## Formulas:

$\sin 30^{\circ}=\cos 60^{\circ}=1 / 2, \quad \cos 30^{\circ}=\sin 60^{\circ}=\sqrt{3} / 2, \quad \sin 45^{\circ}=\cos 45^{\circ}=\sqrt{2} / 2$
$F=k \frac{q_{1} q_{2}}{r^{2}}$ Coulomb's law $; k=9 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2} \quad ; \quad \vec{F}_{12}=\frac{k q_{1} q_{2}}{\left|\vec{r}_{2}-\vec{r}_{1}\right|^{3}}\left(\vec{r}_{2}-\vec{r}_{1}\right)$
Electric field due to charge q at distance $\mathrm{r}: \quad \vec{E}=\frac{k q}{r^{2}} \hat{r}$; Force on charge $\mathrm{Q}: \vec{F}=Q \vec{E}$
Electric field of_dipole: along dipole axis / perpendicular: $E=\frac{2 k p}{x^{3}} / \quad E=\frac{k p}{y^{3}}(\mathrm{p}=\mathrm{qd})$
Energy of and torque on dipole in E-field: $U=-\vec{p} \cdot \vec{E} \quad, \vec{\tau}=\vec{p} \times \vec{E}$
Linear, surface, volume charge density : $d q=\lambda d s, \quad d q=\sigma d A \quad, d q=\rho d V$
Electric field of infinite: line of charge : $E=\frac{2 k \lambda}{r} ;$ sheet of charge : $E=2 \pi k \sigma=\sigma /\left(2 \varepsilon_{0}\right)$
Gauss law: $\quad \Phi=\oint \vec{E} \cdot d \vec{A}=\frac{q_{\text {enc }}}{\varepsilon_{0}} \quad ; \quad \Phi=$ electric flux $; k=\frac{1}{4 \pi \varepsilon_{0}} ; \varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}$
$U_{B}-U_{A}=\Delta U_{A B}=-W_{A B}=-\int_{A}^{B} \overrightarrow{\mathrm{~F}} \cdot \overrightarrow{d l}=-\int_{A}^{B} \overrightarrow{q E} \cdot \overrightarrow{d l} \quad=q \Delta V_{A B}=q\left(V_{B}-V_{A}\right) \quad \mathrm{V}=\mathrm{N} / \mathrm{C}$
$V=\frac{k q}{r} ; \mathrm{V}=\int \frac{k d q}{r} ; V=\frac{k p \cos \theta}{r^{2}}$ (dipole) ; $\quad E_{l}=-\frac{\partial V}{\partial l} \quad ; \quad \overrightarrow{\mathrm{E}}=-\overrightarrow{\mathrm{V}} \mathrm{V}$
Electrostatic energy: $U=k \frac{q_{1} q_{2}}{r}$; Capacitors : $Q=C V$; with dielectric: $\mathrm{C}=\kappa \mathrm{C}_{0} ; \varepsilon_{0}=8.85 \mathrm{pF} / \mathrm{m}$ $C=\frac{\varepsilon_{0} A}{d}$ parallel plates ; $C=\frac{2 \pi \varepsilon_{0} L}{\ln (b / a)}$ cylindrical ; $C=4 \pi \varepsilon_{0} \frac{a b}{b-a}$ spherical
Energy stored in capacitor : $U=\frac{Q^{2}}{2 C}=\frac{1}{2} Q V=\frac{1}{2} C V^{2} ; \quad U=\int d v u_{E} ; u_{E}=\frac{1}{2} \varepsilon_{0} E^{2}$
Capacitors in parallel: $C=C_{1}+C_{2} \quad$; in series: $\quad C=C_{1} C_{2} /\left(C_{1}+C_{2}\right)$
Elementary charge: $e=1.6 \times 10^{-19} \mathrm{C}$
$I=\frac{d q}{d t}=\int \vec{J} \cdot d \vec{A} ; \vec{J}=n e \vec{v}_{d} ; v_{d}=\frac{e E \tau}{m} ; \rho=\frac{m}{n e^{2} \tau} ; R=\rho \frac{\ell}{A} ; \vec{E}=\rho \vec{J}, \vec{J}=\sigma \vec{E}$
$V=I R ; \quad P=V I=I^{2} R=V^{2} / R ; P_{e m f}=\varepsilon I ; R_{e q}=R_{1}+R_{2}$ (series) $; R_{e q}^{-1}=R_{1}^{-1}+R_{2}^{-1}$ (parallel)
Charging capacitor : $Q(t)=C \varepsilon\left(1-e^{-t / R C}\right) \quad ; \quad$ Discharging capacitor : $Q(t)=Q_{0} e^{-t / R C}$
Force on moving charge : $\vec{F}=q(\vec{E}+\vec{v} \times \vec{B}) \quad ;$ force on wire : $d \vec{F}=I \overrightarrow{d \ell} \times \vec{B}$
Circular motion : $a=\frac{v^{2}}{r} ;$ radius $r=\frac{m v}{q B} ;$ period $T=\frac{2 \pi \mathrm{~m}}{\mathrm{qB}}$
Magnetic dipole : $\vec{\mu}=\mathrm{I} \overrightarrow{\mathrm{A}}$; torque : $\vec{\tau}=\vec{\mu} \times \vec{B}$; energy: $U=-\vec{\mu} \cdot \vec{B}$
Biot-Savart law : $d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{I \overrightarrow{\ell \ell} \times \hat{r}}{r^{2}} ; \mu_{0}=4 \pi \times 10^{-7} \frac{N}{A^{2}}$; Ampere's law : $\oint \vec{B} \cdot \overrightarrow{\ell \ell}=\mu_{0} I_{\text {enc }}$ Long wire : $B=\frac{\mu_{0} I}{2 \pi r} \quad ; \quad$ loop, along axis : $B=\frac{\mu_{0} I a^{2}}{2\left(a^{2}+z^{2}\right)^{3 / 2}} \quad ; \quad$ dipole : $\vec{B}=\frac{\mu_{0}}{2 \pi} \frac{\vec{\mu}}{x^{3}}$
solenoid : $B=\mu_{0}$ In ; toroid: $B=\frac{\mu_{0} N I}{2 \pi r} ; \quad$ Gauss law for magnetism: $\quad \oint \vec{B} \cdot d \vec{A}=0$

There are 8 problems. You get 1 point for correct answer, 0 points for incorrect answers, 0.2 points for no answer (up to 5 non-answers). This is Test Form A

## Problem 1



A particle of mass 3 kg and charge 2C starts at the origin $(\mathrm{x}, \mathrm{y}, \mathrm{z})=(0,0,0)$ with velocity v in the yz plane at $45^{0}$ with respect to the $y$ axis as shown in the figure. There is a uniform magnetic field $B$ pointing along the $y$ direction everywhere. After 3 seconds, the $x-$ coordinate of the particle is -12 m . After another 3 seconds the particle hits the $y$-axis again for the first time after it left the origin, i.e. is at the point $\left(0, y_{0}, 0\right)$. The value of $y_{0}$ expressed in meters is
(a) $6 \pi$; (b) $18 \pi \quad$; (c) $24 \pi \quad$; (d) $12 \pi \quad$; (e) $2 \pi$

## Problem 2



The loop in the figure has two sides of length $\mathrm{a}=0.7 \mathrm{~m}$ that join at a 90 degree angle and is in a uniform magnetic field $\mathrm{B}=2 \mathrm{~T}$ oriented as shown in the figure. A current $\mathrm{I}=1.4 \mathrm{~A}$ circulates through the loop in the direction shown. The magnitude of the force on the long side of the loop is
(a) 2 N ; (b) 1.4 N ; (c) 2.8 N ; (d) 2.4 N ; (e) 1.7 N

## Problem 3

For the situation in Problem 2, the magnetic field generated by the current loop at a distance $\mathrm{d}=10 \mathrm{~m}$ in direction perpendicular to the paper has magnitude approximately (a) $0.2 \times 10^{-10} \mathrm{~T}$; (b) $0.4 \times 10^{-10} \mathrm{~T}$; (c) $0.5 \times 10^{-10} \mathrm{~T}$; (d) $0.6 \times 10^{-10} \mathrm{~T}$; (e) $0.7 \times 10^{-10} \mathrm{~T}$

## Problem 4



4I• $\quad$ XI
The four long wires shown are oriented perpendicular to the paper in a square arrangement and carry currents of magnitude shown in or out of the paper as shown. The magnetic field at point P located at the center of the square points:
(a) to the right ; (b) to the left ; (c) up ; (d) down ; (e) at $45^{\circ}$ angle to horizontal

## Problem 5



The loop shown is formed of two concentric quarter circles of radii a and 2 a , and straight segments of length a, and carries current I in the direction shown. With the convention that the magnetic field at point P at the center of the circles is positive if it points out of the paper and negative if it points into the paper, it is given by $\mathrm{B}=$
(a) $\mu_{0} \mathrm{I} / 8 \mathrm{a}$; (b) $-\mu_{0} \mathrm{I} / 8 \mathrm{a}$; (c) $\mu_{0} \mathrm{I} / 16 \mathrm{a}$; (d) $-\mu_{0} \mathrm{I} / 16 \mathrm{a}$; (e) $-\mu_{0} \mathrm{I} / 32 \mathrm{a}$

## Problem 6

Consider the loop of problem 5, and assume a long straight wire goes through the point P in direction perpendicular to the paper, carrying current $I_{1}$ into the paper. The total force exerted on the segment of the loop in the vertical direction (of length a, directly above P ) by the magnetic field of the long straight wire, is
(a) $0.1 \mu_{0} \mathrm{I}_{1}$; (b) $0.2 \mu_{0} \mathrm{I} \mathrm{I}_{1}$; (c) $0.3 \mu_{0} \mathrm{I} \mathrm{I}_{1}$; (d) $0.4 \mu_{0} \mathrm{I} \mathrm{I}_{1}$; (e) $0.5 \mu_{0} \mathrm{I} \mathrm{I}_{1}$

## Problem 7



The infinite flat sheet of thickness 4 cm shown carries a uniform current density in direction perpendicular to the paper (that you're holding). At point $\mathrm{P}_{1}$ inside the sheet at distance 1 cm from the bottom surface the magnetic field has magnitude 2 G ( $\mathrm{G}=$ gauss) and points to the right. At point $P_{2}$ which is outside the sheet at distance 2 cm above the top surface of the sheet the magnetic field has magnitude? and points? (right or left)
(a) 1G, right ; (b) 2G, left ; (c) 4G, left ; (d) 8G, right ; (e) 2G, right

## Problem 8



The figure shows 8 long wires pointing perpendicular to the paper each carrying current I in direction out of the paper, distributed over a 2 m region, very close and above the x axis. The integral along the x -axis shown $\int_{0}^{22 m} B d x=10^{-5} \mathrm{~T} m$. Each wire carries a current I of approximately
(a) 1 A
; (b) 2 A ; (c) 3 A ; (d
(d) 4 A ; (e) 5 A

