#### <u>Formulas</u>:

 $\sin 30^{\circ} = \cos 60^{\circ} = 1/2, \ \cos 30^{\circ} = \sin 60^{\circ} = \sqrt{3}/2, \ \sin 45^{\circ} = \cos 45^{\circ} = \sqrt{2}/2$ 

 $F = k \frac{q_1 q_2}{r^2}$  Coulomb's law ;  $k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$  ;  $\vec{F}_{12} = \frac{k q_1 q_2}{|\vec{r} - \vec{r}|^3} (\vec{r}_2 - \vec{r}_1)$ Electric field due to charge q at distance r:  $\vec{E} = \frac{kq}{r^2}\hat{r}$ ; Force on charge Q:  $\vec{F} = Q\vec{E}$ Electric field of\_dipole: along dipole axis / perpendicular:  $E = \frac{2kp}{r^3}$  /  $E = \frac{kp}{r^3}$  (p=qd) Energy of and torque on dipole in E-field:  $U = -\vec{p} \cdot \vec{E}$ ,  $\vec{\tau} = \vec{p} \times \vec{E}$ Linear, surface, volume charge density :  $dq = \lambda ds$ ,  $dq = \sigma dA$ ,  $dq = \rho dV$ Electric field of infinite: line of charge:  $E = \frac{2k\lambda}{r}$ ; sheet of charge:  $E = 2\pi k\sigma = \sigma/(2\varepsilon_0)$ Gauss law:  $\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ ;  $\Phi = \text{electric flux}$ ;  $k = \frac{1}{4\pi\varepsilon_0}$ ;  $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / Nm^2$  $U_B - U_A = \Delta U_{AB} = -W_{AB} = -\int_{a}^{B} \vec{F} \cdot \vec{dl} = -\int_{a}^{B} \vec{qE} \cdot \vec{dl} = q\Delta V_{AB} = q(V_B - V_A)$ V=N/C  $V = \frac{kq}{r}$ ;  $V = \int \frac{kdq}{r}$ ;  $V = \frac{kp\cos\theta}{r^2}$  (dipole);  $E_l = -\frac{\partial V}{\partial l}$ ;  $\vec{E} = -\vec{\nabla}V$ Electrostatic energy:  $U = k \frac{q_1 q_2}{r}$ ; Capacitors: Q = CV; with dielectric:  $C = \kappa C_0$ ;  $\varepsilon_0 = 8.85 \, pF / m$  $C = \frac{\varepsilon_0 A}{d}$  parallel plates ;  $C = \frac{2\pi\varepsilon_0 L}{\ln(h/a)}$  cylindrical ;  $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$  spherical Energy stored in capacitor:  $U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$ ;  $U = \int dv \, u_E$ ;  $u_E = \frac{1}{2}\varepsilon_0 E^2$ Capacitors in parallel:  $C = C_1 + C_2^{-1}$ ; in series:  $C = C_1 C_2 / (C_1 + C_2)$ Elementary charge:  $e = 1.6 \times 10^{-19} C$  $I = \frac{dq}{dt} = \int \vec{J} \cdot d\vec{A} \quad ; \quad \vec{J} = ne\vec{v}_d \quad ; \quad v_d = \frac{eE\tau}{m} \quad ; \quad \rho = \frac{m}{ne^2\tau} \quad ; \quad R = \rho \frac{\ell}{\Lambda} \quad ; \quad \vec{E} = \rho \vec{J}, \quad \vec{J} = \sigma \vec{E}$  $V = IR ; P = VI = I^2 R = V^2 / R ; P_{emf} = \varepsilon I ; R_{eq} = R_1 + R_2 \text{ (series)}; R_{eq}^{-1} = R_1^{-1} + R_2^{-1} \text{ (parallel)}$ Charging capacitor:  $Q(t) = C\varepsilon(1 - e^{-t/RC})$ ; Discharging capacitor:  $Q(t) = Q_0 e^{-t/RC}$ Force on moving charge :  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ ; force on wire :  $d\vec{F} = Id\vec{\ell} \times \vec{B}$ Circular motion:  $a = \frac{v^2}{r}$ ; radius  $r = \frac{mv}{aB}$ ; period  $T = \frac{2\pi \text{ m}}{aB}$ Magnetic dipole :  $\vec{\mu} = I\vec{A}$  ; torque :  $\vec{\tau} = \vec{\mu} \times \vec{B}$  ; energy :  $U = -\vec{\mu} \cdot \vec{B}$ Biot - Savart law :  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2}$ ;  $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$ ; Ampere's law :  $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$ Long wire:  $B = \frac{\mu_0 I}{2\pi r}$ ; loop, along axis:  $B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$ ; dipole:  $\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{x^3}$ solenoid:  $B = \mu_0 In$ ; toroid:  $B = \frac{\mu_0 NI}{2\pi r}$ ; Gauss law for magnetism:  $\oint \vec{B} \cdot d\vec{A} = 0$ 

**Faraday law**:  $\varepsilon = -\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{s}$ ;  $\Phi_B = \int \vec{B} \cdot d\vec{A}$  magnetic flux

# <u>There are 8 problems. You get 1 point for correct answer, 0 points for incorrect</u> <u>answers, 0.2 points for no answer (up to 5 non-answers). This is Test Form A</u>

#### Problem 1

$\times \times \times \times$	$\times \times \times \times$
$\times \times \times \times_{\text{perimeter p}}$	$\times$ ( $\times$ ) × perimeter p
XXX	$\times$ $\times$ $\times$ $\times$
$\times \times \times \times$	$\times \times \times \times$

The square loop on the left figure has perimeter p and is in a time-dependent magnetic field B(t). At time  $t_0$  it dissipates energy at a rate of 100W. The circular loop on the right figure was made from an identical square loop of the same metal by deforming it to a circular shape, so it has the same perimeter p, and it is in an identical time-dependent magnetic field B(t). At the same time  $t_0$  the circular loop dissipates (a) 100W ; (b) 263W ; (c) 162W ; (d) 121W ; (e) 314W

### Problem 2

Suppose you have a square loop of side length a=2m, that has resistance  $50\Omega$ , and want to get a uniform magnetic field B=10T going through it perpendicularly. How long will it take you to increase the magnetic field from 0 to 10T if you want the total energy dissipated in the process to be less than 0.5J? You may assume that the magnetic field is increased at a constant rate. The time you need is at least: (a) 16s ; (b) 32s ; (c) 64s ; (d) 8s ; (e) 4s

#### Problem 3



In the figure, loop 1 and loop 2 have side lengths a and 2a oriented as shown. Their resistance is  $R_1=100\Omega$ ,  $R_2=200\Omega$ . They are being pulled out of a region of uniform magnetic field at the same speed v by applied forces  $F_1$  and  $F_2$ . If  $F_1=10N$ ,  $F_2=$  (a) 5N ; (b) 10N ; (c) 20N ; (d) 2.5N ; (e) 40N

#### Problem 4



The current in the outer loop  $I_1(t)$  is increasing at a constant rate, it is 2A at t=1s and 10A at t=5s. The current induced in the inner loop is 1µA at t=1s; at t=2s it is (a) 2µA ; (b) 4µA ; (c) 0 ; (d) 0.5µA ; (e) 1µA

## Problem 5



The long wire in the vertical direction carries current  $I(t)=I_0t/t_0$ , with  $I_0=20A$ ,  $t_0=1s$ . The circular loop of wire shown has its center at the long wire and is on the plane perpendicular to the long wire, has radius a=1m and resistance 1 $\Omega$ . The current induced in the circular loop of wire at time t=1s is, in  $\mu A$  (10<sup>-6</sup>A): (a)  $\pi$ ; (b)  $2\pi$ ; (c)  $4\pi$ ; (d) 0; (e)  $0.5\pi$ 

## Problem 6



The solenoid shown is oriented perpendicular to the paper. The time-dependent magnetic field inside the solenoid points into the paper and is given by

$$B(t) = B_0 e^{-t/\tau}$$

with  $\tau$ =2s and B<sub>0</sub>=10T. The radius of the solenoid is a=0.3m. Point P is at distance 2a=0.6m to the right from the center of the solenoid in the horizontal direction along a line that goes through the center of the solenoid.

The induced electric field at point P at time t=1s points (a) up (b) down (c) right (d) left (c) into the per

(a) up ; (b) down ; (c) right ; (d) left ; (e) into the paper

## Problem 7

For the situation of problem 6, the magnitude of the induced electric field at point P at time t=1s is

(a) 0.13V/m; (b) 0.23V/m; (c) 0.33V/m; (d) 0.43V/m; (e) 0.53V/m

#### Problem 8



For the triangular conducting loop shown in the figure that is being pushed into a region of uniform magnetic field at constant speed v, the power dissipated when half the horizontal side is in the region of the magnetic field is 100W. Right before the entire loop is in the region of uniform magnetic field the power dissipated is (a) 200W ; (b) 400W ; (c) 100W ; (d) 800W ; (e) 50W