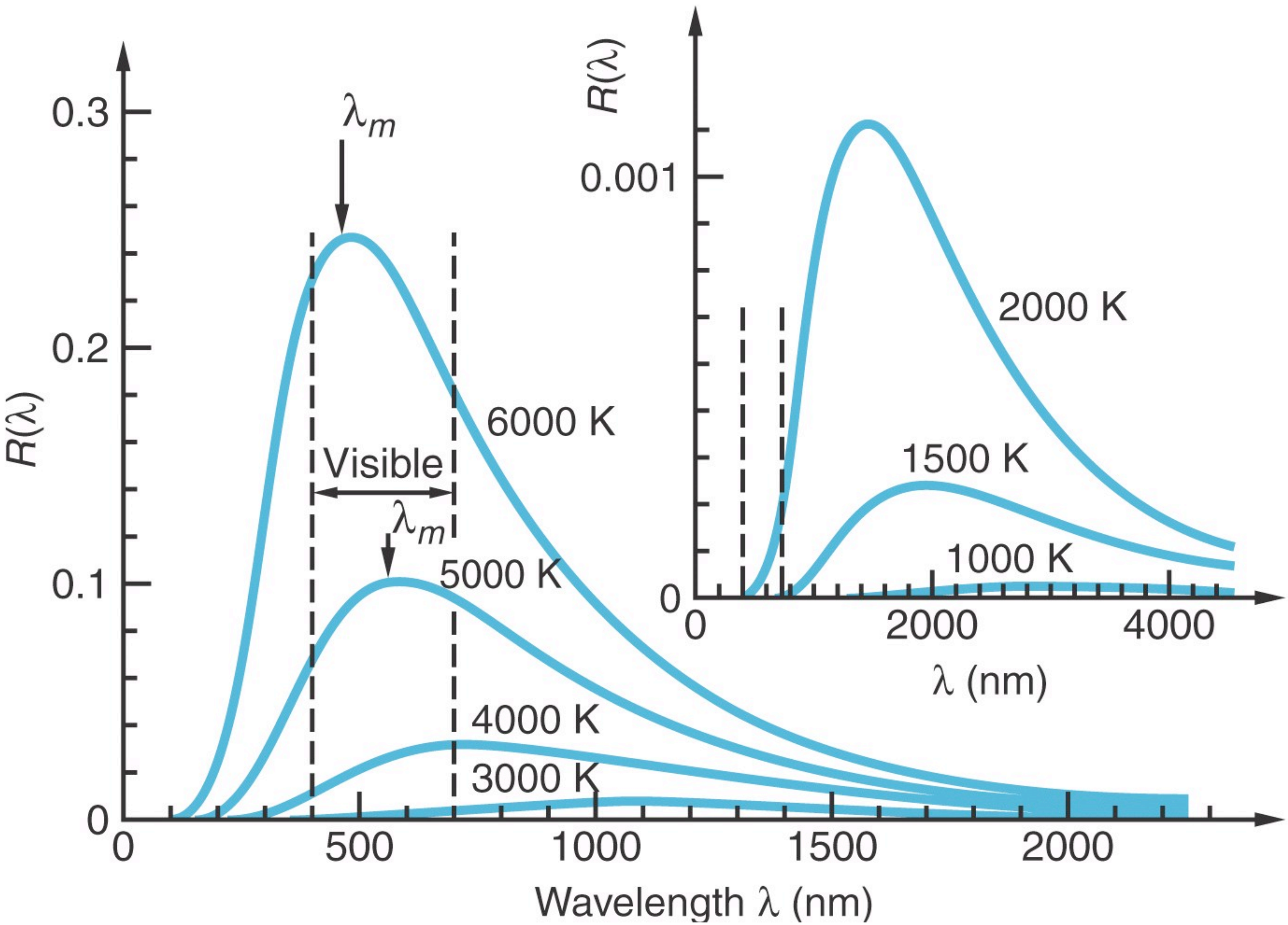
Physics 2D Lecture Slides
Lecture 10

Jan.25 , 2010

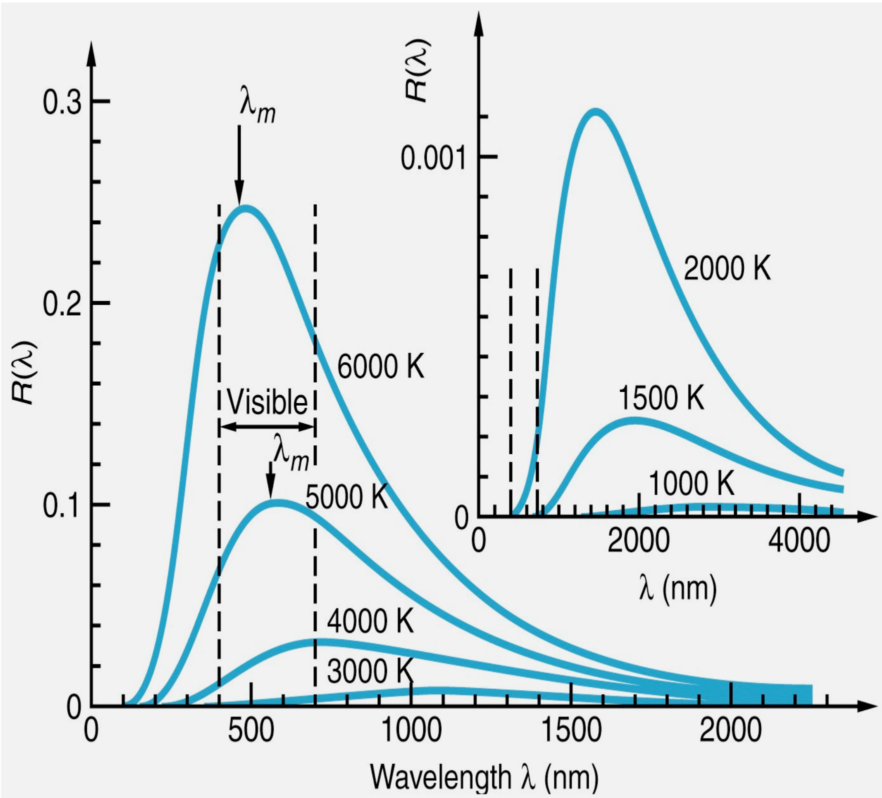
Radiation from A Blackbody



(a) **Intensity of Radiation** $\mathbf{I} = \int R(\lambda)d\lambda \propto T^4$

$$I = \sigma\, T^4 \text{(Area under curve)}$$

Stephan-Boltzmann Constant σ = 5.67 10^{-8} W / m^2 K^4



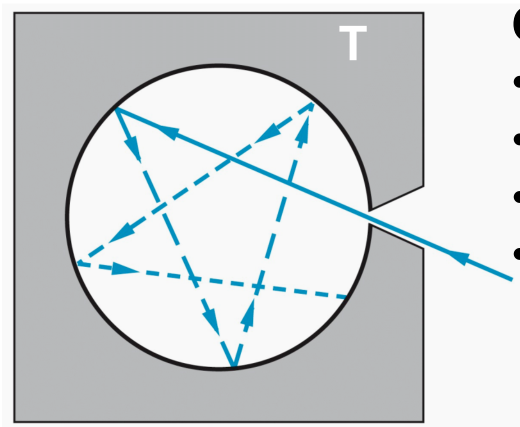
(b) Higher the temperature of BBQ Lower is the λ of PEAK intensity

$$\lambda_{MAX} \propto 1/T$$

Wien's Law λ_{MAX} T = const = 2.898 10^{-3} mK

Reason for different shape of R(λ) Vs λ for different temperature?
Can one explain in on basis of Classical Physics (2A,2B,2C) ??

Blackbody Radiator: An Idealization



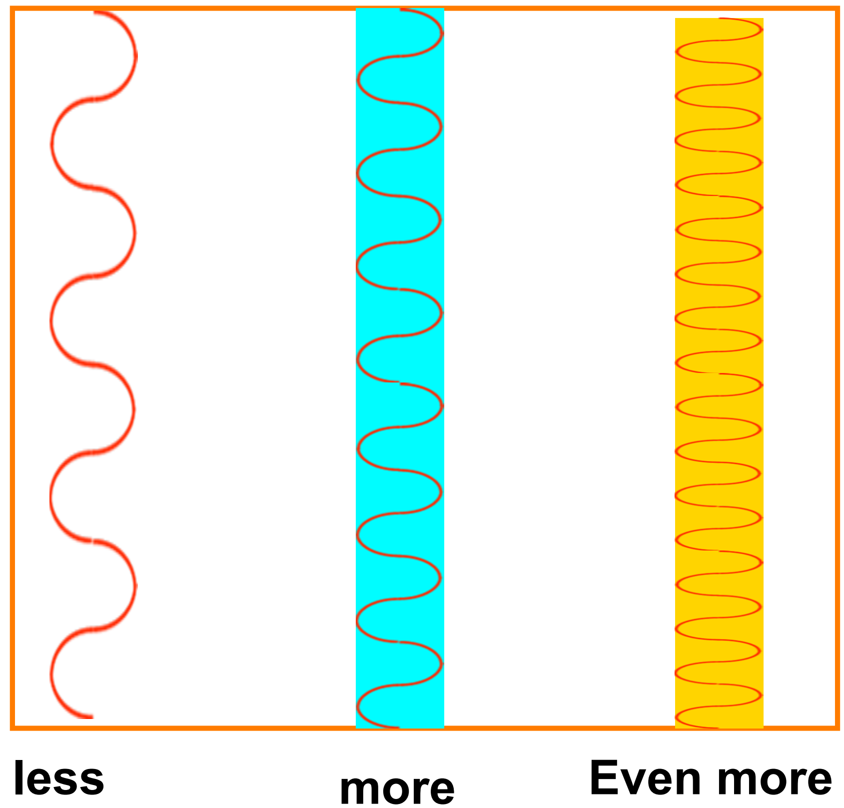
Classical Analysis:
- Box is filled with EM standing waves
- Radiation reflected back-and-forth between walls
- Radiation in thermal equilibrium with walls of Box
- How may waves of wavelength λ can fit inside the box ?

Blackbody Absorbs everything
Reflects nothing
All light entering opening gets absorbed (ultimately) by the cavity wall

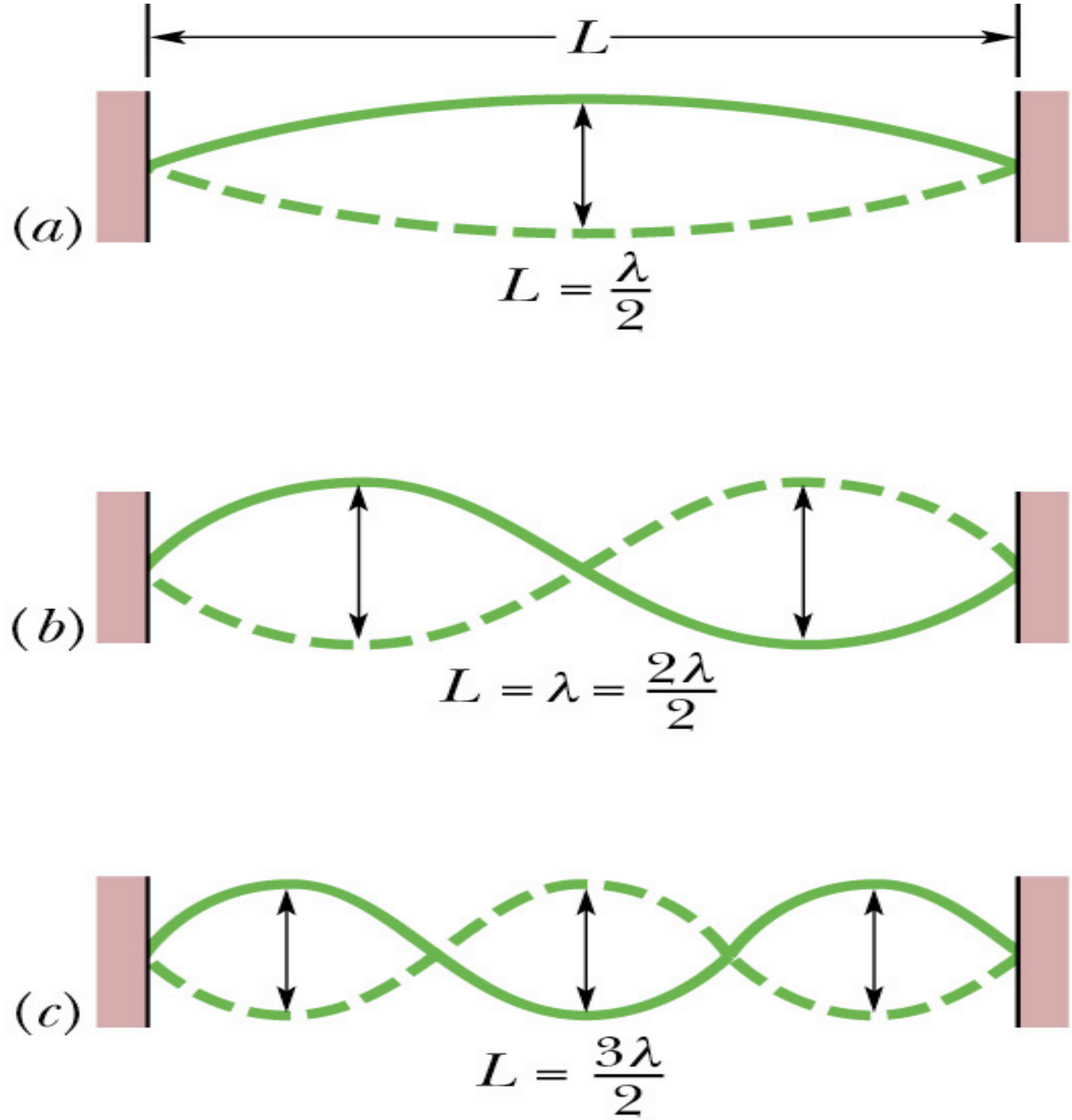
Cavity in equilibrium T w.r.t. surrounding. So it radiates everything It absorbs

Emerging radiation is a sample of radiation inside box at temp T

Predict nature of radiation inside Box ?



Standing Waves



Calculation of Number of Allowed modes/ Unit Volume in a Cavity

Assume cavity is cube of side 2L. **E**-field = 0 at walls ($x,y,z = \pm L$)

Construct wave solutions out of forms $\mathbf{E} = \mathbf{E}_0 \exp(ik_x x + k_y y + k_z z)$

Electromagnetic Wave (Soln. of Maxwell's Equations inside cavity)

Must be of form $\mathbf{E} = \mathbf{E}_0 \,\mathrm{Sin}(n_1\pi x/L)\, \mathrm{Sin}(n_2\pi y/L)\, \mathrm{Sin}(n_3\pi z/L)$

i.e. $k_x = n_1\pi/L$, etc. ($n_1\ n_2\ n_3$ = integers > 0)

k points lie on a cubic mesh of spacing (π/L) along kx, ky, kz axes

i.e. one **k** point per volume $(\pi/2L)^3$

So density of **k** points is $(2L/\pi)^3$ per unit volume in **k**-space

Volume of k space between **k** vectors of magnitude k and k + dk

is $4\pi k^2 dk$ so no. of allowed **k** points in that volume

= (1/8) x Density of **k** points x $4\pi k^2 dk$ = $(1/8)\,(2L/\pi)^3\,4\pi k^2 dk$

Factor of (1/8) is because only positive values of $k_x\ k_y\ k_z$ allowed--> positive octant of volume only.

Multiply by 2 for 2 possible polarizations of **E** field and remember $(2L)^3 = V$ (volume of cavity)

---->No. of allowed modes/unit volume of cavity with k between k and k + dk

= $n(k)dk = (k^2/\pi^2)dk$

Now $k = 2\pi/\lambda$ so $dk = -\,2\pi/\lambda^2\, d\lambda$ ------> $n(\lambda)d\lambda = (8\pi/\lambda^4)d\lambda$

and $\lambda = c/f$ so $d\lambda = -c/f^2\, df$ ----> $n(f)\, df = (8\pi f^2/c^3)\, df$

The above formulae give the no. of modes in k-intervals, wavelength intervals

and frequency intervals for EM radiation in a cavity per unit volume of cavity.

EM Energy/unit volume at Temperature T

for wavelengths between λ and $\lambda + d\lambda$ is $u(\lambda,T)\, d\lambda$

$u(\lambda,T)d\lambda = \langle E(\lambda)\rangle\, n(\lambda)d\lambda$

Classical Physics ------> $\langle E(\lambda)\rangle = k_B T$

So get $u(\lambda,T)\, d\lambda = (8\pi/\lambda^4)\, k_B T\, d\lambda$

$R(\lambda) = c/4\ u\,(\lambda,T)$ ------> Rayleigh-Jeans Law

The Beginning of The End !

Classical Calculation

of standing waves between Wavelengths λ and λ+dλ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \bullet d\lambda \text{ ; V = Volume of box = } L^3$$

Each standing wave contributes energy E= kT to radiation in Box

Energy density $u(\lambda)$= [# of standing waves/volume]$\times$ Energy/Standing Wave

$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

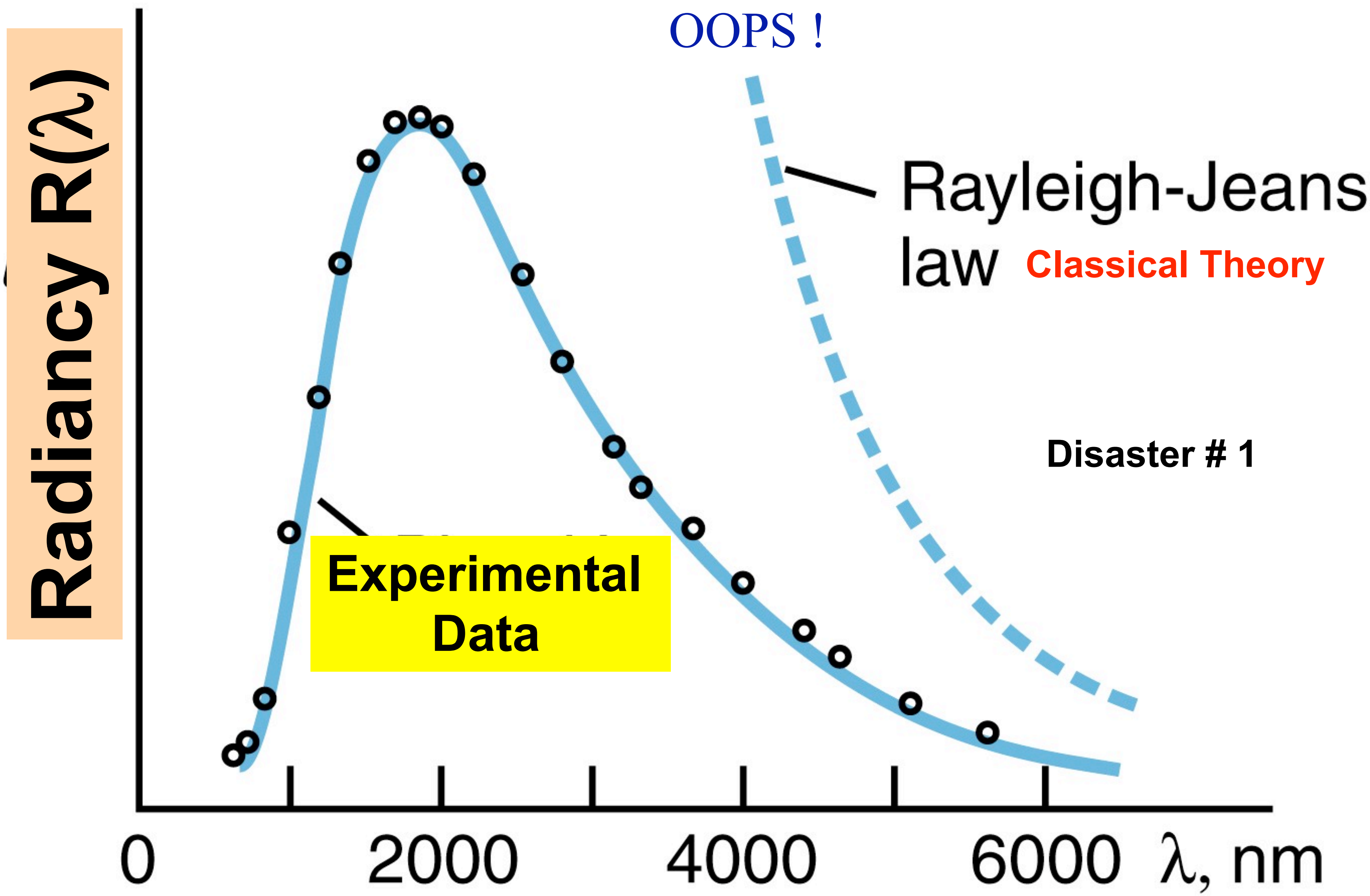
$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

Prediction : as $\lambda \rightarrow 0$ (high frequency) $\Rightarrow$ $R(\lambda) \rightarrow$ Infinity !

Oops !

Ultra Violet (Frequency) Catastrophe



Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$R(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$

$f \rightarrow \infty$

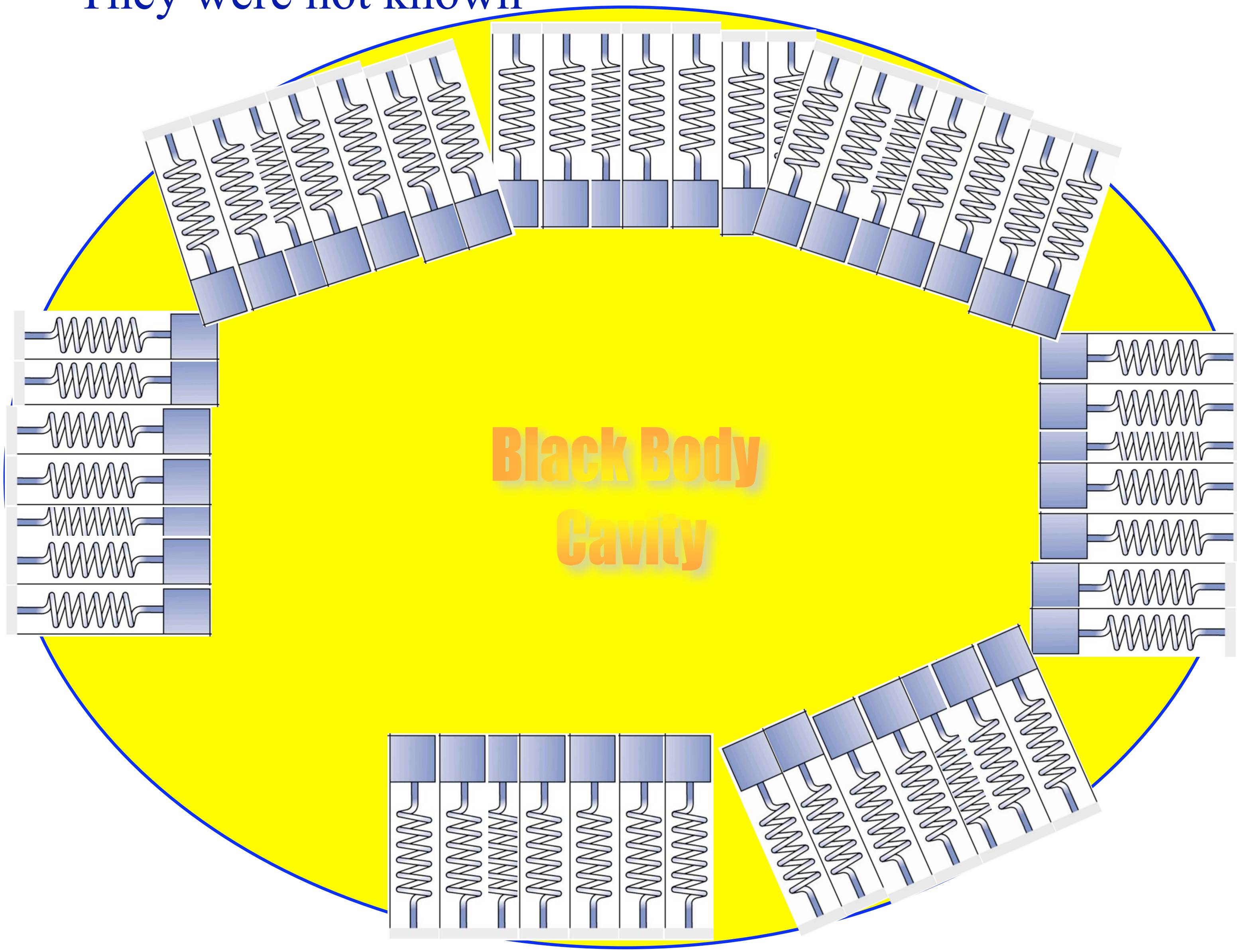
- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous and arbitarary…it is discrete …in packets of same amount
- $E = n\,hf$, with $n = 1,2\,3\ldots.\ \infty$

  h = constant he invented, a very small number he made up

Planck's "Charged Oscillators" in a Black Body Cavity

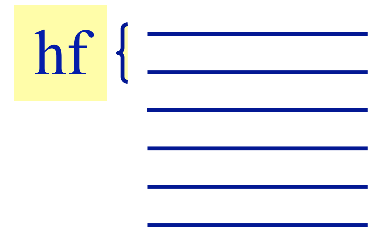
Planck did not know about electrons, Nucleus etc:

They were not known

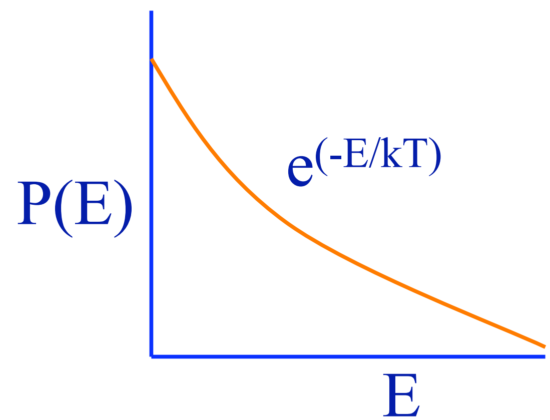


Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy
  - $E = n\,hf$ ($n = 1,2,3 \ldots 10 \ldots 1000 \ldots$)



- Probability Distribution: At an equilibrium temp T, possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy $E=hf$ = favored
  - More energy $E=hf$ = disfavored



By this statistics, large energy, high f modes of EM disfavored

Planck

Difference is in calculation of $\langle E \rangle$

Consider a mode of frequency f. Planck assumed it was emitted by a set of harmonic oscillators in walls of cavity which could only have energies $E = nhf$ (h= constant now known as Planck's constant).
Probability of oscillator having energy nhf by statistical mechanics

$$P(n) = (\exp - nhf/ k_BT) / \{ \sum_m (\exp - mhf/ k_BT) \}$$

Sum can be evaluated by writing $\exp(-hf/k_BT) = x$, so it can be written as

$$1 + x + x^2 + x^3 + x^4 + \ldots\ldots\ldots = [1 - x]^{-1}$$

so $P(n) = (\exp - nhf/ k_BT)[1 - \exp(-hf/k_BT)]$

Now $E = nhf$ = Energy of oscillator so average energy

So $\langle E(f,T)\rangle = \{\sum_n nhf \exp - nhf/ k_BT\} [1 - \exp(-hf/k_BT)]$

This can be evaluated as $\langle E(f,T)\rangle = hf / [\exp(hf/k_BT) - 1]$**

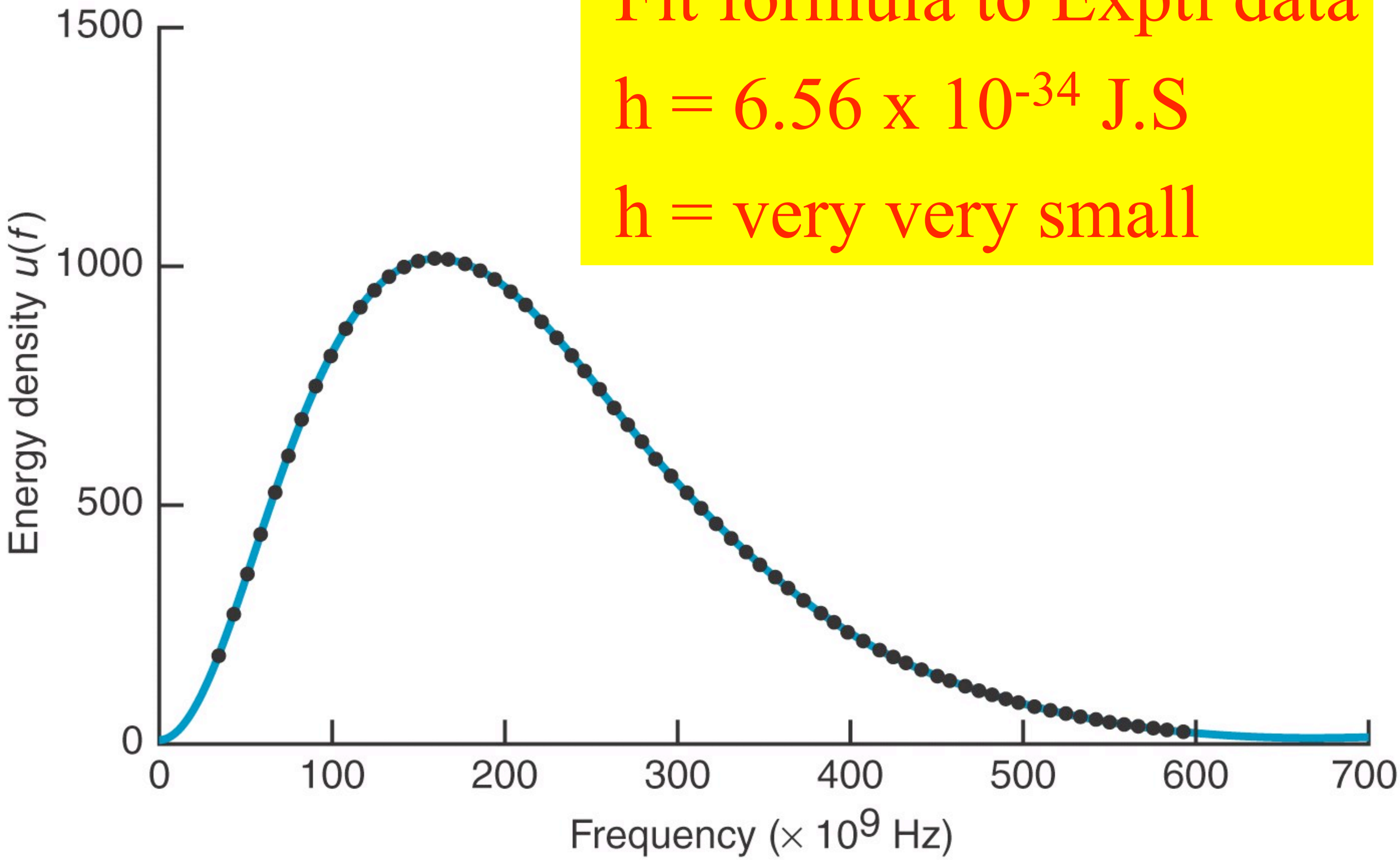
yields $u(f, T) = (8\pi f^3/c^3)/ [\exp(hf/k_BT) - 1]$---> Planck's formula

Planck's Explanation of BB Radiation

Fit formula to Exptl data

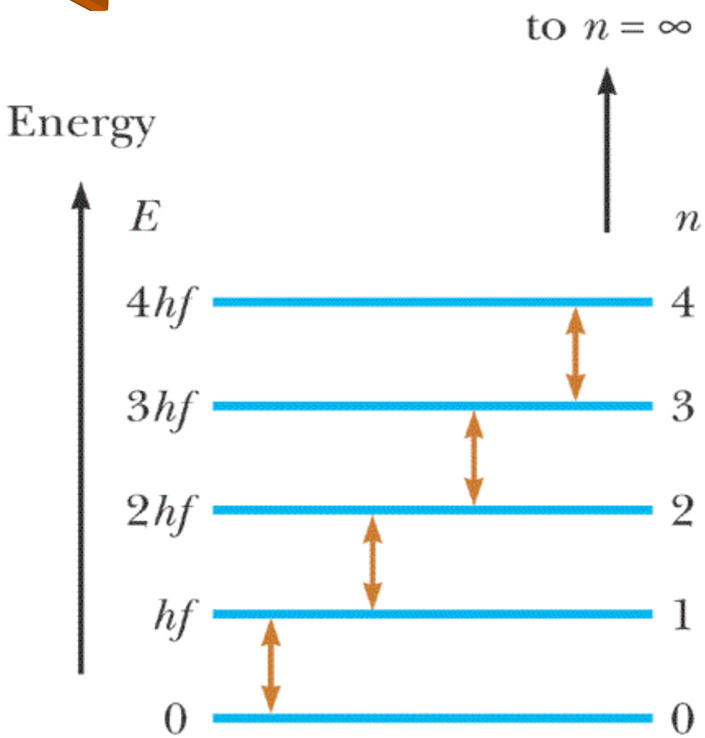
$h = 6.56 \times 10^{-34}$ J.S

h = very very small



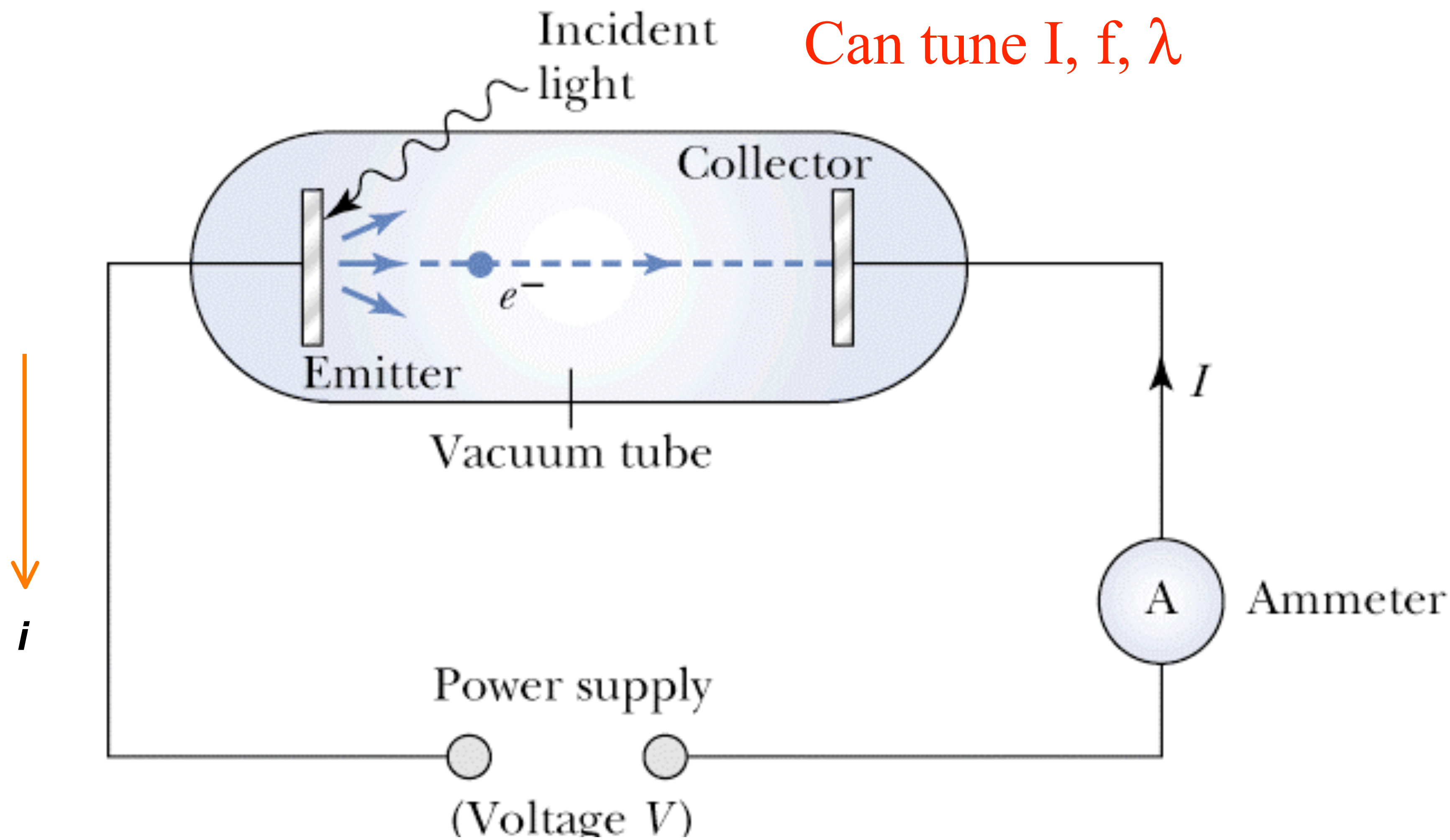
Major Consequence of Planck's Formula

Quantization of Energy !



Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency ν incident on a photo-cathode



Measure characteristics of current in the circuit as a fn of I, f, λ

Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
  - From current *i* seen in ammeter

- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - » **$K_{MAX} = eV_S$ (V_S = Stopping voltage)**
    - » **Stopping voltage → no current flows**
- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit

Observations : Current Vs Intensity of Incident Light

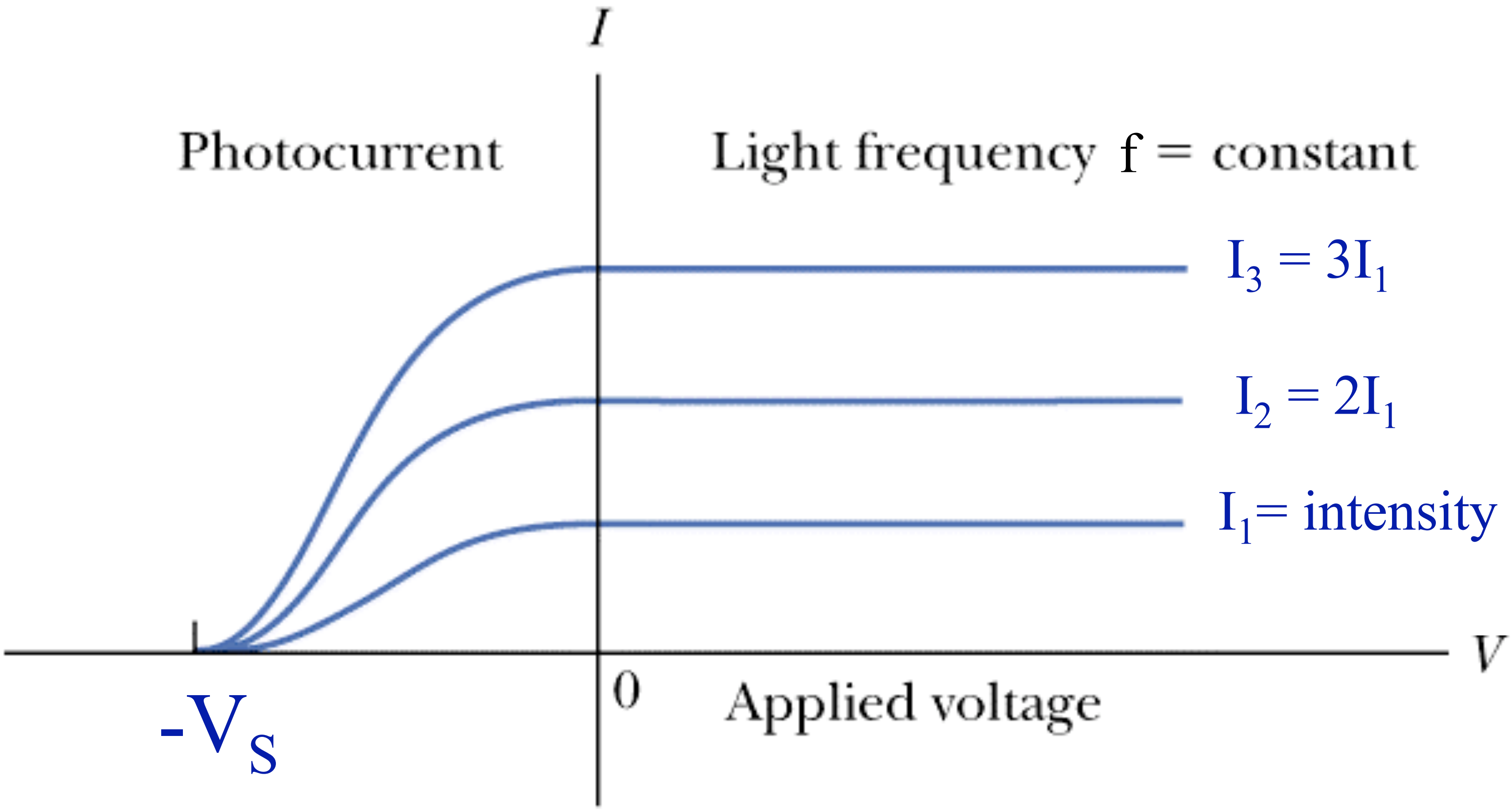
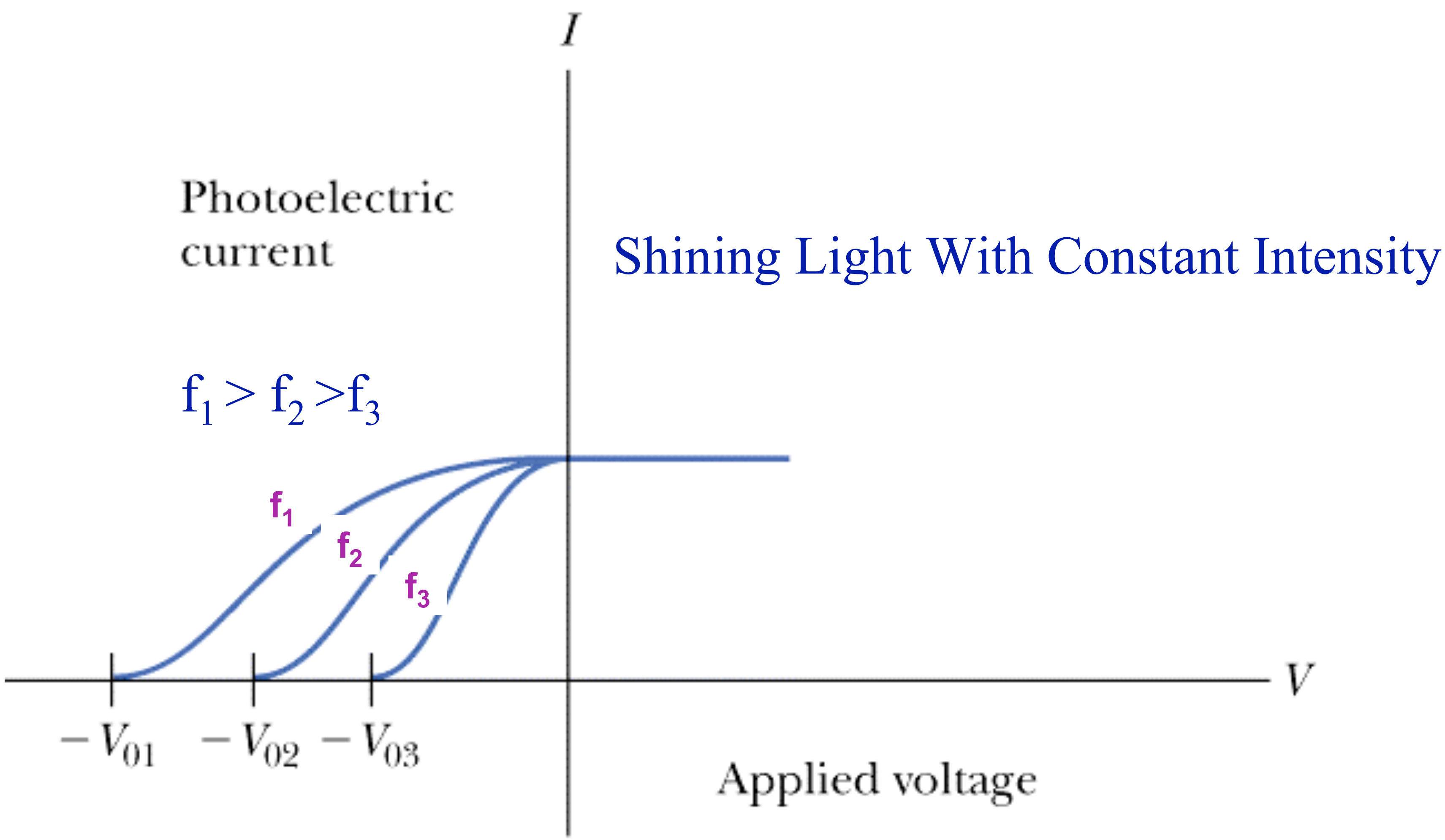


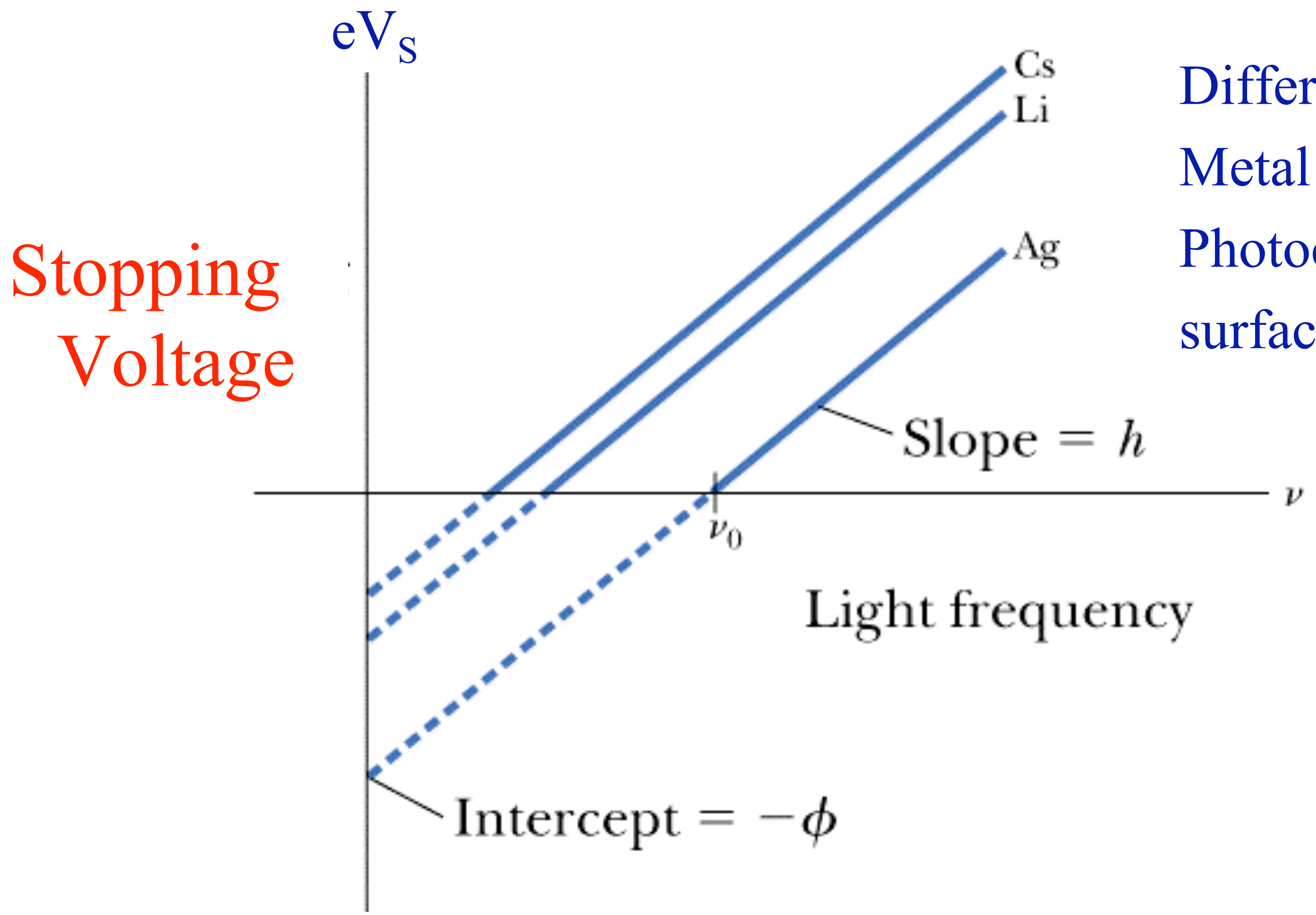
Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Quantum of Energy $E = hf = KE + \varphi \quad \Rightarrow KE = hf - \varphi$

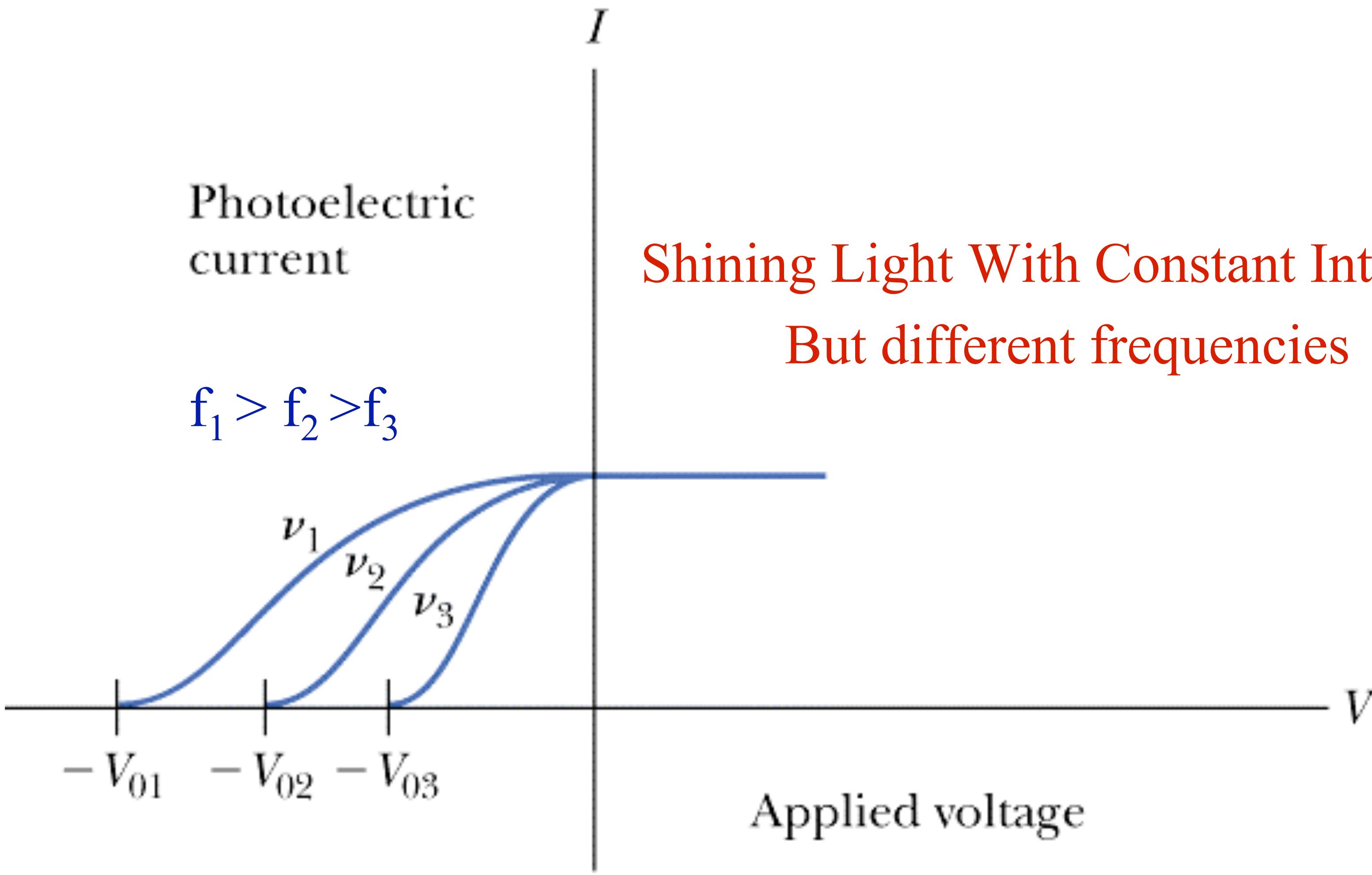


Stopping Voltage V_s Vs Incident Light Frequency



Different Metal Photocathode surfaces

Retarding Potential Vs Light Frequency



Shining Light With Constant Intensity

But different frequencies

$f_1 > f_2 > f_3$

Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow$ $\vec{E}$ field amplitude larger
  - E field and electrical force seen by the “charged subatomic oscillators” Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More energy transferred to it
    - $\Rightarrow$ Charged particle “hooked to the atom” should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is <u>intense enough</u> , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a noticeable time lag ΔT between time it is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity $I = 120W/m^2$ on Na metal
  - Binding energy = 2.3 eV= "Work Function"
  - Electron confined in Na atom, size ≅ 0.1nm ..how long before ejection ?
  - Average Power Delivered $P_{AV} = \mathbf{I} . \mathbf{A}$, $A = \pi r^2 \cong 3.1 \times 10^{-20}\ m^2$
  - If all energy absorbed then $\Delta E = P_{AV} . \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6 \times 10^{-19} J / eV)}{(120W / m^2)(3.1 \times 10^{-20} m^2)} = 0.10\ S$$

  - Classical Physics predicts Measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results & goes to DOGHOUSE !

Einstein's Explanation of Photoelectric Effect

- Energy associated with EM waves in not uniformly distributed over wave-front, rather is contained in packets of "stuff" $\Rightarrow$ PHOTON
- E= hf = hc/λ [but is it the same h as in Planck's th.?]
- Light shining on metal emitter/cathode is a stream of photons of energy which depends on frequency f
- Photons knock off electron from metal instantaneously
  - Transfer all energy to electron
  - Energy gets used up to pay for Work Function Φ (Binding Energy)
    - Rest of the energy shows up as KE of electron KE = hf- Φ
- Cutoff Frequency $hf_0 = \Phi$ (pops an electron, KE = 0)
- Larger intensity I → more photons incident
- Low frequency light f → not energetic enough to overcome work function of electron in atom