Homework 10

12.1A

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (12.26)$$

$$\frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \quad (12.27)$$

Differentiate (12.26) with respect to t:

$$\frac{\partial^2 E_y}{\partial x \partial t} = -\frac{\partial^2 B_z}{\partial t} \qquad (*)$$

and (12.27) with respect to x:

$$\frac{\partial^2 B_z}{\partial x^2} = -\mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t \partial x}$$

plug in $(*)$ above:

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

✓

12.1B Let $B_z(x,t) = B_0 \sin(kx + \omega t)$

with $\frac{\omega}{k} = c \equiv \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

then $$\frac{\partial^2 B_z}{\partial x^2} = -B_0 k^2 \sin(kx+\omega t)$$

$$\frac{\partial^2 B_z}{\partial t^2} = -B_0 \omega^2 \sin(kx+\omega t) =$$

$$= \frac{\omega^2}{k^2} \frac{\partial^2 B_z}{\partial x^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{k^2}{\omega^2} \frac{\partial^2 B_z}{\partial t^2} = \varepsilon_0 \mu_0 \frac{\partial^2 B_z}{\partial t^2}$$ ✓

12.1C $q = q_0 \sin \omega t$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\Delta V = \frac{q}{C} = \frac{q_0 d}{\varepsilon_0 A} \sin \omega t$$

$$E = \frac{\Delta V}{d} = \frac{q_0}{\varepsilon_0 A} \sin \omega t$$

$$\frac{\partial E}{\partial t} = \frac{q_0 \omega}{\varepsilon_0 A} \cos \omega t$$

Consider a circle of radius r around the axis of symmetry;

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S}$$

$$B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \cdot \pi r^2$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{\partial E}{\partial t}$$

$$B = \mu_0 \frac{q_0 r \omega}{2A} \cos \omega t$$

12.2 A $\quad f = \frac{c}{\lambda}$

a. $\lambda = 10^3 m$ $\quad f = \underline{3 \cdot 10^5 Hz}$

b. $\lambda = 1 m$ $\quad f = \underline{3 \cdot 10^8 Hz}$

c. $\lambda = 3 cm = 3 \cdot 10^{-2} m$ $\quad f = \underline{10^{10} Hz}$

d. $\lambda = 10^{-4} m$ $\quad f = \underline{3 \cdot 10^{12} Hz}$

e. $\lambda = 5000 A = 5 \cdot 10^{-7} m$ $\quad f = \underline{6 \cdot 10^{14} Hz}$

f. $\lambda = 0{,}1 A = 10^{-11} m$ $\quad f = \underline{3 \cdot 10^{19} Hz}$

g. $\lambda = 10^{-2} A = 10^{-12} m$ $\quad f = \underline{3 \cdot 10^{20} Hz}$

12.3A

$$z = z_0 \sin \frac{2\pi}{\lambda}(x - ut) =$$

$$= z_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi u}{\lambda}t\right)$$

$$z \equiv z_0 \sin(kx - \omega t)$$

so $\underline{k = \frac{2\pi}{\lambda}}$, $\underline{\omega = \frac{2\pi u}{\lambda} = ku}$

12.4A

$$f = 10^6 \, Hz$$

$$I = 20 \, W/m^2$$

a. $\lambda = \frac{c}{f} = \underline{300 \, m}$

b. $I = \frac{1}{2} \frac{E_0 B_0}{\mu_0}$

$$B_0 = \sqrt{\mu_0 \varepsilon_0} E_0$$

$$I = \frac{1}{2} E_0^2 \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$E_0^2 = 2I\sqrt{\frac{\mu_0}{\varepsilon_0}} = 15070 \left(\frac{N}{C}\right)^2$$

$$E_0 = 122.7 \frac{N}{C}$$

$$B_0 = 4.1 \cdot 10^{-7} T$$

12.4 B $\quad I = 2 \ cal/cm^2 \cdot min =$

$$= 2 \cdot 4.184 \ J/(10^{-2} m)^2 \cdot 60 s = 1.39 \cdot 10^3 \ W/m^2$$

$$I = \frac{1}{2} \frac{E_0 B_0}{\mu_0} = \frac{1}{2} E_0^2 \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$E_0^2 = 2I\sqrt{\frac{\mu_0}{\varepsilon_0}} = 1.05 \cdot 10^6 \left(\frac{N}{C}\right)^2$$

$$E_0 = 1.02 \cdot 10^3 \frac{N}{C}$$

$$B_0 = \frac{E_0}{c} = 3.41 \cdot 10^{-6} T$$

12.5A

$$A = 1\,cm^2 = 10^{-4}\,m^2$$

$$d = 5\,mm = 5\cdot 10^{-3}\,m$$

$$I_{el} = 10\,mA = 10^{-2}\,A \quad (rms)$$

$$f = 10^6\,Hz$$

$$V_{rms} = 100\,V$$

$$j_{el} = \frac{I_{el}}{A} = 10^2\,\frac{A}{m^2} \quad (rms)$$

$$E_{rms} = \frac{V_{rms}}{d} = 2\cdot 10^4\,\frac{V}{m}$$

$$j_{dis} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$j_{dis,rms} = \varepsilon_0 E_{rms}\cdot\omega = \varepsilon_0 E_{rms}\cdot 2\pi f$$

$$j_{dis,rms} = 1{,}11\,\frac{A}{m^2}$$

$$j_{dis} \ll j_{el} \quad (1{,}11 \ll 10^2) \quad \checkmark$$

$$I_{dis,rms} = j_{dis,rms}\cdot A = \underline{1.11\cdot 10^{-4}\,A}$$