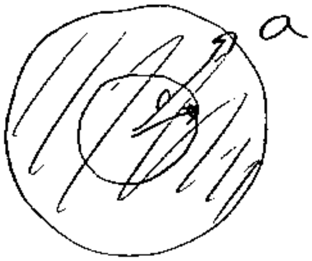
Homework 7

8.2 B



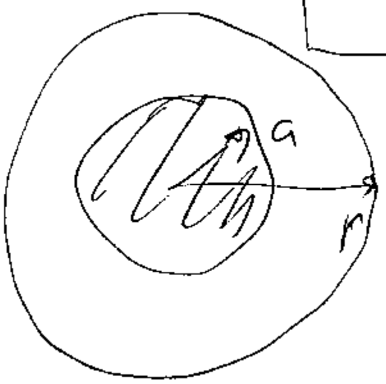
For $r < a$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$I = \pi r^2 j$$

$$2\pi r B = \mu_0 \pi r^2 j$$

$$\boxed{B = \frac{\mu_0 r j}{2}}$$



For $r \geq a$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

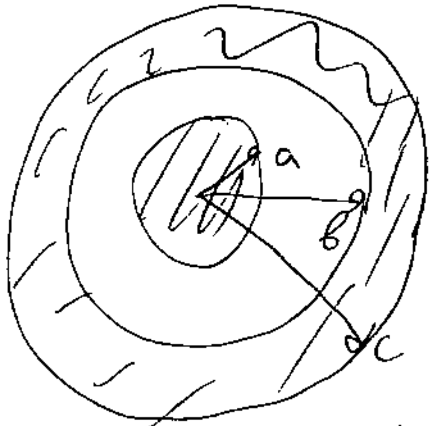
$$I = \pi a^2 j$$

$$2\pi r B = \mu_0 \pi a^2 j$$

$$\boxed{B = \frac{\mu_0 a^2 j}{2r}}$$

8.2E

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$



For $r < a$:

$$I_{in} = j_1 \pi r^2$$

where j_1 is the current density of the inner wire;

$$j_1 = \frac{I}{\pi a^2}$$

$$2\pi r B = \mu_0 \frac{I}{\pi a^2} \pi r^2$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

For $a < r < b$: $I_{in} = I$

$$2\pi r B = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

For $b \leq r < c$:

$$I_{in} = I - j_2 (\pi r^2 - \pi b^2)$$

where j_2 is the current density of the outer wire:

$$j_2 = \frac{I}{\pi c^2 - \pi b^2}$$

$$2\pi r B = \mu_0 I \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)}$$

For $r \geq c$: $I_{in} = I - I = 0$ so

$$\boxed{B = 0}$$

8.2F $l = 20\,cm = 0.2\,m$

$r = 2\,cm = 0.02\,m$

$N = 3000$

$I = 2A$

a. $j^s = \frac{NI}{l} = 3 \cdot 10^4 \frac{A}{m}$

b. $B_{mid} = \mu_0 j^s = 3.77 \cdot 10^{-2}\,T$

c. $B_{end} = \frac{\mu_0 j^s}{2} = 1.88 \cdot 10^{-2}\,T$

d. $\phi_{mid} = B_{mid} \cdot \pi r^2 = 4.74 \cdot 10^{-5}\,Wb$

e. $\phi_{end} = B_{end} \cdot \pi r^2 = 2.37 \cdot 10^{-5}\,Wb$

8.2G $Q = \sigma \cdot 2\pi a$

$T = \frac{2\pi}{\omega}$

$I = \frac{Q}{T} = \frac{\sigma \cdot 2\pi a}{2\pi / \omega} = \sigma \omega a$

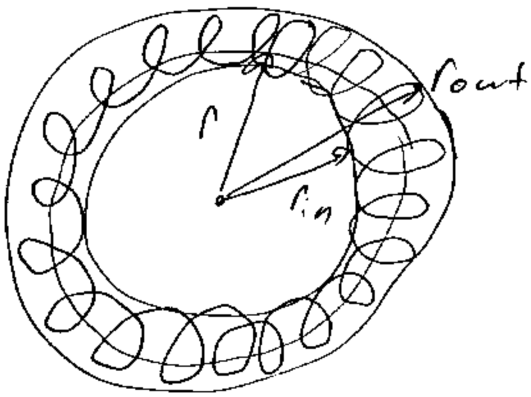
$B = \frac{\mu_0 I}{2a} = \frac{\mu_0 \sigma \omega a}{2a} = \frac{\mu_0 \sigma \omega}{2}$

8.2 I

$N = 1000$; $I = 5A$

$r_{in} = 5cm = 0{,}05m$

$r_{out} = 6cm = 0{,}06m$



At radius r

$\oint \vec{B} d\vec{l} = \mu_0 N I$

$2\pi r \cdot B = \mu_0 N I$

$B = \dfrac{\mu_0 N I}{2\pi r}$

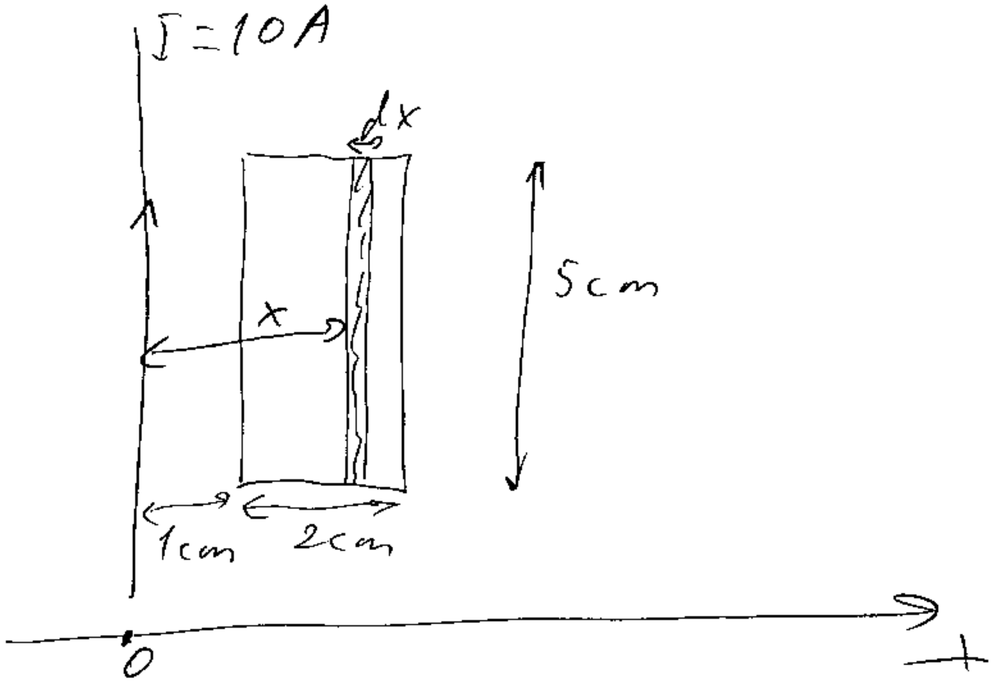
a. For average we take $r = \dfrac{r_{in} + r_{out}}{2} = 0{,}055m$

$$\boxed{B = 1{,}82 \cdot 10^{-2}\, T}$$

b. For $r = 0{,}51cm$

$$\boxed{B = 1{,}96 \cdot 10^{-2}\, T}$$

8.2 J



The flux through a region of width dx is

$$d\phi = B \cdot dx \cdot 0.05m$$

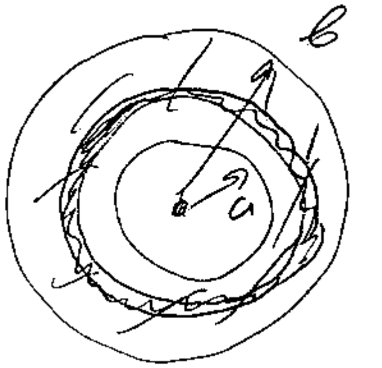
$$B = \frac{\mu_0 I}{2\pi x}$$

$$d\phi = \frac{\mu_0 I}{2\pi x} \cdot 0{,}05m\, dx$$

$$\phi = \int_{0{,}01m}^{0{,}03m} \frac{\mu_0 I}{2\pi x} \cdot 0{,}05m\, dx = \frac{\mu_0 I \cdot 0{,}05m}{2\pi} \ln \frac{0{,}03}{0{,}01}$$

$$\phi = 1{,}1 \cdot 10^{-7}\ Wb$$

8.20



Consider a ring of radius r and thickness dr. The number of turns is

$$dn = Z \cdot dr$$

The current is $dI = I \cdot dn$

$$dB = \frac{\mu_0 \, dI}{2r} = \frac{\mu_0 I \, dn}{2r} = \frac{\mu_0 I Z \, dr}{2r}$$

$$B = \int_a^b \frac{\mu_0 I Z \, dr}{2r} = \frac{\mu_0 I Z}{2} \ln \frac{b}{a}$$

8.2P

$r = 0{,}5 \cdot 10^{-10}\ m$

$f = 10^{13}\ Hz$

$T = \frac{1}{f} = 10^{-13}\ s$

$I = \frac{e}{T} = 1{,}6 \cdot 10^{-6}\ A$

$$B = \frac{\mu_0 I}{2r} = 2 \cdot 10^{-2}\ T$$

8.3B

$B = 0.1\,T$

$r = 0.3\,m$

$\Delta V = 1000\,V$

$$\frac{mv^2}{2} = e\Delta V$$

$$v^2 = \frac{2e\Delta V}{m} \qquad (1)$$

$$a = \frac{v^2}{r} = \frac{2e\Delta V}{mr}$$

$$F = ma = \frac{2e\Delta V}{r}$$

$$F = evB$$

$$evB = \frac{2e\Delta V}{r} \Rightarrow v = \frac{2\Delta V}{Br}$$

$$v^2 = \frac{4(\Delta V)^2}{B^2 r^2}$$

From (1):

$$\frac{2e\Delta V}{m} = \frac{4(\Delta V)^2}{B^2 r^2} \Rightarrow m = \frac{eB^2 r^2}{2\Delta V}$$

$$\boxed{m = 7.2 \cdot 10^{-26}\,kg}$$

8.3 C $T = 10^{-8}$ s

$$F = ma$$

$$e\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$$

$$eB = m\omega = m\frac{2\pi}{T}$$

$$B = \frac{2\pi m}{eT}$$

$$\boxed{B = 3{,}57 \cdot 10^{-4}\ T}$$

8.4 A $r = 10$ cm $= 0{,}1$ m

$N = 10$

$I = 5$ A

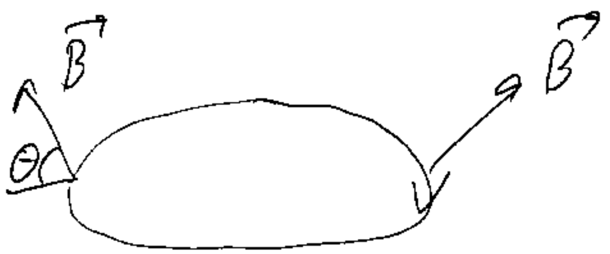
$$\mu = NI \cdot A = NI \cdot \pi r^2$$

$$\tau = \vec{\mu} \times \vec{B}$$

$$\tau = \mu B \sin\theta = NI\pi r^2 B \sin\theta$$

$$\boxed{\tau = 1{,}57\, B \sin\theta \ \ N\cdot m}$$

8.4B



The vertical component of the magnetic field is homogeneous and creates a total force equal to 0. The horizontal component creates a force directed upwards at every point of the loop.

$B_{hor.} = B_0 \cos\theta$

On an element dl

$dF = N I_0 B_{hor} \, dl = N I_0 B_0 \cos\theta \, dl$

For the whole loop

$l = 2\pi r$

$$\boxed{F = 2\pi r N I_0 B_0 \cos\theta \quad \text{upwards}}$$