Problem

4. A 2-g ping-pong ball rubbed against a wool jacket acquires a net positive charge of $1\ \mu\text{C}$. Estimate the fraction of the ball's electrons that have been removed.

Solution

If half the ball's mass is protons, their number (equal to the original number of electrons) is $1\ \text{g}/m_p$. The number of electrons removed is $1\ \mu\text{C}/e$, so the fraction removed is

$$\frac{(1\ \mu\text{C}/e)}{(1\ \text{g}/m_p)} = \frac{10^{-6}\ \text{C} \times 1.67 \times 10^{-24}\ \text{g}}{1.6 \times 10^{-19}\ \text{C} \times 1\ \text{g}} = 1.04 \times 10^{-11}$$

(a hundred billionth).

Problem

6. Find the ratio of the electrical force between a proton and an electron to the gravitational force between the two. Why doesn't it matter that you aren't told the distance between them?

Solution

At all distances (for which the particles can be regarded as classical point charges), the Coulomb force is stronger than the gravitational force by a factor of:

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \left(\frac{ke^2}{r^2}\right)\left(\frac{r^2}{Gm_pm_e}\right)$$

$$= \frac{(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.67 \times 10^{-27}\ \text{kg})(9.11 \times 10^{-31}\ \text{kg})} \approx 2.3 \times 10^{39}.$$

The spacial dependence of both forces is the same, and cancels out.

Problem

9. Two charges, one twice as large as the other, are located 15 cm apart and experience a repulsive force of 95 N. What is the magnitude of the larger charge?

Solution

The product of the charges is $q_1q_2 = r^2F_{\text{Coulomb}}/k = (0.15\ \text{m})^2(95\ \text{N})/(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 2.38 \times 10^{-10}\ \text{C}^2$. If one charge is twice the other, $q_1 = 2q_2$, then $\frac{1}{2}q_1^2 = 2.38 \times 10^{-10}\ \text{C}$ and $q_1 = \pm 21.8\ \mu\text{C}$.

Problem

11. A proton is on the x-axis at $x = 1.6$ nm. An electron is on the y-axis at $y = 0.85$ nm. Find the net force the two exert on a helium nucleus (charge $+2e$) at the origin.

Solution

A unit vector from the proton's position to the origin is $-\hat{\mathbf{i}}$, so the Coulomb force of the proton on the helium nucleus is $\mathbf{F}_{\text{P,He}} = k(e)(2e)(-\hat{\mathbf{i}})/(1.6\ \text{nm})^2 = -0.180\hat{\mathbf{i}}$ nN. (Use Equation 23-1, with q_1 for the proton, q_2 for the helium nucleus, and the approximate values of k and e given.) A unit vector from the electron's position to the origin is $-\hat{\mathbf{j}}$, so its force on the helium nucleus is $\mathbf{F}_{\text{e,He}} = k(-e)(2e)(-\hat{\mathbf{j}})/(0.85\ \text{nm})^2 = 0.638\hat{\mathbf{j}}$ nN. The net Coulomb force on the helium nucleus is the sum of these. (The vector form of Coulomb's law and superposition, as explained in the solution to Problems 15 and 19, provides a more general approach.)

Problem

14. A proton is at the origin and an electron is at the point $x = 0.41$ nm, $y = 0.36$ nm. Find the electric force on the proton.

Solution

The magnitude of the force is

$$|\mathbf{F}_p| = \frac{ke^2}{r^2} = \frac{(9\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})^2}{(0.41^2+0.36^2)\times10^{-18}\ \mathrm{m}^2} = 7.74\times10^{-10}\ \mathrm{N},$$

and its direction is from the proton (at $\mathbf{r}_p = 0$) to the electron (at $\mathbf{r}_e = (0.41\hat{\mathbf{i}} + 0.36\hat{\mathbf{j}})$ nm), for an attractive force, at an angle $\theta = \tan^{-1}(0.36/0.41) = 41.3°$ to the x-axis. The vector form of Coulomb's law, $\mathbf{F}_p = -ke^2(\mathbf{r}_p - \mathbf{r}_e)/|\mathbf{r}_p - \mathbf{r}_e|^3$ (see solution of next problem) gives the same result:

$$\mathbf{F}_p = -(9\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})^2(-0.41\hat{\mathbf{i}} - 0.36\hat{\mathbf{j}})/(0.41^2+0.36^2)^{3/2}(10^{-9}\ \mathrm{m})^2$$

$$= (5.82\hat{\mathbf{i}} + 5.11\hat{\mathbf{j}})\times10^{-10}\ \mathrm{N}.$$

Problem

16. A charge $3q$ is at the origin, and a charge $-2q$ is on the positive x-axis at $x = a$. Where would you place a third charge so it would experience no net electric force?

Solution

The reasoning of Example 23-3 implies that for the force on a third charge Q to be zero, it must be placed on the
x-axis to the right of the (smaller) negative charge, i.e., at $x > a$. The net Coulomb force on a third charge so placed is $F_x = kQ[3qx^{-2} - 2q(x-a)^{-2}]$, so $F_x = 0$ implies that $3(x-a)^2 = 2x^2$, or $x^2 - 6xa + 3a^2 = 0$. Thus, $x = 3a \pm \sqrt{9a^2 - 3a^2} = (3 \pm \sqrt{6})a$. Only the solution $(3+\sqrt{6})a = 5.45a$ is to the right of $x = a$.

Problem

19. In Fig. 23-39 take $q_1 = 68\ \mu$C, $q_2 = -34\ \mu$C, and $q_3 = 15\ \mu$C. Find the electric force on q_3.

Solution

Denote the positions of the charges by $\mathbf{r}_1 = \hat{\mathbf{j}}$, $\mathbf{r}_2 = 2\hat{\mathbf{i}}$, and $\mathbf{r}_3 = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (distances in meters). The vector form of Coulomb's law (in the solution to Problem 15) and the superposition principle give the net electric force on q_3 as:

$$\mathbf{F}_3 = \mathbf{F}_{13} + \mathbf{F}_{23} = \frac{kq_1q_3(\mathbf{r}_3 - \mathbf{r}_1)}{|\mathbf{r}_3 - \mathbf{r}_1|^3} + \frac{kq_2q_3(\mathbf{r}_3 - \mathbf{r}_2)}{|\mathbf{r}_3 - \mathbf{r}_2|^3} = (9\times10^9\ \mathrm{N})(15\times10^{-6})[(68\times10^{-6})(2\hat{\mathbf{i}} + \hat{\mathbf{j}})/5\sqrt{5} + (-34\times10^{-6})2\hat{\mathbf{j}}/8]$$

$$= (1.64\hat{\mathbf{i}} - 0.326\hat{\mathbf{j}})\ \mathrm{N},$$

or $F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = 1.67$ N at an angle of $\theta = \tan^{-1}(F_{3y}/F_{3x}) = -11.2°$ to the x-axis.

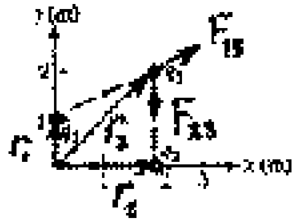


FIGURE 23-39 Problem 19 Solution.

Problem

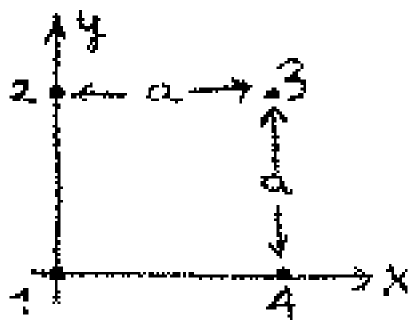
21. Four identical charges q form a square of side a. Find the magnitude of the electric force on any of the charges.

Solution

By symmetry, the magnitude of the force on any charge is the same. Let's find this for the charge at the lower left corner, which we take as the origin, as shown. Then $\mathbf{r}_1 = 0$, $\mathbf{r}_2 = a\hat{\mathbf{j}}$, $\mathbf{r}_3 = a(\hat{\mathbf{i}} + \hat{\mathbf{j}})$, $\mathbf{r}_4 = a\hat{\mathbf{i}}$, and

$$\mathbf{F}_1 = kq^2\left[\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} + \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3} + \frac{\mathbf{r}_1 - \mathbf{r}_4}{|\mathbf{r}_1 - \mathbf{r}_4|^3}\right] = kq^2\left[\frac{-a\hat{\mathbf{j}}}{a^3} - \frac{a(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{2\sqrt{2}a^3} - \frac{a\hat{\mathbf{i}}}{a^3}\right] = -\frac{kq^2}{a^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})\left(1 + \frac{1}{2\sqrt{2}}\right),$$

(Use the vector form of Coulomb's law in the solution to Problem 15, and the superposition principle.) Since $|\hat{\mathbf{i}} + \hat{\mathbf{j}}| = \sqrt{2}$, $|\mathbf{F}_1| = (kq^2/a^2)\sqrt{2}(1 + 1/2\sqrt{2}) = (kq^2/a^2)(\sqrt{2} + \frac{1}{2}) = 1.91kq^2/a^2$.



Problem 21 Solution.

Problem

22. Three identical charges $+q$ and a fourth charge $-q$ form a square of side a. (a) Find the magnitude of the electric force on a charge Q placed at the center of the square. (b) Describe the direction of this force.

Solution

The magnitudes of the forces on Q from each of the four charges are equal to $kqQ/(\sqrt{2}a/2)^2 = 2kqQ/a^2$. But the forces from the two positive charges on the same diagonal are in opposite directions, and cancel, while the forces from the positive and negative charges on the other diagonal are in the same direction (depending on the sign of Q) and add. Thus, the net force on Q has magnitude $2(2kqQ/a^2)$ and is directed toward (or away from) the negative charge for $Q > 0$ (or $Q < 0$).

Problem

24. Two identical small metal spheres initially carry charges q_1 and q_2, respectively. When they're 1.0 m apart they experience a 2.5-N attractive force. Then they're brought together so charge moves from one to the other until they have the same net charge. They're again placed 1.0 m apart, and now they repel with a 2.5-N force. What were the original values of q_1 and q_2?

Solution

The charges initially attract, so q_1 and q_2 have opposite signs, and $2.5\text{ N} = -kq_1q_2/1\text{ m}^2$. When the spheres are brought together, they share the total charge equally, each acquiring $\frac{1}{2}(q_1 + q_2)$. The magnitude of their repulsion is $2.5\text{ N} = k\frac{1}{4}(q_1 + q_2)^2/1\text{ m}^2$. Equating these two forces, we find a quadratic equation $\frac{1}{4}(q_1 + q_2)^2 = -q_1q_2$, or $q_1^2 + 6q_1q_2 + q_2^2 = 0$, with solutions $q_1 = (-3 \pm \sqrt{8})q_2$. Both solutions are possible, but since $3 + \sqrt{8} = (3 - \sqrt{8})^{-1}$, they merely represent a relabeling of the charges. Since $-q_1q_2 = 2.5\text{ N}\cdot\text{m}^2/(9\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2) = (16.7\,\mu\text{C})^2$, the solutions are $q_1 = \pm\sqrt{3 + \sqrt{8}}(16.7\,\mu\text{C}) = \pm 40.2\,\mu\text{C}$ and $q_2 = \mp 40.2\,\mu\text{C}/(3 + \sqrt{8}) = \mp 6.90\,\mu\text{C}$, or the same values with q_1 and q_2 interchanged.

Problem

30. A $65\text{-}\mu\text{C}$ point charge is at the origin. Find the electric field at the points (a) $x = 50$ cm, $y = 0$; (b) $x = 50$ cm, $y = 50$ cm; (c) $x = -25$ cm, $y = 75$ cm.

Solution

The electric field from a point charge at the origin is $\mathbf{E}(\mathbf{r}) = kq\hat{\mathbf{r}}/r^2 = kq\mathbf{r}/r^3$, since $\hat{\mathbf{r}} = \mathbf{r}/r$. (a) For $\mathbf{r} = 0.5\hat{\mathbf{\imath}}$ m and $q = 65\,\mu\text{C}$, $\mathbf{E} = (9\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(65\,\mu\text{C})\hat{\mathbf{\imath}}/(0.5\text{ m})^2 = 2.34\hat{\mathbf{\imath}}\text{ MN/C}$. (b) At $\mathbf{r} = 0.5\text{ m}(\hat{\mathbf{\imath}} + \hat{\mathbf{\jmath}})$, $\mathbf{E} = (9\times 65\times 10^3\text{ N}\cdot\text{m}^2/\text{C})(0.5\text{ m})(\hat{\mathbf{\imath}} + \hat{\mathbf{\jmath}})/(0.5\sqrt{2}\text{ m})^3 = (827\text{ kN/C})(\hat{\mathbf{\imath}} + \hat{\mathbf{\jmath}})$. (The field strength is 1.17 MN/C at 45° to the x axis.) (c) When $\mathbf{r} = (-0.25\hat{\mathbf{\imath}} + 0.75\hat{\mathbf{\jmath}})$ m, $\mathbf{E} = (5.85\times 10^5\text{ N}\cdot\text{m}^2/\text{C})(-0.25\hat{\mathbf{\imath}} + 0.75\hat{\mathbf{\jmath}})\text{ m}/[(-0.25)^2 + (0.75)^2]^{3/2}\text{ m}^3 = (-296\hat{\mathbf{\imath}} + 888\hat{\mathbf{\jmath}})\text{ kN/C}$ ($|\mathbf{E}| = 936$ kN/C, $\theta_x = 108°$).

Problem

32. A $1.0\text{-}\mu\text{C}$ charge and a $2.0\text{-}\mu\text{C}$ charge are 10 cm apart, as shown in Fig. 23-41. Find a point where the electric field is zero.

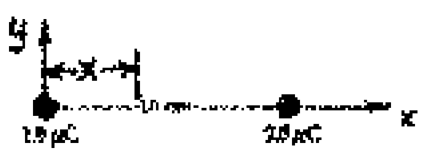


FIGURE 23-41 Problem 32 Solution.

Solution

The field can be zero only along the line joining the charges (the x-axis). To the left or right of both charges, the fields due to each are in the same direction, and cannot add to zero. Between the two, a distance $x > 0$ from the $1\,\mu\text{C}$ charge, the electric field is $\mathbf{E} = k[q_1\hat{\mathbf{\imath}}/x^2 + q_2(-\hat{\mathbf{\imath}})/(10\text{ cm} - x)^2]$, which vanishes when $1\,\mu\text{C}/x^2 = 2\,\mu\text{C}/(10\text{ cm} - x)^2$, or $x = 10\text{ cm}/(\sqrt{2} + 1) = 4.14$ cm.

Problem

37. A dipole lies on the y axis, and consists of an electron at $y = 0.60$ nm and a proton at $y = -0.60$ nm. Find the electric field (a) midway between the two charges, (b) at the point $x = 2.0$ nm, $y = 0$, and (c) at the point $x = -20$ nm, $y = 0$.

Solution

We can use the result of Example 23-6, with y replaced by x, and x by $-y$ (or equivalently, $\hat{\mathbf{j}}$ by $\hat{\mathbf{i}}$, and $\hat{\mathbf{i}}$ by $-\hat{\mathbf{j}}$). Then $\mathbf{E}(x) = 2kqa\,\hat{\mathbf{j}}(a^2 + x^2)^{-3/2}$, where $q = e = 1.6 \times 10^{-19}$ C and $a = 0.6$ nm. (Look at Fig. 23-18 rotated 90° CW.) The constant
$2kq = 2(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C}) = (2.88\ \mathrm{GN/C})(\mathrm{nm})^2$. (a) At $x = 0$, $\mathbf{E}(0) = 2kq\hat{\mathbf{j}}/a^2 =$
$(2.88\ \mathrm{GN/C})\hat{\mathbf{j}}/(0.6)^2 = (8.00\ \mathrm{GN/C})\hat{\mathbf{j}}$. (b) For $x = 2$ nm,
$\mathbf{E} = (2.88\ \mathrm{GN/C})\hat{\mathbf{j}}(0.6)(0.6^2 + 2^2)^{-3/2} = (190\ \mathrm{MN/C})\hat{\mathbf{j}}$.
(c) At $x = 20$ nm, $\mathbf{E} = (2.88\ \mathrm{GN/C})\hat{\mathbf{j}}(0.6)(0.6^2 + 20^2)^{-3/2} = (216\ \mathrm{kN/C})\hat{\mathbf{j}}$.

Problem

39. The dipole moment of the water molecule is 6.2×10^{-30} C·m. What would be the separation distance if the molecule consisted of charges $\pm e$? (The effective charge is actually less because electrons are shared by the oxygen and hydrogen atoms.)

Solution

The distance separating the charges of a dipole is $d = p/q = 6.2 \times 10^{-30}\ \mathrm{C \cdot m}/1.6 \times 10^{-19}\ \mathrm{C} = 38.8$ pm.

Problem

40. You're 1.5 m from a charge distribution whose size is much less than 1 m. You measure an electric field strength of 282 N/C. You move to a distance of 2.0 m and the field strength becomes 119 N/C. What is the net charge of the distribution? *Hint:* Don't try to calculate the charge. Determine instead how the field decreases with distance, and from that infer the charge.

Solution

Taking the hint, we suppose that the field strength varies with a power of the distance, $E \propto r^n$. Then $282/119 = (1.5/2)^n$, or $n = \ln(282/119)/\ln(0.75) = -3.00$. A dipole field falls off like r^{-3}, hence the net charge is zero.

Problem

43. A 30-cm-long rod carries a charge of 80 μC spread uniformly over its length. Find the electric field strength on the rod axis, 45 cm from the end of the rod.

Solution

Applying the result of Example 23-7, at a distance $a = 0.45$ m from the near end of the rod, we get
$E = kQ/a(a + \ell) = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(80\ \mu\mathrm{C})/(0.45\ \mathrm{m})(0.45\ \mathrm{m} + 0.30\ \mathrm{m}) = 2.13$ MN/C.

Problem

46. Two identical rods of length ℓ lie on the x-axis and carry uniform charges $\pm Q$, as shown in Fig. 23-43. (a) Find an expression for the electric field strength as a function of position x for points to the right of

the right-hand rod. (b) Show that your result has the $1/x^3$ dependence of a dipole field for $x \gg \ell$. (c) What is the dipole moment of this configuration? *Hint:* See Equation 23-7b.

FIGURE 23-43 Problem 46 Solution.

Solution

(a) The field due to each rod, for a point on their common axis, can be obtained from Example 23-7: $E_+ = kQ/x(x - \ell)$, to the right, and $E_- = kq/x(x + \ell)$, to the left. The resultant field (positive right) is

$$E = E_+ - E_- = \frac{kQ}{x}\left(\frac{1}{x - \ell} - \frac{1}{x + \ell}\right) = \frac{2kQ\ell}{x(x^2 - \ell^2)}.$$

(b) For $x \gg \ell$, $E \approx 2kQ\ell/x^3$. (c) Comparison with Equation 23-7b shows that the rods appear like a dipole with moment $p = Q\ell$.

Problem

48. Figure 23-44 shows a thin, uniformly charged disk of radius R. Imagine the disk divided into rings of varying radii r, as suggested in the figure. (a) Show that the area of such a ring is very nearly $2\pi r\,dr$. (b) If the surface charge density on the disk is σ C/m^2, use the result of (a) to write an expression for the charge dq on an infinitesimal ring. (c) Use the result of (b) along with the result of Example 23-8 to write the infinitesimal electric field dE of this ring at a point on the disk axis, taken to be the positive x axis. (d) Integrate over all such rings (that is, from $r = 0$ to $r = R$), to show that the net electric field on the disk axis is

$$E = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right).$$

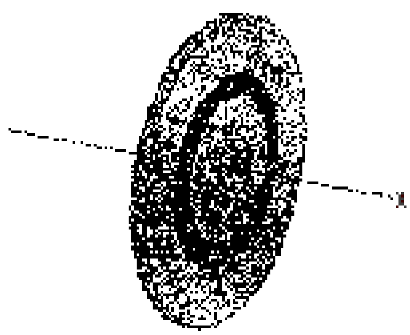


FIGURE 23-44 Problem 48.

Solution

(a) The area of an anulus of radii $R_1 < R_2$ is just $\pi(R_2^2 - R_1^2)$. For a thin ring, $R_1 = r$ and $R_2 = r + dr$, so the area is $\pi[(r + dr)^2 - r^2] = \pi(2r\,dr + dr^2)$. When dr is very small, the square term is negligible, and $dA = 2\pi r\,dr$. (This is equal to the circumference of the ring times its thickness.) (b) For surface charge density σ, $dq = \sigma\,dA = 2\pi\sigma r\,dr$. (c) From Example 23-8,

$dE_x = k(dq)x(x^2 + r^2)^{-3/2} = 2\pi k\sigma xr(x^2 + r^2)^{-3/2}dr$, which holds for x positive away from the ring's center. (d) Integrating from $r = 0$ to R, one finds $E_x = \int_0^R dE_x$, or

$$E_x = 2\pi k\sigma x\int_0^R \frac{r\,dr}{(x^2 + r^2)^{3/2}} = 2\pi k\sigma x\left|\frac{-1}{\sqrt{x^2 + r^2}}\right|_0^R = 2\pi k\sigma\left[\frac{x}{|x|} - \frac{x}{(x^2 + R^2)^{1/2}}\right].$$

(Note: For $x > 0$, $|x| = x$ and the field is $E_x = 2\pi k\sigma[1 - x(x^2 + R^2)^{-1/2}]$. However, for $x < 0$, $|x| = -x$ and $E_x = 2\pi k\sigma[-1 + |x|(x^2 + R^2)^{-1/2}]$. This is consistent with symmetry on the axis, since $E_x(x) = -E_x(-x)$.)

Problem

50. A semicircular loop of radius a carries positive charge Q distributed uniformly over its length. Find the electric field at the center of the loop (point P in Fig. 23-45). *Hint:* Divide the loop into charge elements dq as shown in Fig. 23-45, and write dq in terms of the angle $d\theta$. Then integrate over θ to get the net field at P.

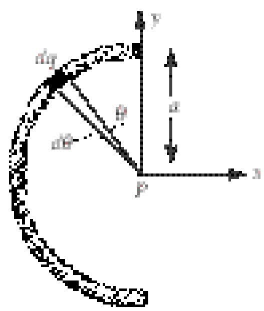


FIGURE 23-45 Problem 50 Solution.

Solution

This problem is the same as Problem 73, with $\theta_0 = 0$. Thus, $\mathbf{E}(P) = 2kQ\hat{\mathbf{i}}/\pi a^2$.

Problem

51. The electric field 22 cm from a long wire carrying a uniform line charge density is 1.9 kN/C. What will be the field strength 38 cm from the wire?

Solution

For a very long wire $(\ell \gg 38\text{ cm})$, Example 23-9 shows that the magnitude of the radial electric field falls off like $1/r$. Therefore, $E(38\text{ cm})/E(22\text{ cm}) = 22\text{ cm}/38\text{ cm}$; or $E(38\text{ cm}) = (22/38)1.9\text{ kN/C} = 1.10\text{ kN/C}$.