PHYSICS 140A : STATISTICAL PHYSICS
MIDTERM EXAM SOLUTIONS

Consider a classical gas of indistinguishable particles in three dimensions with Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \left\{ A \, |\boldsymbol{p}_i|^3 - \mu_0 H S_i \right\} ,$$

where A is a constant, and where $S_i \in \{-1\,,\,0\,,\,+1\}$ (*i.e.* there are three possible spin polarization states).

(a) Compute the free energy $F_{\text{gas}}(T, H, V, N)$.

(b) Compute the magnetization density $m_{\text{gas}} = M_{\text{gas}}/V$ as a function of temperature, pressure, and magnetic field.

The gas is placed in thermal contact with a surface containing N_{s} adsorption sites, each with adsorption energy $-\Delta$. The surface is metallic and shields the adsorbed particles from the magnetic field, so the field at the surface may be approximated by $H = 0$.

(c) Find the Landau free energy for the surface, $\Omega_{\text{surf}}(T, N_{\text{s}}, \mu)$.

(d) Find the fraction $f_0(T, \mu)$ of empty adsorption sites.

(e) Find the gas pressure $p^*(T, H)$ at which $f_0 = \frac{1}{2}$.