2.1 Standing Waves

Standing waves (waves on a string)

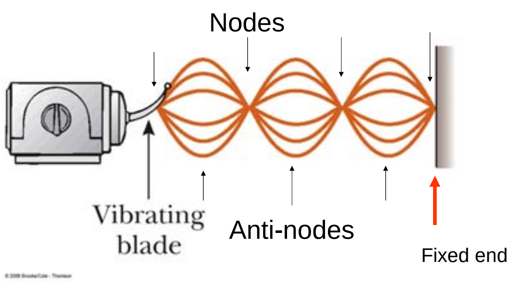
Forced vibrations /Resonance

Standing waves in air columns.

Standing Wave

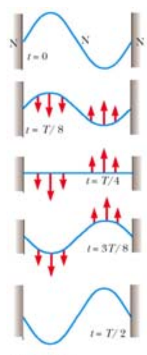
- A standing wave is formed by reflections back and forth at the boundaries of a media.
- The standing wave does not carry energy but serves to store energy.
- The standing wave stores energy of waves with specific wavelengths.

Standing wave on a string



Standing Wave with 3 nodes
– At different times.

The Standing wave doesn't "move" it just stands in one place



Standing Waves

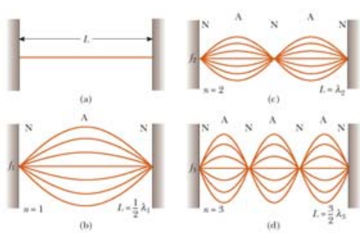
- A standing wave is generated by superposition of two waves with the same frequency and wavelength traveling in opposite directions.

Simulation of a standing wave.

http://www.walter-fendt.de/ph14e/stwaverefl.htm

Boundary Conditions

- For a wave on a string the two ends must be nodes.
- In addition there can be other nodes in the string.
- The higher the number of nodes the shorter the wavelength.
- The distance between nodes is $d_{NN}=\lambda/2$
- The distance between a node and anti-node is $\lambda/4$



Standing wave frequencies and wavelengths

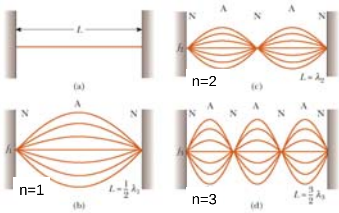
For a string of length L only a specified number of wavelengths and frequencies are allowed.

$$L = \frac{\lambda}{2} n$$

where n = 1, 2, 3......

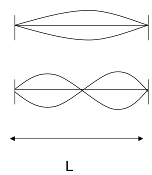
$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda} = \frac{v}{2L} n$$



For n=1 f_1 is called the fundamental frequency or first harmonic.
For n=2 f_2 is called the second harmonic, (or first overtone)

14.8
Find the fundamental and second harmonics of a steel wire 1.00 m long with mass/length = 2.00×10^{-3} kg/m under tension of 80.0 N. v=200 m/s



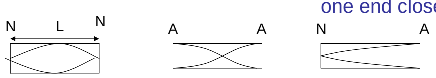
$$\lambda_n = \frac{2}{n} L \qquad f_n = \frac{v}{\lambda_n}$$

$$\lambda_1 = 2L \qquad f_1 = \frac{v}{2L} = \frac{200}{2(1)} = 100\text{Hz}$$

$$\lambda_2 = L \qquad f_2 = \frac{v}{L} = 2f_1 = 200 \text{ Hz}$$

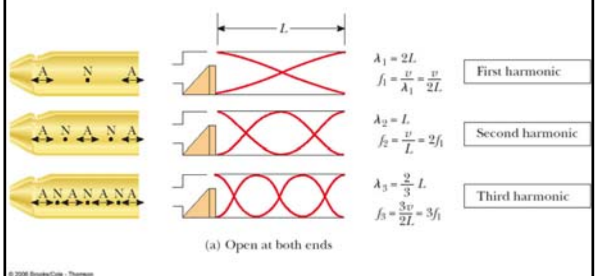
Standing waves in air columns Fundamental Frequency

2 ends closed	2 ends open	one end open one end closed
$L = \frac{\lambda_1}{2}$	$L = \frac{\lambda_1}{2}$	$L = \frac{\lambda_1}{4}$
$\lambda_1 = 2L$	$\lambda_1 = 2L$	$\lambda_1 = 4L$
$F_1 = \frac{v}{2L}$	$F_1 = \frac{v}{2L}$	$F_1 = \frac{v}{4L}$



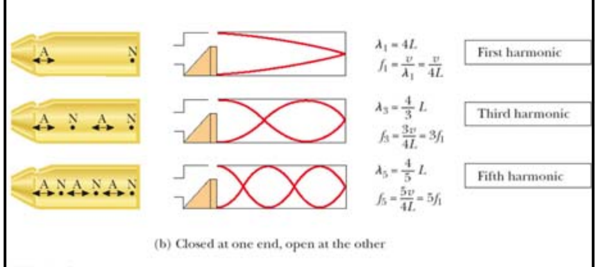
F_1 lower by a factor of 2

Cylinder open at both ends Harmonics



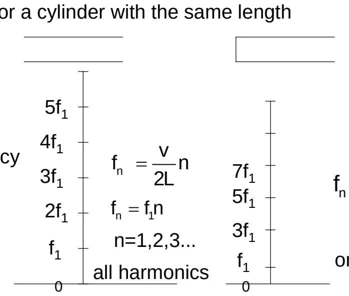
$f_n = n f_1$ n = 1, 2, 3, 4 .... All harmonics

Cylinder open at one end closed at one end - Harmonics



$f_n = nf_1$ n= 1, 3, 5, 7 .... Only odd harmonics

Summary
For a cylinder with the same length



$f_n = \frac{v}{2L} n$

$f_n = f_1 n$

n=1,2,3...
all harmonics

both ends open/ closed

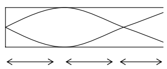
$f_n = \frac{v}{4L} n = f_1 n$

n=1, 3, 5.......
only odd harmonics

one open one closed

Question 1

A cylinder 2.5 cm in length is closed at one end and open at the other end. Find the frequency of the third harmonic of the standing wave in the column. v_{air} =340 m/s



$$\frac{\lambda}{4} \quad \frac{\lambda}{4} \quad \frac{\lambda}{4}$$

$$L = 3x\left(\frac{\lambda}{4}\right)$$

$$\lambda = \frac{4L}{3}$$

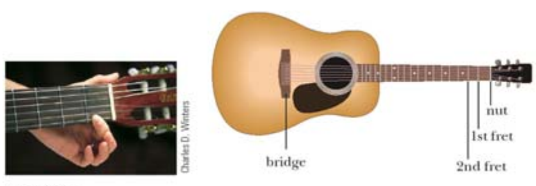
$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{4L} = \frac{3(340)m/s}{4(0.025)m} = 10{,}200Hz$$

Energy in Standing Waves

- Standing waves store energy.
- Energy in standing waves decays.
- Energy can be put into the standing wave by different means
  - large pulse
  - many small pulses (resonance)

Musical Instruments

String Instruments

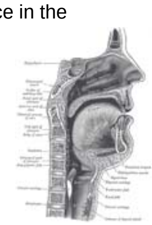


Frequency due to standing waves on the string.
The body of the instrument acts as a resonator to move air to amplify the sound.

Musical Instruments

Wind instruments

The sound is produced by vibrating air and the frequency is enhanced by resonance in the air column



Complex waves

- In general sound waves are a combination of different frequencies.
- The superposition of waves with different frequencies gives rise to the characteristic quality (timbre) of the sound.
- The different frequencies can be determined by mathematical procedure called a Fourier Transform.

Complex waves consist of different frequency components , i.e. harmonics.

displacement