

**Formulas:**

$$\sin 30^\circ = \cos 60^\circ = 1/2, \quad \cos 30^\circ = \sin 60^\circ = \sqrt{3}/2, \quad \sin 45^\circ = \cos 45^\circ = \sqrt{2}/2$$

$$F = k \frac{q_1 q_2}{r^2} \quad \text{Coulomb's law} ; k = 9 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2 ; \vec{F}_{12} = \frac{k q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

$$\text{Electric field due to charge } q \text{ at distance } r: \quad \vec{E} = \frac{kq}{r^2} \hat{r} ; \text{ Force on charge } Q: \quad \vec{F} = Q\vec{E}$$

$$\text{Electric field of dipole, along dipole axis:} \quad E = \frac{2kp}{x^3} \quad (p=qd)$$

$$\text{Electric field of dipole, along direction perpendicular to dipole axis:} \quad E = \frac{kp}{y^3}$$

$$\text{Energy of and torque on dipole in E-field:} \quad U = -\vec{p} \cdot \vec{E} , \quad \vec{\tau} = \vec{p} \times \vec{E}$$

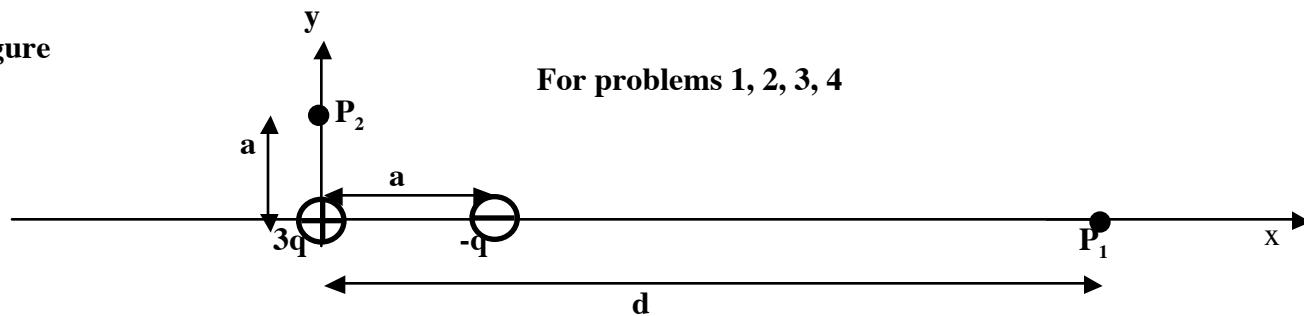
$$\text{Linear, surface, volume charge density:} \quad dq = \lambda ds , \quad dq = \sigma dA , \quad dq = \rho dV$$

$$\text{Electric field of infinite: line of charge:} \quad E = \frac{2k\lambda}{r}; \quad \text{sheet of charge:} \quad E = 2\pi k\sigma$$

**Solutions are in bold**

**There are 7 problems in this quiz, all are worth the same. Mark the answer closest to yours. This is Test Form A**

Figure



The charge  $3q$  is at the origin  $(x,y)=(0,0)$ , the charge  $-q$  is at  $(a,0)$ . Point  $P_1$  is at  $(d,0)$ , point  $P_2$  is at  $(0,a)$ .  $q > 0$ . Problems 1 to 4 refer to this figure.

**Problem 1**

A point on the x-axis where the electric field is 0 is  $(x_0,0)$ , with  $x_0 =$

- (a)  $1.8a$  ; (b)  $2.4a$  ; (c)  $4a$  ; (d)  $2.8a$  ; (e)  $-2a$

**On the x axis, to the right of the charge  $-q$ , set  $3q/x^2=q/(x-a)^2$ , solve for x:  $x^2=3(x-a)^2 \implies x=\sqrt{3}(x-a)$ , solve, gives  $x=\sqrt{3}/(\sqrt{3}-1)$  times a**

**Problem 2**

The point on the positive x-axis to the right of the charge  $-q$  where the force on a positive test charge points to the right and its maximum is at

- (a)  $3.3a$  ; (b)  $3.5a$  ; (c)  $3.7a$  ; (d)  $3.9a$  ; (e)  $4.1a$

E-field is  $3q/x^2 - q/(x-a)^2$ , take derivative, set it =0 to locate maximum, solve for x.  
 Equation is  $6(x-a)^3 = 2x^3 \implies x = 3^{1/3}(x-a)$ , etc.

**Problem 3**

The electric field at point  $P_1$  (see figure) at a distance  $d \gg a$  is approximately:

$$E_{P_1} = \frac{kq\alpha}{d^2} + \frac{kqa\beta}{d^3}$$

with

- (a)  $\alpha=2, \beta=2$ ; (b)  $\alpha=2, \beta=1$  ; (c)  $\alpha=3, \beta=-2$  ; (d)  $\alpha=3, \beta=-1$  ; (e)  $\alpha=2, \beta=-2$

Hint: use superposition

**$1/d^2$  dependence comes from net charge,  $1/d^3$  from dipole. At a large distance, this charge arrangement looks like a dipole ( $q, -q$ ) with dipole moment  $p=qd$  pointing in the negative x direction, plus a charge  $2q$ . Use formulas for field of charge and field of dipole.**

**Problem 4**

The electric field at point  $P_2$  (see figure) has components

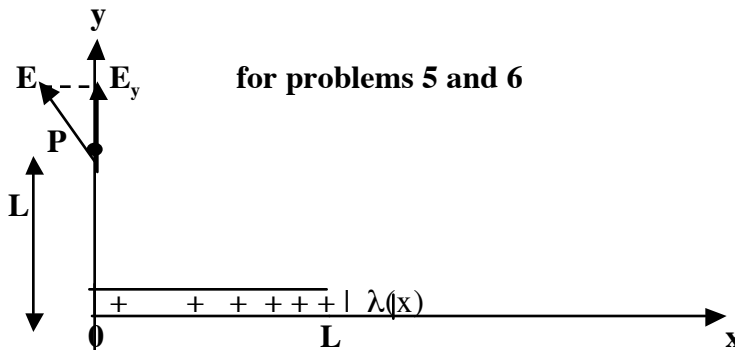
$$E_x = \frac{kq}{a^2}\alpha, \quad E_y = \frac{kq}{a^2}\beta$$

with

- (a)  $\alpha=0.5, \beta=2.5$ ; (b)  $\alpha=1, \beta=2$  ; (c)  $\alpha=1.5, \beta=-2.5$  ; (d)  $\alpha=0.5, \beta=3$  ; (e)  $\alpha=1, \beta=2.5$

**Charge  $3q$  gives a positive y-component only, charge  $-q$  gives a positive x-component and a negative y-component that are equal in magnitude since the angle is 45 degrees. Add y-components from both contributions (with their sign).**

**Figure**



A charged rod of length  $L$  on the  $x$ -axis has a nonuniform linear charge distribution

$\lambda(x) = \lambda_0 \frac{x}{L}$ , where  $x$  is measured from the left end. At the point  $P$  shown in the figure, at

distance  $L$  from the left end of the rod in the vertical direction, the electric field points approximately in the direction shown, and its  $y$ -component,  $E_y$ , is positive. It can be calculated by doing an easy integral.

**Problem 5**

The magnitude of  $E_y$  shown in the figure is  $\frac{\lambda_0}{L}\alpha$ , with

- (a)  $\alpha=0.1$ ; (b)  $\alpha=0.2$  ; (c)  $\alpha=0.3$  ; (d)  $\alpha=0.4$  ; (e)  $\alpha=0.5$

It is like the integral discussed in class in connection with example 23-9 in the book, except that the charge density is  $x$ -dependent. That makes the integral a lot easier, since it involves  $x/(x^2+L^2)^{3/2}$  and can be done simply by substitution.

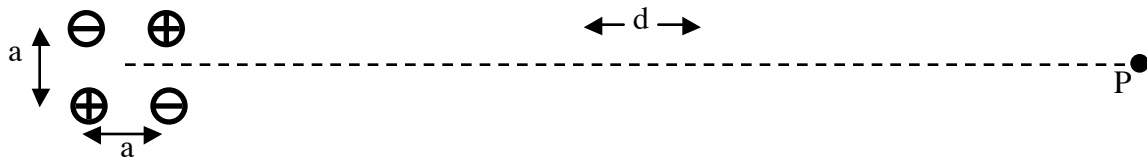
**Problem 6**

At another point very far away from this rod, at distance  $d \gg L$ , the magnitude of the electric field is  $E = \frac{\lambda_0 L}{d^2} \beta$ , with

- (a)  $\beta=0.25$ ; (b)  $\beta=0.5$  ; (c)  $\beta=1$  ; (d)  $\beta=1.5$  ; (e)  $\beta=2$

Far from the rod, the electric field is  $kq/d^2$  with  $q$  the total charge of the rod, which is obtained by integrating the linear charge density from 0 to  $L$ , giving  $\lambda_0 L/2$ .

**Problem 7**



The charges in the square arrangement in the figure with side  $a$  have all magnitude  $q$ , and their sign is shown. At the point P shown, very far away from this charge arrangement (at a distance  $d \gg a$ ), the electric field:

- (a) points to the right ; (b) points to the left ; (c) points up ; (d) points down ; (e) is exactly zero

**This can be seen as 2 dipoles, one pointing in the + y direction and the other in the -y direction, the latter one is a little further away (a further away). The first one gives an E-field pointing down, the second pointing up but a little smaller, so the net E-field points down.**