Problem

5. A 1.5-V battery stores 4.5 kJ of energy. How long can it light a flashlight bulb that draws 0.60 A?

Solution

The average power, supplied by the battery to the bulb, multiplied by the time equals the energy capacity of the battery. For an ideal battery, $\mathcal{P} = \mathcal{E}I$, therefore $\mathcal{E}It = 4.5\text{ kJ}$, or $t = 4.5\text{ kJ}/(1.5\text{ V})(0.60\text{ A}) = 5 \times 10^3\text{ s} = 1.39\text{ h}$.

Problem

10. In Fig. 28-49 all resistors have the same value, *R*. What will be the resistance measured (a) between *A* and *B* or (b) between *A* and *C*?

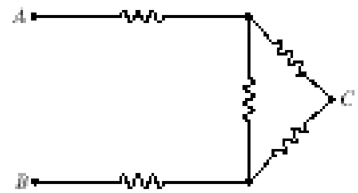


FIGURE 28-49 Problems 10 and 11.

Solution

(a) The resistance between *A* and *B* is equivalent to two resistors of value *R* in series with the parallel combination of resistors of values *R* and 2*R*. Thus, $R_{AB} = R + R + R(2R)/(R + 2R) = 8R/3$. (b) R_{AC} is equivalent to just one resistor of value *R* in series with the parallel combination of *R* and 2*R* (since the resistor at point *B* carries no current, i.e., its branch is an open circuit). Thus $R_{AC} = R + R(2R)/3R = 5R/3$.

Problem

12. A defective starter motor in a car draws 300 A from the car's 12-V battery, dropping the battery terminal voltage to only 6 V. A good starter motor should draw only 100 A. What will the battery terminal voltage be with a good starter?

Solution

The starter circuit contains all the resistances in series, as in Fig. 28-10. (We assume R_L includes the resistance of the cables, connections, etc., as well as that of the motor.) With the defective starter, $V_T = \mathcal{E} - IR_{\text{int}} = 6\text{ V} = 12\text{ V} - (300\text{ A})R_{\text{int}}$, so $R_{\text{int}} = 0.02\ \Omega$. With a good starter, $V_T = 12\text{ V} - (100\text{ A})(0.02\ \Omega) = 10\text{ V}$.

Problem

19. What is the equivalent resistance between *A* and *B* in each of the circuits shown in Fig. 28-50? *Hint:* In (*c*), think about symmetry and the current that would flow through R_2.

Solution

(a) There are two parallel pairs $(\frac{1}{2}R_1)$ in series, so $R_{AB} = \frac{1}{2}R_1 + \frac{1}{2}R_1 = R_1$. (b) Here, there are two series pairs $(2R_1)$ in parallel, so $R_{AB} = (2R_1)(2R_1)/(2R_1 + 2R_1) = R_1$. (c) Symmetry requires that the current divides equally on the right and left sides, so points *C* and *D* are at the same potential. Thus, no current flows through R_2, and the circuit is equivalent to (b). (Note that the reasoning in parts (a) and (b) is easily generalized to resistances of different values; the generalization in part (c) requires the equality of ratios of resistances which are mirror images in the plane of symmetry.)

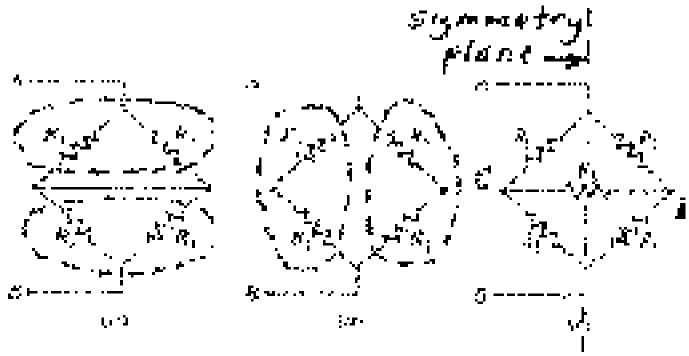


FIGURE 28-50 Problem 19 Solution.

Problem

20. A 6.0-V battery has an internal resistance of 2.5 Ω. If the battery is short circuited, what is the rate of energy dissipation in its internal resistance?

Solution

For a short-circuited battery, $I = \mathcal{E}/R_{int}$, so the dissipated power is $P = I^2 R_{int} = \mathcal{E}^2/R_{int} = (6\ \text{V})^2/2.5\ \Omega = 14.4\ \text{W}$.

Problem

22. What is the current through the 3-Ω resistor in the circuit of Fig. 28-51? *Hint:* This is trivial. Can you see why?

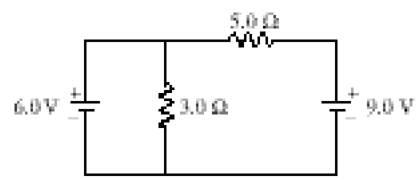


FIGURE 28-51 Problem 22.

Solution

The current is $I_{3\Omega} = V_{3\Omega}/R_{3\Omega} = 6\ \text{V}/3\ \Omega = 2\ \text{A}$, from Ohm's law. The answer is trivial because the potential difference across the 3 Ω resistor is evident from the circuit diagram. (However, if the 6 V battery had internal resistance, an argument like that in Example 28-5 must be used.)

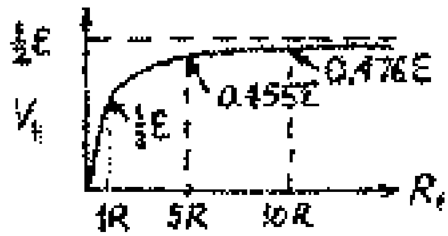
Problem

25. In the circuit of Fig. 28-52, R_1 is a variable resistor, and the other two resistors have equal resistances *R*. (a) Find an expression for the voltage across R_1, and (b) sketch a graph of this quantity as a function of R_1 as R_1 varies from 0 to 10*R*. (c) What is the limiting value as $R_1 \to \infty$?

FIGURE 28-52 Problem 25.

Solution

(a) The resistors in parallel have an equivalent resistance of $R_{\parallel} = RR_1/(R + R_1)$. The other R, and $R_{\parallel}$, is a voltage divider in series with $\mathcal{E}$, so Equation 28-2 gives $V_{\parallel} = \mathcal{E}R_{\parallel}/(R + R_{\parallel}) = \mathcal{E}R_1/(R + 2R_1)$. (b) and (c) If $R_1 = 0$ (the second resistor shorted out), $V_{\parallel} = 0$, while if $R_1 = \infty$ (open circuit), $V_{\parallel} = \frac{1}{2}\mathcal{E}$ (the value when R_1 is removed). If $R_1 = 10R$, $V_{\parallel} = (10/21)\mathcal{E}$ (as in Problem 24).



Problem 25 Solution.

Problem

29. In the circuit of Fig. 28-54 it makes no difference whether the switch is open or closed. What is $\mathcal{E}_3$ in terms of the other quantities shown?

Solution

If the switch is irrelevant, then there is no current through its branch of the circuit. Thus, points A and B must be at the same potential, and the same current flows through R_1 and R_2. Kirchhoff's voltage law applied to the outer loop, and to the left-hand loop, gives $\mathcal{E}_1 - IR_1 - IR_2 + \mathcal{E}_2 = 0$, and $\mathcal{E}_1 - IR_1 + \mathcal{E}_3 = 0$, respectively. Therefore,

$$\mathcal{E}_3 = IR_1 - \mathcal{E}_1 = \left(\frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2}\right)R_1 - \mathcal{E}_1 = \frac{\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2}{R_1 + R_2}.$$

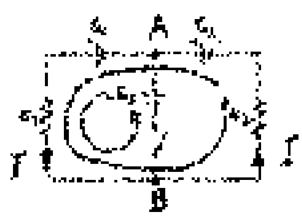


FIGURE 28-54 Problem 29 Solution.

Problem

30. What is the current through the ammeter in Fig. 28-55?

Solution

If the ammeter has zero resistance, the potential difference across it is zero, or nodes C and D are at equal potentials. If I is the current through the battery, $\frac{1}{2}I$ must go through each of the $2\ \Omega$-resistors connected at node A (because $V_A - V_C = \frac{1}{2}I(2\ \Omega) = V_A - V_D$). At node B, the $2\ \Omega$-resistor inputs twice the current of the $4\ \Omega$-resistor, or $\frac{2}{3}I$ and $\frac{1}{3}I$ respectively (because $V_C - V_B = \frac{2}{3}I(2\ \Omega) = \frac{1}{3}I(4\ \Omega) = V_D - V_B$). Therefore $\frac{1}{6}I$ must go through the ammeter from D to C, as required by Kirchhoff's current law. To find the value of I, note that the upper pair of resistors are effectively in parallel $(V_C = V_D)$ as is the lower pair. The effective resistance between A and B is $R_{\text{eff}} = 2 \times 2\ \Omega/(2 + 2) + 2 \times 4\ \Omega \div (2 + 4) = 1\ \Omega + (\frac{4}{3})\ \Omega = (\frac{7}{3})\ \Omega$. Thus $I = V/R_{\text{eff}}$, and the ammeter current is $\frac{1}{6}I = \frac{1}{6}(6\ \text{V})/(\frac{7}{3})\ \Omega = (\frac{3}{7})\ \text{A} = 0.429\ \text{A}$.

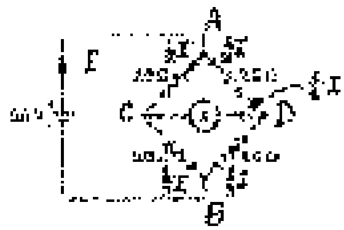


FIGURE 28-55 Problem 30 Solution.

Problem

40. An ammeter with $100\text{-}\Omega$ resistance is inserted in the circuit of Fig. 28-59. By what percentage is the measurement in error because of the nonzero meter resistance?

Solution

The current in the circuit of Fig. 28-59 is $I = (150\text{ V})/(5 + 10)\text{ k}\Omega = 10\text{ mA}$. With the ammeter inserted, the resistance is increased and the current drops to $(150\text{ V})/(5 + 10 + 0.1)\text{ k}\Omega = 9.93\text{ mA}$, about 0.662% lower.

Problem

42. The voltage across the $30\text{-k}\Omega$ resistor in Fig. 28-60 is measured with (a) a $50\text{-k}\Omega$ voltmeter, (b) a $250\text{-k}\Omega$ voltmeter, and (c) a digital meter with $10\text{-M}\Omega$ resistance. To two significant figures, what does each read?

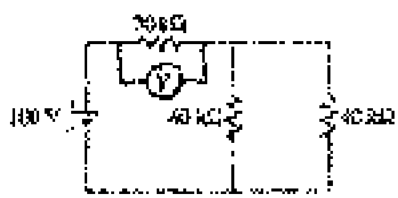


FIGURE 28-60 Problem 42 Solution.

Solution

With a meter of resistance R_m connected as indicated, the circuit reduces to two pairs of parallel resistors in series. The total resistance is $R_{\text{tot}} = (30\text{ k}\Omega)R_m/(30\text{ k}\Omega + R_m) + 40\text{ k}\Omega/2$. The voltage reading is $V_m = R_m I_m = R_m(30\text{ k}\Omega)I_{\text{tot}} \div (30\text{ k}\Omega + R_m)$, where $I_{\text{tot}} = (100\text{ V})/R_{\text{tot}}$ (the expression for V_m follows from Equation 28-2, with R_1 and R_2 as the above pairs, or from I_m as a fraction of I_{tot}, as in the solution to Problem 65). For the three voltmeters specified, $I_{\text{tot}} = 2.58\text{ mA}$, 2.14 mA, and 2.00 mA, while $V_m = 48.4\text{ V}$, 57.3 V, and 59.9 V, respectively. (After checking the calculations, round off to two figures. Of course, 60 V is the ideal voltmeter reading.)

Problem

48. An uncharged $10\text{-}\mu\text{F}$ capacitor and a $470\text{-k}\Omega$ resistor are connected in series, and 250 V applied across the combination. How long does it take the capacitor voltage to reach 200 V?

Solution

For the *RC* circuit described, Equation 28-6 gives the voltage across the capacitor, as a function of time. Thus, $V_C = \mathcal{E}(1 - e^{-t/RC})$ or $t = RC\ln[\mathcal{E}/(\mathcal{E} - V_C)] = (10\ \mu\text{F})(470\text{ k}\Omega)\ln(250/50) = 7.56\text{ s}$.

Problem

51. A $1.0\text{-}\mu\text{F}$ capacitor is charged to 10.0 V. It is then connected across a $500\text{-k}\Omega$ resistor. How long does it take (a) for the capacitor voltage to reach 5.0 V and (b) for the energy stored in the capacitor to decrease to half its initial value?

Solution

A capacitor discharging through a resistor is described by exponential decay, with time constant RC (see Equation 28-8), and, of course, $U_C(t) = \frac{1}{2}CV(t)^2 = \frac{1}{2}CV_0^2e^{-2t/RC} = U_C(0)e^{-2t/RC}$ is the energy stored (see Equation 26-8b).
(a) $V(t)/V(0) = 1/2$ implies $t = RC\ln 2 = (500\ \text{k}\Omega)(1\ \mu\text{F})(0.693) = 347\ \text{ms}$. (b) $U_c(t)/U_c(0) = 1/2$ implies $t = \frac{1}{2}RC\ln 2 = 173\ \text{ms}$.

Problem

54. A $2.0\text{-}\mu\text{F}$ capacitor is charged to 150 V. It is then connected to an uncharged $1.0\text{-}\mu\text{F}$ capacitor through a $2.2\text{-k}\Omega$ resistor, by closing switch *S* in Fig. 28-63. Find the total energy dissipated in the resistor as the circuit comes to equilibrium. *Hint:* Think about charge conservation.

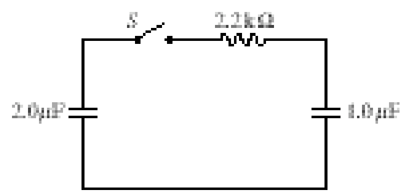


FIGURE 28-63 Problem 54.

Solution

When current stops flowing (at $t = \infty$), the potential difference across the capacitors is equal, but the total charge is just the initial charge. Thus, $V_1(\infty) = V_2(\infty) \equiv V(\infty)$, and $Q_2(0) = Q_1(\infty) + Q_2(\infty)$. Since $Q = CV$, $C_1V_1(\infty) + C_2V_2(\infty) = C_2V_2(0)$ or $V(\infty) = V_2(0)C_2/(C_1 + C_2) = \frac{2}{3}V_2(0)$. The energy stored in the capacitors is initially $U(0) = \frac{1}{2}C_2V_2^2(0) = \frac{1}{2}(2\ \mu\text{F})(150\ \text{V})^2 = 22.5\ \text{mJ}$, and finally $U(\infty) = \frac{1}{2}(C_1 + C_2)V^2(\infty) = \frac{1}{2}(3\ \mu\text{F})(100\ \text{V})^2 = 15.0\ \text{mJ}$. The difference, $|\Delta U| = 7.50\ \text{mJ}$, is dissipated in the resistor (except for a negligible amount of radiated energy).

Problem

56. In the circuit of Fig. 28-64 the switch is initially open and both capacitors initially uncharged. All resistors have the same value R. Find expressions for the current in R_2 (a) just after the switch is closed and (b) a long time after the switch is closed. (c) Describe qualitatively how you except the current in R_3 to behave after the switch is closed.

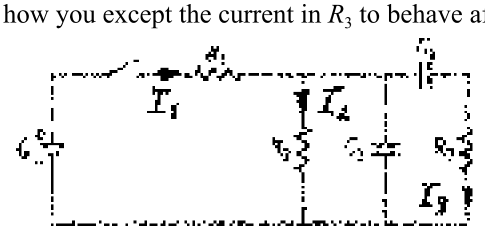


FIGURE 28-64 Problem 56 Solution.

Solution

(a) An uncharged capacitor acts instantaneously like a short circuit (see Example 28-9), so initially $(t = 0)$ all of the current from the battery goes through R_1 and C_1, and none goes through R_2 and R_3. Thus, $I_1(0) = \mathcal{E}/R$, and $I_2(0) = I_3(0) = 0$.
(b) A fully charged capacitor acts like an open circuit (when responding to a constant applied emf), so after a long time $(t = \infty)$, all of the current goes through R_1 and R_2 in series, and none goes through R_3. Thus $I_1(\infty) = I_2(\infty) = \mathcal{E}/2R$, and $I_3(\infty) = 0$. (c) One can easily guess that I_1 and I_2 respectively decrease and increase monotonically from their initial to their

final values, and that I_3 first increases from, and then decreases to zero. (One can use the loop and node equations to solve for the currents. They turn out to be linear combinations of two decaying exponentials with different time constants.)