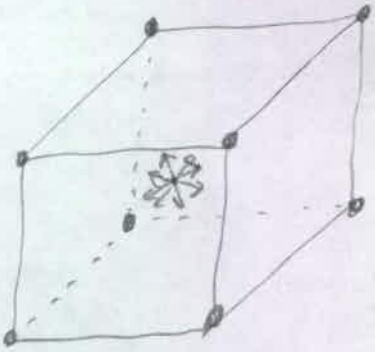
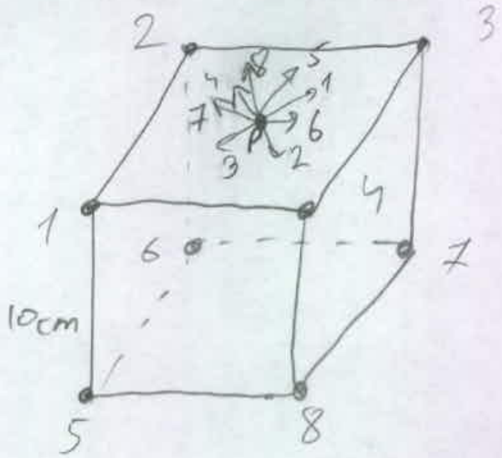
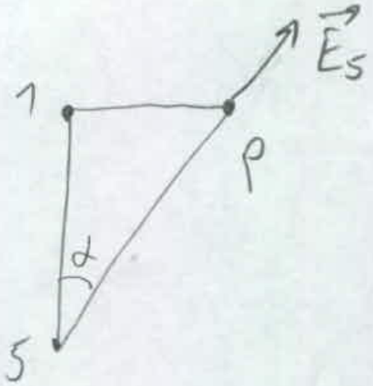
2.1 A



a. The electric field at the center of the cube created by charges at opposite corners is totally canceled, so <u>the total electric field is 0.</u>



b. The electric field at the center of the upper face created by charges on the upper face is canceled ($\vec{E}_1$ cancels $\vec{E}_3$, $\vec{E}_2$ cancels $\vec{E}_4$). The horizontal components of field created by charges 5 and 7, as well as 6 and 8 cancel, the vertical components are all the same.



$$r_{1P} = \frac{1}{2} r_{13} = \frac{1}{2}\sqrt{2}\cdot 10\text{ cm}$$

$$r_{5P}^2 = (10\text{cm})^2 + \left(\frac{\sqrt{2}}{2}\,10\text{ cm}\right)^2 =$$

$$= 150\text{ cm}^2 = 0{,}015\text{ m}^2$$

$$E_5 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{5P}^2} = 6\cdot 10^{11}\, Q\ \frac{N}{C}$$

$$E_{5y} = E_5 \cos\alpha = E_5 \cdot \frac{10\text{ cm}}{\sqrt{150}\text{ cm}} = 4{,}9\cdot 10^{11}\, Q\ \frac{N}{C}$$

The total electric field at point P is 4 times the vertical component of the field from one charge:

$$\underline{E_y = 4E_{5y} = 1{,}96\cdot 10^{12}\, Q\ \frac{N}{C}}$$

c. Suppose charge 1 is removed. Then all the opposite pairs cancel and the field is just that created by charge 7.

$$r_{17}^2 = r_{15}^2 + r_{57}^2 = r_{15}^2 + r_{58}^2 + r_{87}^2$$

$r_{17}^2 = 3 \cdot (10\,cm)^2$

$r_{17} = \sqrt{3} \cdot 0{,}1\,m$

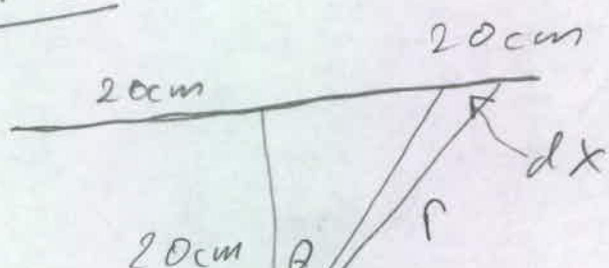
The distance from charge 7 to center is half of r_{17}:

$$r = \frac{1}{2} r_{17} = \frac{\sqrt{3}}{2} 0{,}1\,m$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\frac{3}{4} 0{,}01\,m^2}$$

$$\boxed{E = 1{,}2 \cdot 10^{12}\, Q\, \frac{N}{C}}$$

2.1. C



The horizontal components of the fields from both sides cancel (because of symmetry), the vertical components add.

Consider a small element dx:

$$dQ = \mu\, dx$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\mu\, dx}{(0.2m/\cos\theta)^2}$$

$$x = 0.2m \tan\theta$$

$$dx = 0.2m \frac{1}{\cos^2\theta} d\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\mu\, d\theta}{0.2m}$$

$$dE_\perp = dE\cos\theta = \frac{1}{4\pi\varepsilon_0} \frac{\mu}{0.2m} \cos\theta\, d\theta$$

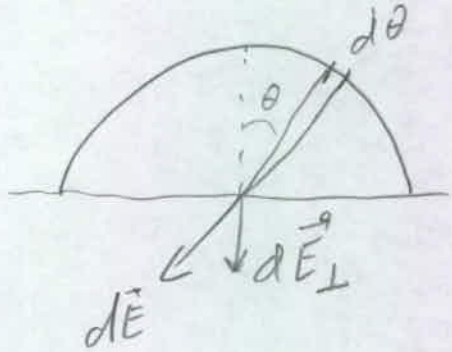
$$E_\perp = \int_{-45^\circ}^{45^\circ} \frac{1}{4\pi\varepsilon_0} \frac{\mu}{0.2m} \cos\theta\, d\theta =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\mu}{0.2m} \sin\theta \Big|_{-45^\circ}^{45^\circ} = \frac{1}{4\pi\varepsilon_0} \frac{\mu}{0.2m} \sqrt{2}$$

$$\mu = \frac{Q}{0.4m}$$

$$\boxed{E = 1.6 \cdot 10^{11}\, Q\, \frac{N}{C}}$$

2.1E



The horizontal component of the electric field cancels (because of symmetry), while the vertical component is in the same direction from all parts of the rod and has to be added. Consider an element of angular size $d\theta$. The charge is

$$dQ = \frac{Q}{\pi} d\theta$$

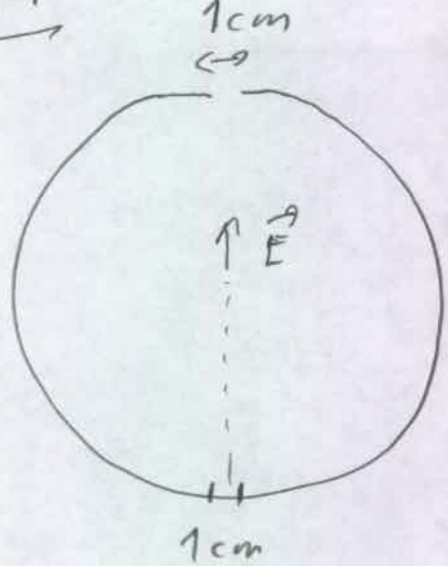
$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} = \frac{1}{4\pi^2\varepsilon_0} \frac{Q\,d\theta}{r^2}$$

$$dE_\perp = dE\cos\theta = \frac{1}{4\pi^2\varepsilon_0} \frac{Q}{r^2} \cos\theta\, d\theta$$

$$E_\perp = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4\pi^2\varepsilon_0} \frac{Q}{r^2} \cos\theta\, d\theta = \frac{1}{4\pi^2\varepsilon_0} \frac{Q}{r^2} \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\boxed{E_\perp = \frac{1}{2\pi^2\varepsilon_0} \frac{Q}{r^2}}$$

2.1 G



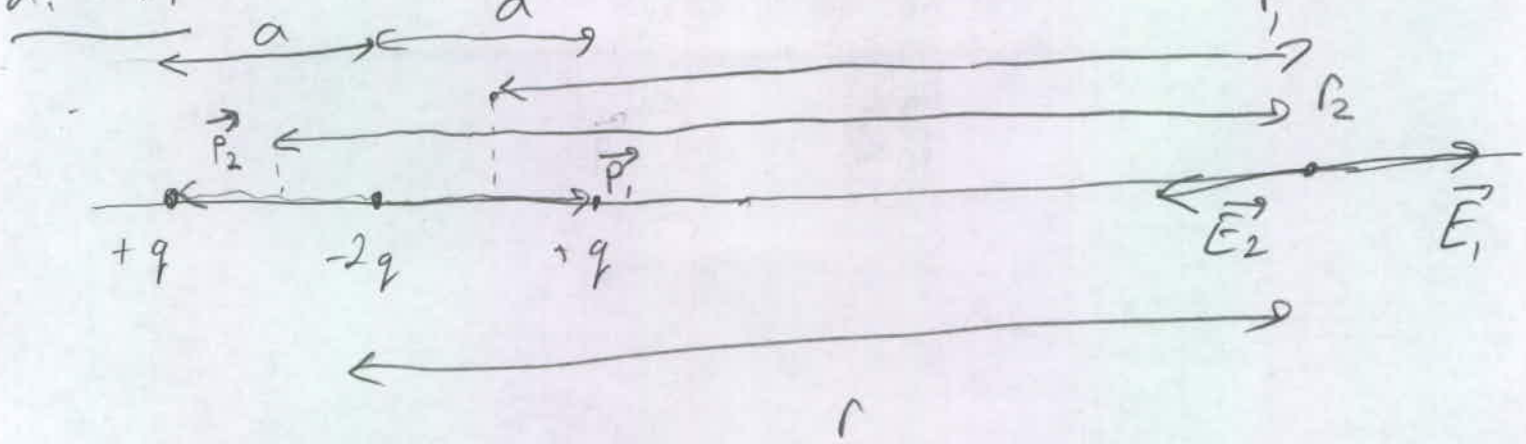
For a full ring, the electric field at the center is canceled from opposite parts of the ring and is 0. If a small section is removed, then still the field from opposite parts is canceled and all is left is the field created by an identical section which is opposite to the removed section. Since the section is small compared to the radius we can treat it as a point with charge

$$q = \mu \cdot 0.01 m$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\mu \cdot 0.01m}{(0.2m)^2}$$

$$\boxed{E = 2.25 \cdot 10^9 \mu \frac{N}{C}}$$

2.7A



$$\vec{p}_2 = -\vec{p}_1$$

$$r_1 = r - \frac{a}{2}$$

$$r_2 = r + \frac{a}{2}$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}_1}{r_1^3}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}_2}{r_2^3} = -\frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}_1}{r_2^3}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\cdot 2\vec{p}_1\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) =$$

$$= \frac{2\vec{p}_1}{4\pi\varepsilon_0}\cdot\left(\frac{1}{(r-\frac{a}{2})^3} - \frac{1}{(r+\frac{a}{2})^3}\right) =$$

$$= \frac{2\vec{p}_1}{4\pi\varepsilon_0}\cdot\frac{1}{r^3}\left(\left(1-\frac{a}{2r}\right)^{-3} - \left(1+\frac{a}{2r}\right)^{-3}\right)$$

We use the Taylor expansion

$$(1+x)^n \simeq 1+nx \quad \text{for} \quad x \ll 1$$

to get

$$\vec{E} = \frac{2\vec{p}_1}{4\pi\varepsilon_0} \cdot \frac{1}{r^3}\left(\left(1+\frac{3a}{2r}\right)-\left(1-\frac{3a}{2r}\right)\right) =$$

$$= \frac{2\vec{p}_1}{4\pi\varepsilon_0} \cdot \frac{1}{r^3} \frac{6a}{2r} = \frac{6a\vec{p}_1}{4\pi\varepsilon_0 r^4}$$

$$\boxed{E = \frac{6a^2 q}{4\pi\varepsilon_0 r^4}}$$