

# PHYS 2B Final Brief Solutions

Aris

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## 1 Problem 1

Gauss' Law tells us that the electric field is

$$E = \frac{Q(a^3 - 1)}{4\pi\epsilon_0 R^2 (7a^2)} \quad (1)$$

where I define the distance as  $aR$  so  $a$  is a dimensionless number. Plug in  $a = 1.5$  to determine  $Q$ . Then plug in  $a = 2$ . Ans:  $E = 1.66$ .

## 2 Problem 2

$E_1 = 0$  because it is a conductor. All the charges will reside at the outer surface.

Apply Gauss' Law for a spherical surface touching  $P_2$ . It encloses the same amount of charge  $Q$ . The electric field  $E_2$  is the same as it was in problem 1.

## 3 Problem 3

Equality of potential demands

$$V = \frac{kQ_{small}}{R} = \frac{kQ_{big}}{3R} \quad (2)$$

This is satisfied when  $Q_{small} = 1 = 2/2 = Q/2$  and  $Q_{big} = 3$ .

## 4 Problem 4

The difference in potential is, from Coulomb's law,  $k2/R - k2/(3R)$ . Use  $V = IR$ .  $I = 0.89kA$ .

## 5 Problem 5

The plate has charge density  $\lambda = Q/(\pi(10h)^2)$ . The electric field felt by the charge on top of the plate is, by Gauss' law,  $Q/2\epsilon_0$ . Compare that to the Coulomb electric field experienced on the right pair. We find  $Q/q = 50$ .

## 6 Problem 6

For the plate, electric field is independent of distance. For the two point charges, field goes as  $1/r^2$  and will go up by a factor of 4. Ans: 0.25.

## 7 Problem 7

On the charge  $2q$ , by vector addition we have that

$$E \sim \sqrt{(2 + \sqrt{2})^2 + (2 + \sqrt{2})^2} \quad (3)$$

On charge  $4q$ ,

$$E \sim \sqrt{(2 + 1/\sqrt{2})^2 + (2 + 1/\sqrt{2})^2} \quad (4)$$

Compare the ratios of the electric fields in this manner. Ans  $1.6N/C$ .

## 8 Problem 8

The dominant contribution is just a monopole charge of  $10q$  that falls off as  $1/r^2$ . Of course there are dipole and quadrupole contributions but to leading order, focus on the monopole. Find the coulomb field of a point charge of  $10q$ . Ans:  $10^{-3}q/(4\pi\epsilon_0 a^2)$

## 9 Problem 9

Use  $B = \mu_0 I / 2\pi r$  with  $I = 100$  and  $r = 3a/2$ . Ans  $B = 13.3\mu T$  into paper

## 10 Problem 10

$$M = \Phi/I = \frac{\mu_0 a}{2\pi} \int_a^{2a} \frac{1}{r} dr = 8\pi 2 \quad (5)$$

$dI/dt = 200$ . The current induced is  $I = \epsilon/R = M dI/dt / R = 6.9\mu A$

## 11 Problem 11

The strength of the magnetic field into the paper is increasing. Lenz's law says the induced magnetic field goes out of the paper, which requires a counterclockwise current.

## 12 Problem 12

Apply Faraday's law. The current is  $I = \varepsilon/R = Blv/R = 0.67A$ .

## 13 Problem 13 or use $F = I\ell B$

External work rate is equal to the power dissipated in the resistor. Hence  $P = Fv = \varepsilon^2/R = (Blv)^2/R$ . Obtain  $F = 0.33N$ .

## 14 Problem 14 or use $W = \int P dt$

Work done is force times distance covered in direction of force. That distance is  $0.2m$ , the horizontal length of the loop. The force we know. Ans  $0.066J$ .

## 15 Problem 15

In the long time limit,  $dI/dt = 0$  and the potential across the inductor is 0. It presents the path of least resistance and all current flows through the inductor, leaving none to go through the resistor.

## 16 Problem 16

From information given, deduce that the emf is  $60V$ . The initial current is then  $6A$  and decays with time constant  $L/R_2 = 0.25s$ . In 1 second, there are four e-foldings, i.e.  $I = 6e^{-4} = 0.11A$ .

## 17 Problem 17

Impedance at resonance (60) is just the resistance. From the formula for resonant frequency  $\omega = 1/\sqrt{LC}$ , obtain  $C = 2.5 * 10^{-5}$ . Now that we know  $R$ ,  $L$ ,  $C$ , use the impedance formula at  $\omega = 4000$  to obtain  $Z = 67\Omega$ .

## 18 Problem 18

Phase difference formula.

$$\phi = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{30}{60} = 27 \quad (6)$$

(in degrees)

## 19 Problem 19

Charge decays with time constant  $RC = 0.75$ . Hence current also decays as

$$I(t) = \frac{5 \exp(-t/0.75)}{0.75} \quad (7)$$

$$I(1) = 1.76A \quad (8)$$

## 20 Problem 20

Apply the generalized Ampere's law on a circular loop through point P. By area consideration, the displacement current is  $1/16$  of total current  $I$ . The distance is  $a/4$ . Hence  $B = \mu_0 I / (16 * 2\pi * (a/4))$  or  $\beta = 0.25$ .

## 21 Problem 21

Displacement current flows to the left. Using the right hand rule, magnetic field points into the paper.

## 22 Problem 22

The electric field is constant in the plane of the paper and is perpendicular to the magnetic field. The same electric field acting on positive and negative charges will result in opposite directed forces in the horizontal direction.