $q = 40.0 \text{ cm}$, $M = -1.00$, real and inverted; (b) $q \to \infty$, no image is formed; (c) $q = -20.0 \text{ cm}$, $M = +2.00$, virtual and upright

P26.32 (a) 6.40 cm; (b) –0.250; (c) converging

P26.34 (a) $f = 39.0 \text{ mm}$; (b) $p = 39.5 \text{ mm}$

P26.36 (a) 5.36 cm; (b) –18.8 cm; (c) virtual, right side up, enlarged; (d) A magnifying glass with focal length 7.50 cm is used to form an image of a stamp, enlarged 3.50 times. Find the object distance. Locate and describe the image.

P26.38 (a) $f = 13.3 \text{ cm}$; (b) see the solution; (c) 224 cm^2

P26.40 2.18 mm away from the film plane

P26.42 (a) the bottom half; (b) 1.50 mm; (c) 48.2°; (d) see the solution, 96.4°; (e) see the solution

P26.44 160 cm to the left of the lens, inverted, $M = -0.800$

P26.46 see the solution

P26.48 25.3 cm to the right of the mirror, virtual, upright, enlarged 8.05 times

P26.50 (a) 1.40 kW/m^2; (b) 6.91 mW/m^2; (c) 0.164 cm; (d) 58.1 W/m^2

P26.52 11.7 cm

P26.54 (a) 0.334 m or larger; (b) R_a must be at least 0.025 5 R

P26.56 Align the lenses on the same axis and 9.00 cm apart. Let the light pass first through the diverging lens and then through the converging lens. The diameter increases by a factor of 1.75.

P26.58 (a) 1.99; (b) 10.0 cm to the left of the lens, –2.50; (c) inverted

P26.60 (a) 20.0 cm to the right of the second lens, –6.00; (b) inverted; (c) 6.67 to the right of the second lens, –2.00, inverted