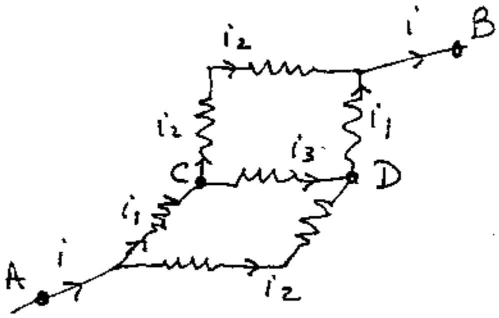
Physics 4C — **Quiz 3 Solutions** — Fall 2011

Problem 1



$i_1 = i_2 + i_3$

$i_1 R + i_3 R = 2 i_2 R \Rightarrow i_1 + i_3 = 2 i_2$

$\Rightarrow i_3 = 2 i_2 - i_1$

$i_1 = i_2 + 2 i_2 - i_1 \Rightarrow 2 i_1 = 3 i_2 \Rightarrow \boxed{i_2 = \frac{2}{3} i_1 \,,\; i_3 = \frac{1}{3} i_1}$

$i = i_1 + i_2 = \frac{5}{3} i_1 \Rightarrow \boxed{i_1 = \frac{3}{5} i}$

if V is the potential difference between points A and B,

$V = i_1 R + 2 i_2 R$ (or equivalently $V = i_1 R + i_3 R + i_1 R$)

$\Rightarrow V = i_1 R + \frac{4}{3} i_1 R = \frac{7}{3} i_1 R = \frac{7}{3} \cdot \frac{3}{5} i R = \frac{7}{5} R \cdot i$

$\Rightarrow \frac{V}{i} = \frac{7}{5} R \Rightarrow \boxed{R_{eq} = \frac{7}{5} R}$ equivalent resistance

(b) Voltage drop between A and C: $V_{AC} = i_1 R = \frac{3}{5} i R$

Total voltage drop between A and B: $V_{AB} = i R_{eq} = \frac{7}{5} i R$

So $V_{AC} = \frac{3}{7} V_{AB}$; given that $V_{AB} = 14V \Rightarrow \boxed{V_{AC} = 6V}$

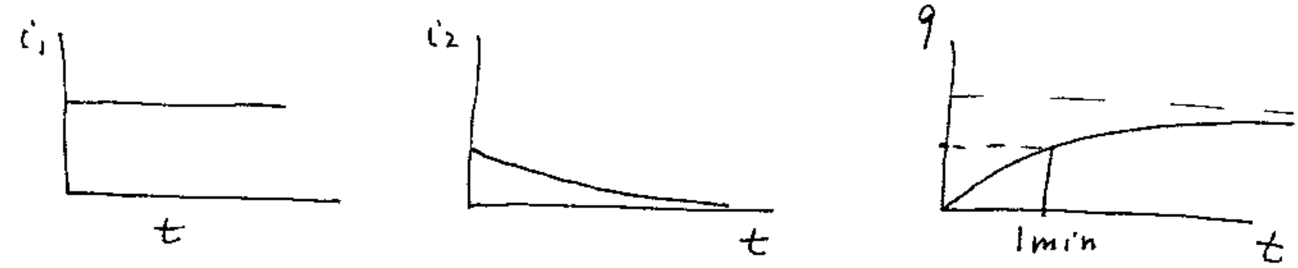
Voltage drop between D and B = 6 V, by symmetry $\Rightarrow$ between C and D = 2 V

Or: $V_{CD} = i_3 R = \frac{1}{3} i_1 R = \frac{1}{5} i R = \frac{1}{7} V_{AB} = 2V$

Hence, $\boxed{V_A = 14V, \quad V_C = 8V, \quad V_D = 6V, \quad V_B = 0}$

Problem 2

(a)



(b) right after switch is closed:

$\boxed{i_1 = \frac{\varepsilon}{R}}$, $\boxed{i_2 = \frac{\varepsilon}{2R}}$ (capacitor acts as a short-circuit, no potential drop)

(c) 1 minute after S is closed, the charge in capacitor is $\frac{1}{2}$ of what it is after a long time (1 hour). For i_1, no change. $\boxed{i_1 = \frac{\varepsilon}{R}}$ still.

For i_2, we have a circuit with emf ε and resistance $2R$. So the charge in capacitor is

$$q(t) = C\varepsilon\left(1 - e^{-t/2RC}\right) \; ; \; \text{for } t = t_0 = 1\text{min},\; q(t_0) = \tfrac{1}{2} q(t=\infty)$$

$$\Rightarrow \boxed{e^{-t_0/2RC} = \frac{1}{2}}$$. The current is

$$i_2(t_0) = \frac{dq}{dt} = -\frac{\varepsilon}{2R} e^{-t_0/2RC} = -\frac{\varepsilon}{4R} \Rightarrow \boxed{i_2 = \frac{\varepsilon}{4R}}$$ half of value at $t=0$

(d) When the switch is opened again, capacitor discharges across a resistance $= 3R$. So charge as function of time is:

$$q(t) = q_0 e^{-t/3RC} = \frac{q_0}{2}$$ (charge goes from $q_0 = 100C$ to $50C$)

$$\Rightarrow e^{-\frac{t}{3RC}} = \frac{1}{2} = e^{-\frac{t_0}{2RC}} \Rightarrow \boxed{t = \frac{3}{2} t_0}$$

since $t_0 = 1$ min $\Rightarrow$ $\boxed{t = 90 \text{ seconds}}$

Problem 3

Magnetic field at center of circular loop: $B = \frac{\mu_0 i}{2R}$ (R = radius)

Mag. field of ∞ straight wire at distance r: $B = \frac{\mu_0 i}{2\pi r}$

(a) We need to add field from $\frac{1}{2}$ circular loops of radii b and $2b \Rightarrow$

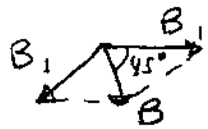
$$B = \frac{1}{2}\left(\frac{\mu_0 i}{2b} + \frac{\mu_0 i}{4b}\right) \Rightarrow \boxed{B = \frac{3\mu_0 i}{8b} \text{ points \underline{into} paper}}$$

(b) We have half of ∞ wire twice, distances a and $2a$, in opposite directions:

$$B = \frac{1}{2}\left(\frac{\mu_0 i}{2\pi a} - \frac{\mu_0 i}{4\pi a}\right) \Rightarrow \boxed{B = \frac{\mu_0 i}{8\pi a} \text{ points \underline{out of} paper}}$$

(since closer wire gives bigger field)

(c) The fields are of equal magnitude at $90°$ angle



$B_1 = \frac{\mu_0 i}{2R}$; $B = \sqrt{2}\, B_1 \Rightarrow$

$\boxed{B = \frac{\mu_0 i}{\sqrt{2} R}}$ | points to the right and out of the paper at 45° as shown above |