

# PHYS 2B Quiz 3 Solutions

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## 1 Problem 1

At  $P_1$ , the contribution to the potential due to the charges above and below cancel out. The net contribution is due solely to the charge on the left, at distance  $d_1 = ((a/2)^2 + a^2)^{1/2}$  away. This gives a potential of  $kq/d_1$ .

At  $P_2$ , all three charges are equidistant with separation  $d_2 = ((a/2)^2 + (a/2)^2)^{1/2}$ . The contributions from the negative and EITHER positive charge cancel. The contribution due to one positive charge is  $kq/d_2$ .

$$\frac{V_2}{V_1} = \frac{d_1}{d_2} = 1.58 \quad (1)$$

## 2 Problem 2

Sum over all possible pair interactions, while being careful not to double count. Each interaction, by Coloumbs law, gives a contribution of  $kq_1q_2/r$  to the electrostatic energy. There are 3 pairs in total. The two pairs that each separated by distance  $a$  cancel out because the products of the charges have opposite signs. The only pair that remains is  $k(q)(-q)/(2^{1/2}a)$ . This gives  $\alpha = -0.71$ .

## 3 Problem 3

$$U = \frac{CV^2}{2} = \frac{QV}{2} \quad (2)$$

$$Q = \frac{2U}{V} = \frac{2(2)}{3} = 1.33C \quad (3)$$

In the second equality of the first line, we used the definition of capacitance:  $C = Q/V$ . The second line is just rearranging.

## 4 Problem 4

Before the potential difference was applied across the two capacitors, all conductors were charge neutral. When the potential difference is maintained, we know there is a charge  $-Q_1$  on the bottom plate of the top capacitor. By charge conservation, there must be charge  $Q_1$  on the top plate of the bottom capacitor. In other words, the charge stored on capacitor 2 equals the charge stored on capacitor 1. The charge stored on the top capacitor is  $Q_1 = C_1 V_1 = 3\mu C = Q_2 = C_2 V_2 = (4\mu F)V_2$ . Hence  $V_2 = 0.75V$ .

## 5 Problem 5

Look first at the right arm with 3 capacitors. The two capacitors in parallel have effective capacitance  $2C$ . The whole right arm has effective capacitance  $2C/3$ , using the formula for capacitors in series. Combine the left and right arms using the the formula for capacitors in parallel. The combined capacitance is  $C + 2C/3 = 5C/3$ .

## 6 Problem 6

The dielectric capacitor has now capacitance  $2C$ , because capacitance is proportional to the dielectric constant. In the process that the dielectric slab is inserted, the total energy stored in the capacitor need not be conserved, because work can be done by an external agent during the insertion. However, the total charge stored  $2Q$  ( $Q$  for each capacitor) is conserved, because charge has no where to go in an isolated circuit; there has been a charge redistribution. Since the capacitors are in parallel, there is the same potential difference  $V$  across both of them. Call  $x$  the charge stored on the dielectric capacitor. By charge conservation, the charge stored on the left capacitor is  $2Q - x$ . Equality of potential difference demands:

$$V = \frac{2Q - x}{C} = \frac{x}{2C} \quad (4)$$

Solving for  $x$ , we obtain  $x = 4Q/3$ .

## 7 Problem 7

Initially, the sum of the capacitor energies is

$$U = \frac{Q^2}{2C} + \frac{Q^2}{2C} = \frac{Q^2}{C} \quad (5)$$

Now, the sum is

$$U = \frac{(2Q/3)^2}{2C} + \frac{(4Q/3)^2}{2(2C)} = \frac{2Q^2}{3C} = 0.67U \quad (6)$$

There is a loss in electrostatic energy because as the dielectric is inserted, the fringe electric fields polarize the dielectric and there is a net attractive force towards the center of the capacitor (see the textbook for a nice picture). Hence, the capacitor does positive work in pulling in the dielectric slab; it loses its electrostatic energy.

## 8 Problem 8

The gain in kinetic energy from R to 2R is the loss in electrostatic potential energy:

$$\frac{1}{2}mv_1^2 = kQq\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{kQq}{2R} \quad (7)$$

Similarly for R to 4R,

$$\frac{1}{2}mv_2^2 = kQq\left(\frac{1}{R} - \frac{1}{4R}\right) = \frac{3kQq}{4R} \quad (8)$$

Hence,

$$\frac{v_2^2}{v_1^2} = \frac{\frac{3}{4}}{\frac{1}{2}} \quad (9)$$

Taking the square root, we obtain  $v_2 = 1.22v_1$ .