## Formulas:

$\sin 30^{\circ}=\cos 60^{\circ}=1 / 2, \quad \cos 30^{\circ}=\sin 60^{\circ}=\sqrt{3} / 2, \sin 45^{\circ}=\cos 45^{\circ}=\sqrt{2} / 2$
$F=k \frac{q_{1} q_{2}}{r^{2}}$ Coulomb's law $; k=9 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2} \quad ; \quad \vec{F}_{12}=\frac{k q_{1} q_{2}}{\left|\vec{r}_{2}-\vec{r}_{1}\right|^{3}}\left(\vec{r}_{2}-\vec{r}_{1}\right)$
Electric field due to charge q at distance $\mathrm{r}: \quad \vec{E}=\frac{k q}{r^{2}} \hat{r}$; Force on charge $\mathrm{Q}: \vec{F}=Q \vec{E}$
Electric field of dipole: along dipole axis / perpendicular: $E=\frac{2 k p}{x^{3}} / \quad E=\frac{k p}{y^{3}}(\mathrm{p}=\mathrm{qd})$
Energy of and torque on dipole in E-field: $U=-\vec{p} \cdot \vec{E} \quad, \vec{\tau}=\vec{p} \times \vec{E}$
Linear, surface, volume charge density : $d q=\lambda d s, \quad d q=\sigma d A \quad, d q=\rho d V$
Electric field of infinite : line of charge : $E=\frac{2 k \lambda}{r} ; \quad$ sheet of charge : $E=2 \pi k \sigma=\sigma /\left(2 \varepsilon_{0}\right)$
Gauss law: $\quad \Phi=\oint \vec{E} \cdot d \vec{A}=\frac{q_{\text {enc }}}{\varepsilon_{0}} \quad ; \quad \Phi=$ electric flux $; k=\frac{1}{4 \pi \varepsilon_{0}} ; \varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}$ $U_{B}-U_{A}=\Delta U_{A B}=-W_{A B}=-\int_{A}^{B} \overrightarrow{\mathrm{~F}} \cdot \overrightarrow{d l}=-\int_{A}^{B} q \vec{E} \cdot \overrightarrow{d l} \quad=q \Delta V_{A B}=q\left(V_{B}-V_{A}\right) \quad \mathrm{V}=\mathrm{N} / \mathrm{C}$
$V=\frac{k q}{r} ; \mathrm{V}=\int \frac{k d q}{r} ; \quad V=\frac{k p \cos \theta}{r^{2}}$ (dipole) $; \quad E_{l}=-\frac{\partial V}{\partial l} \quad ; \quad \overrightarrow{\mathrm{E}}=-\vec{\nabla} \mathrm{V}$
Electrostatic energy: $U=k \frac{q_{1} q_{2}}{r}$; Capacitors : $Q=C V$; with dielectric : $\mathrm{C}=\kappa \mathrm{C}_{0} ; \varepsilon_{0}=8.85 \mathrm{pF} / \mathrm{m}$ $C=\frac{\varepsilon_{0} A}{d}$ parallel plates ; $C=\frac{2 \pi \varepsilon_{0} L}{\ln (b / a)}$ cylindrical ; $C=4 \pi \varepsilon_{0} \frac{a b}{b-a}$ spherical
Energy stored in capacitor : $U=\frac{Q^{2}}{2 C}=\frac{1}{2} Q V=\frac{1}{2} C V^{2} ; \quad U=\int d v u_{E} ; u_{E}=\frac{1}{2} \varepsilon_{0} E^{2}$
Capacitors in parallel: $C=C_{1}+C_{2} \quad$; in series: $\quad C=C_{1} C_{2} /\left(C_{1}+C_{2}\right)$
Elementary charge: $e=1.6 \times 10^{-19} \mathrm{C}$
$I=\frac{d q}{d t}=\int \vec{J} \cdot d \vec{A} ; \vec{J}=n e \vec{v}_{d} ; v_{d}=\frac{e E \tau}{m} ; \rho=\frac{m}{n e^{2} \tau} ; R=\rho \frac{\ell}{A} ; \vec{E}=\rho \vec{J}, \vec{J}=\sigma \vec{E}$
$V=I R ; P=V I=I^{2} R=V^{2} / R ; P_{e m f}=\varepsilon I ; R_{e q}=R_{1}+R_{2}$ (series) ; $R_{e q}^{-1}=R_{1}^{-1}+R_{2}^{-1}$ (parallel)
Charging capacitor: $Q(t)=C \varepsilon\left(1-e^{-t / R C}\right) \quad ; \quad$ Discharging capacitor: $Q(t)=Q_{0} e^{-t / R C}$
Force on moving charge : $\vec{F}=q(\vec{E}+\vec{v} \times \vec{B}) ;$ force on wire : $d \vec{F}=I \overrightarrow{d \ell} \times \vec{B}$
Circular motion : $a=\frac{v^{2}}{r} ;$ radius $r=\frac{m v}{q B} ;$ period $T=\frac{2 \pi \mathrm{~m}}{\mathrm{qB}}$
Magnetic dipole: $\vec{\mu}=\mathrm{I} \overrightarrow{\mathrm{A}}$; torque : $\vec{\tau}=\vec{\mu} \times \vec{B}$; energy: $U=-\vec{\mu} \cdot \vec{B}$
Biot-Savart law : $d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{I d \vec{\ell} \times \hat{r}}{r^{2}} ; \mu_{0}=4 \pi \times 10^{-7} \frac{N}{A^{2}} ;$ Ampere's law : $\oint \vec{B} \cdot d \vec{\ell}=\mu_{0} I_{\text {enc }}$ Long wire : $B=\frac{\mu_{0} I}{2 \pi r} \quad ; \quad$ loop, along axis : $B=\frac{\mu_{0} I a^{2}}{2\left(a^{2}+z^{2}\right)^{3 / 2}} \quad ; \quad$ dipole : $\vec{B}=\frac{\mu_{0}}{2 \pi} \frac{\vec{\mu}}{x^{3}}$ solenoid : $B=\mu_{0} I n \quad ;$ toroid: $\quad B=\frac{\mu_{0} N I}{2 \pi r} ; \quad$ Gauss law for magnetism: $\quad \oint \vec{B} \cdot d \vec{A}=0$

Faraday law : $\quad \varepsilon=-\frac{d \Phi_{B}}{d t}=\oint \vec{E} \cdot d \vec{s} ; \quad \Phi_{B}=\int \vec{B} \cdot d \vec{A} \quad$ magnetic flux

There are 8 problems. You get 1 point for correct answer, 0 points for incorrect answers, 0.2 points for no answer (up to 5 non-answers). This is Test Form A

## Problem 1



The square loop on the left figure has perimeter p and is in a time-dependent magnetic field $B(t)$. At time $t_{0}$ it dissipates energy at a rate of 100 W . The circular loop on the right figure was made from an identical square loop of the same metal by deforming it to a circular shape, so it has the same perimeter $p$, and it is in an identical time-dependent magnetic field $B(t)$. At the same time $t_{0}$ the circular loop dissipates
(a) 100 W
; (b) 263 W
; (c) 162 W
; (d) 121 W ; (e) 314 W

## Problem 2

Suppose you have a square loop of side length $\mathrm{a}=2 \mathrm{~m}$, that has resistance $50 \Omega$, and want to get a uniform magnetic field $\mathrm{B}=10 \mathrm{~T}$ going through it perpendicularly. How long will it take you to increase the magnetic field from 0 to 10T if you want the total energy dissipated in the process to be less than 0.5 J ? You may assume that the magnetic field is increased at a constant rate. The time you need is at least:
(a) 16 s ; (b) 32 s ; (c) 64 s ; (d) 8 s ; (e) 4 s

Problem 3


In the figure, loop 1 and loop 2 have side lengths a and 2 a oriented as shown. Their resistance is $R_{1}=100 \Omega, R_{2}=200 \Omega$. They are being pulled out of a region of uniform magnetic field at the same speed $v$ by applied forces $F_{1}$ and $F_{2}$. If $F_{1}=10 \mathrm{~N}, F_{2}=$
(a) 5 N ; (b) 10 N ; (c) 20 N ; (d) 2.5 N ; (e) 40 N

## Problem 4



The current in the outer loop $I_{1}(t)$ is increasing at a constant rate, it is 2 A at $\mathrm{t}=1 \mathrm{~s}$ and 10 A at $t=5 \mathrm{~s}$. The current induced in the inner loop is $1 \mu \mathrm{~A}$ at $\mathrm{t}=1 \mathrm{~s}$; at $\mathrm{t}=2 \mathrm{~s}$ it is
(a) $2 \mu \mathrm{~A}$; (b) $4 \mu \mathrm{~A}$; (c) 0 ; (d) $0.5 \mu \mathrm{~A}$; (e) $1 \mu \mathrm{~A}$

## Problem 5



The long wire in the vertical direction carries current $I(t)=I_{0} t / t_{0}$, with $I_{0}=20 \mathrm{~A}, \mathrm{t}_{0}=1 \mathrm{~s}$. The circular loop of wire shown has its center at the long wire and is on the plane perpendicular to the long wire, has radius $\mathrm{a}=1 \mathrm{~m}$ and resistance $1 \Omega$. The current induced in the circular loop of wire at time $\mathrm{t}=1 \mathrm{~s}$ is, in $\mu \mathrm{A}\left(10^{-6} \mathrm{~A}\right)$ :
(a) $\pi$; (b) $2 \pi$; (c) $4 \pi$; (d) 0 ; (e) $0.5 \pi$

Problem 6

## solenoid



The solenoid shown is oriented perpendicular to the paper. The time-dependent magnetic field inside the solenoid points into the paper and is given by

$$
B(t)=B_{0} e^{-t / \tau}
$$

with $\tau=2 \mathrm{~s}$ and $\mathrm{B}_{0}=10 \mathrm{~T}$. The radius of the solenoid is $\mathrm{a}=0.3 \mathrm{~m}$. Point P is at distance $2 \mathrm{a}=0.6 \mathrm{~m}$ to the right from the center of the solenoid in the horizontal direction along a line that goes through the center of the solenoid.
The induced electric field at point $P$ at time $t=1 \mathrm{~s}$ points
(a) up ; (b) down ; (c) right ; (d) left ; (e) into the paper

## Problem 7

For the situation of problem 6, the magnitude of the induced electric field at point P at time $t=1 \mathrm{~s}$ is
(a) $0.13 \mathrm{~V} / \mathrm{m}$; (b) $0.23 \mathrm{~V} / \mathrm{m}$; (c) $0.33 \mathrm{~V} / \mathrm{m}$; (d) $0.43 \mathrm{~V} / \mathrm{m}$; (e) $0.53 \mathrm{~V} / \mathrm{m}$

## Problem 8



For the triangular conducting loop shown in the figure that is being pushed into a region of uniform magnetic field at constant speed v , the power dissipated when half the horizontal side is in the region of the magnetic field is 100 W . Right before the entire loop is in the region of uniform magnetic field the power dissipated is
(a) 200 W
; (b) 400W ;
(c) 100 W ; (d) 800 W
; (e) 50 W

