Problem
5. A 1.5-V battery stores 4.5 kJ of energy. How long can it light a flashlight bulb that draws 0.60 A?

Solution
The average power, supplied by the battery to the bulb, multiplied by the time equals the energy capacity of the battery. For an ideal battery, \( P = EI \), therefore \( EI t = 4.5 \text{ kJ} \), or \( t = \frac{4.5 \text{ kJ}(1.5 \text{ V})(0.60 \text{ A})}{5 \times 10^7} \approx 1.39 \text{ h} \).

Problem
10. In Fig. 28-49 all resistors have the same value, \( R \). What will be the resistance measured (a) between \( A \) and \( B \) or (b) between \( A \) and \( C \)?

![Figure 28-49 Problems 10 and 11.](image)

Solution
(a) The resistance between \( A \) and \( B \) is equivalent to two resistors of value \( R \) in series with the parallel combination of resistors of values \( R \) and \( 2R \). Thus, \( R_{AB} = R + R + R(2R)R + 2R) = 8R \Omega \). (b) \( R_{AC} \) is equivalent to just one resistor of value \( R \) in series with the parallel combination of \( R \) and \( 2R \) (since the resistor at point \( B \) carries no current, i.e., its branch is an open circuit). Thus \( R_{AC} = R + R(2R)R = 5R \Omega \).

Problem
12. A defective starter motor in a car draws 300 A from the car’s 12-V battery, dropping the battery terminal voltage to only 6 V. A good starter motor should draw only 100 A. What will the battery terminal voltage be with a good starter?

Solution
The starter circuit contains all the resistances in series, as in Fig. 28-10. (We assume \( R_L \) includes the resistance of the cables, connections, etc., as well as that of the motor.) With the defective starter, \( V_T = E - I R_{int} = 6 \text{ V} = 12 \text{ V} - (300 \text{ A})R_{int} \), so \( R_{int} = 0.02 \Omega \). With a good starter, \( V_T = 12 \text{ V} - (100 \text{ A})(0.02 \Omega) = 10 \text{ V} \).

Problem
19. What is the equivalent resistance between \( A \) and \( B \) in each of the circuits shown in Fig. 28-50? \textit{Hint:} In (c), think about symmetry and the current that would flow through \( R_2 \).

Solution
(a) There are two parallel pairs \( \left( \frac{1}{2} R_1 \right) \) in series, so \( R_{AB} = \frac{1}{2} R_1 + \frac{1}{2} R_1 = R_1 \). (b) Here, there are two series pairs \( (2R_1) \) in parallel, so \( R_{AB} = (2R_1)(2R_1) = (2R_1 + 2R_1) = R_1 \). (c) Symmetry requires that the current divides equally on the right and left sides, so points \( C \) and \( D \) are at the same potential. Thus, no current flows through \( R_2 \), and the circuit is equivalent to (b). (Note that the reasoning in parts (a) and (b) is easily generalized to resistances of different values; the generalization in part (c) requires the equality of ratios of resistances which are mirror images in the plane of symmetry.)
Problem
20. A 6.0-V battery has an internal resistance of 2.5 Ω. If the battery is short circuited, what is the rate of energy dissipation in its internal resistance?

Solution
For a short-circuited battery, \( I = \frac{E}{R_{\text{int}}} \) so the dissipated power is \( P = I^2 R_{\text{int}} = \frac{E^2}{R_{\text{int}}} = \frac{(6 \, \text{V})^2}{2.5 \, \Omega} = 14.4 \, \text{W} \).

Problem
22. What is the current through the 3-Ω resistor in the circuit of Fig. 28-51? Hint: This is trivial. Can you see why?

Solution
The current is \( I_{3\Omega} = \frac{V_{3\Omega}}{R_{3\Omega}} = \frac{6 \, \text{V}}{3 \, \Omega} = 2 \, \text{A} \), from Ohm’s law. The answer is trivial because the potential difference across the 3 Ω resistor is evident from the circuit diagram. (However, if the 6 V battery had internal resistance, an argument like that in Example 28-5 must be used.)

Problem
25. In the circuit of Fig. 28-52, \( R_1 \) is a variable resistor, and the other two resistors have equal resistances \( R \). (a) Find an expression for the voltage across \( R_1 \), and (b) sketch a graph of this quantity as a function of \( R_1 \) as \( R_1 \) varies from 0 to 10\( R \). (c) What is the limiting value as \( R_1 \to \infty \)?
Solution
(a) The resistors in parallel have an equivalent resistance of \( R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \). The other \( R \), and \( R_{eq} \) is a voltage divider in series with \( E \), so Equation 28-2 gives \( V_i = E \frac{R_2}{R + R_2} + E \frac{R_1}{R + R_2} \). (b) and (c) If \( R_i = 0 \) (the second resistor shorted out), \( V_i = 0 \), while if \( R_i = \infty \) (open circuit), \( V_i = \frac{1}{2} E \) (the value when \( R_i \) is removed). If \( R_i = 10 R \), \( V_i = (10 \Omega) E \) (as in Problem 24).

![Problem 25 Solution.](image)

Problem
29. In the circuit of Fig. 28-54 it makes no difference whether the switch is open or closed. What is \( E_3 \) in terms of the other quantities shown?

Solution
If the switch is irrelevant, then there is no current through its branch of the circuit. Thus, points \( A \) and \( B \) must be at the same potential, and the same current flows through \( R_1 \) and \( R_2 \). Kirchhoff's voltage law applied to the outer loop, and to the left-hand loop, gives \( E_i - IR_i - IR_0 + E_2 = 0 \), and \( E_i - IR_i + E_3 = 0 \), respectively. Therefore,

\[
E_i = IR_i - E_i = \left( \frac{E_1 + E_2}{R_1 + R_2} \right) R_1 - E_i = \frac{E_0 R_1 - E_1 R_1}{R_1 + R_2}.
\]

![FIGURE 28-54 Problem 29 Solution.](image)

Problem
30. What is the current through the ammeter in Fig. 28-55?

Solution
If the ammeter has zero resistance, the potential difference across it is zero, or nodes \( C \) and \( D \) are at equal potentials. If \( I \) is the current through the battery, \( \frac{1}{2} I \) must go through each of the 2 \( \Omega \) -resistors connected at node \( A \) (because

\[
V_A - V_C = \frac{1}{2} I (2 \Omega) = V_A - V_D.
\]

At node \( B \), the 2 \( \Omega \) -resistor inputs twice the current of the 4 \( \Omega \) -resistor, or \( \frac{1}{2} I \) and \( \frac{1}{2} I \) respectively (because \( V_C - V_B = \frac{2}{3} I (2 \Omega) = \frac{1}{3} I (4 \Omega) = V_D - V_B \)). Therefore \( \frac{1}{6} I \) must go through the ammeter from \( D \) to \( C \), as required by Kirchhoff's current law. To find the value of \( I \), note that the upper pair of resistors are effectively in parallel \( (V_C = V_D) \) as is the lower pair. The effective resistance between \( A \) and \( B \) is \( R_{eff} = 2 \times 2 \Omega + 2 + 2 \times 4 \Omega + (2 + 4) = 1 \Omega + \left( \frac{1}{2} \right) \Omega = \left( \frac{5}{2} \right) \Omega \). Thus \( I = V \cdot R_{eff} \), and the ammeter current is \( \frac{1}{6} I = \frac{1}{6} (6 V) \left( \frac{5}{2} \right) \Omega = \left( \frac{5}{2} \right) A = 0.429 \ A. \)
Problem
40. An ammeter with 100-Ω resistance is inserted in the circuit of Fig. 28-59. By what percentage is the measurement in error because of the nonzero meter resistance?

Solution
The current in the circuit of Fig. 28-59 is \( I = \frac{(150 \text{ V})}{(5 + 10) \text{ kΩ}} = 10 \text{ mA} \). With the ammeter inserted, the resistance is increased and the current drops to \( I = \frac{(150 \text{ V})}{(5 + 10 + 0.1) \text{ kΩ}} = 9.93 \text{ mA} \), about 0.662% lower.

Problem
42. The voltage across the 30-kΩ resistor in Fig. 28-60 is measured with (a) a 50-kΩ voltmeter, (b) a 250-kΩ voltmeter, and (c) a digital meter with 10-MΩ resistance. To two significant figures, what does each read?

Solution
With a meter of resistance \( R_m \) connected as indicated, the circuit reduces to two pairs of parallel resistors in series. The total resistance is \( R_{\text{tot}} = (30 \text{ kΩ})R_m = (30 \text{ kΩ} + R_m) + 40 \text{ kΩ} \). The voltage reading is \( V_m = R_m I_m = R_m (30 \text{ kΩ}) I_{\text{tot}} + (30 \text{ kΩ} + R_m) \), where \( I_{\text{tot}} = \frac{(100 \text{ V})}{R_{\text{tot}}} \) (the expression for \( V_m \) follows from Equation 28-2, with \( R_1 \) and \( R_2 \) as the above pairs, or from \( I_m \) as a fraction of \( I_{\text{tot}} \), as in the solution to Problem 65). For the three voltmeters specified, \( I_{\text{tot}} = 2.58 \text{ mA}, 2.14 \text{ mA}, \) and \( 2.00 \text{ mA} \), while \( V_m = 48.4 \text{ V}, 57.3 \text{ V}, \) and \( 59.9 \text{ V} \), respectively. (After checking the calculations, round off to two figures. Of course, 60 V is the ideal voltmeter reading.)

Problem
48. An uncharged 10-µF capacitor and a 470-kΩ resistor are connected in series, and 250 V applied across the combination. How long does it take the capacitor voltage to reach 200 V?

Solution
For the RC circuit described, Equation 28-6 gives the voltage across the capacitor, as a function of time. Thus, \( V_C = E(1 - e^{-t/RC}) \) or \( t = RC \ln[E(t - V_C)] = (10 \text{ µF})(470 \text{ kΩ}) \ln(250/50) = 7.56 \text{ s} \).

Problem
51. A 1.0-µF capacitor is charged to 10.0 V. It is then connected across a 500-kΩ resistor. How long does it take (a) for the capacitor voltage to reach 5.0 V and (b) for the energy stored in the capacitor to decrease to half its initial value?
Solution
A capacitor discharging through a resistor is described by exponential decay, with time constant $RC$ (see Equation 28-8), and, of course, $U_C(t) = \frac{1}{2} CV(t)^2 = \frac{1}{2} CV_0^2 e^{-2tRC} = U_C(0)e^{-2tRC}$ is the energy stored (see Equation 26-8b).

(a) $V(t)I(0) = 12$ implies $t = RC\ln 2 = (500 \, \text{k}\Omega \, (1 \, \mu\text{F})(0.693) = 347 \, \text{ms}$. (b) $U_C(t)q(t) = 12$ implies $t = \frac{1}{2} RC\ln 2 = 173 \, \text{ms}$.

Problem
54. A 2.0-\mu F capacitor is charged to 150 V. It is then connected to an uncharged 1.0-\mu F capacitor through a 2.2-k\Omega resistor, by closing switch $S$ in Fig. 28-63. Find the total energy dissipated in the resistor as the circuit comes to equilibrium. Hint: Think about charge conservation.

![FIGURE 28-63 Problem 54.](image)

Solution
When current stops flowing (at $t = \infty$), the potential difference across the capacitors is equal, but the total charge is just the initial charge. Thus, $V_0(\infty) = V_2(\infty) = V(\infty)$, and $Q_0(0) = Q(\infty) + Q_2(\infty)$. Since $Q = CV$, $C_1V_1(\infty) + C_2V_2(\infty) = C_2V_2(0)$ or $V(\infty) = V_2(0)C_2(C_1 + C_2) = \frac{2}{3} V_2(0)$. The energy stored in the capacitors is initially $U(0) = \frac{1}{2} C_2V_2^2(0) = \frac{1}{2}(2 \, \mu\text{F})(150 \, \text{V})^2 = 22.5 \, \text{mJ}$, and finally $U(\infty) = \frac{1}{2}(C_1 + C_2)V^2(\infty) = \frac{1}{2}(3 \, \mu\text{F})(100 \, \text{V})^2 = 15.0 \, \text{mJ}$. The difference, $|\Delta U| = 7.50 \, \text{mJ}$, is dissipated in the resistor (except for a negligible amount of radiated energy).

Problem
56. In the circuit of Fig. 28-64 the switch is initially open and both capacitors initially uncharged. All resistors have the same value $R$. Find expressions for the current in $R_1$ (a) just after the switch is closed and (b) a long time after the switch is closed. (c) Describe qualitatively how you expect the current in $R_3$ to behave after the switch is closed.

![FIGURE 28-64 Problem 56 Solution.](image)

Solution
(a) An uncharged capacitor acts instantaneously like a short circuit (see Example 28-9), so initially ($t = 0$) all of the current from the battery goes through $R_1$ and $C_1$, and none goes through $R_2$ and $R_3$. Thus, $I_1(0) = E/R$, and $I_2(0) = I_3(0) = 0$.

(b) A fully charged capacitor acts like an open circuit (when responding to a constant applied emf), so after a long time ($t = \infty$), all of the current goes through $R_1$ and $R_2$ in series, and none goes through $R_3$. Thus $I_1(\infty) = I_2(\infty) = E / (2R)$, and $I_3(\infty) = 0$. (c) One can easily guess that $I_1$ and $I_2$ respectively decrease and increase monotonically from their initial to their
final values, and that $I_3$ first increases from, and then decreases to zero. (One can use the loop and node equations to solve for the currents. They turn out to be linear combinations of two decaying exponentials with different time constants.)