PHY 1B(b) Chapter 16 HW Solutions

4. $\Delta PE_e = q(\Delta V) = q(V_f - V_i)$

$$\Rightarrow q = \frac{\Delta PE_e}{V_f - V_i} = \frac{-1.92\times10^{-17}\,J}{+60.0\,J/C} = \boxed{-3.20\times10^{-19}\,C}$$

7. a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600\,J/C}{5.33\times10^{-3}\,m} = \boxed{1.13\times10^{5}\,N/C}$

b) $F = |q|E = (1.60\times10^{-19}\,C)(1.13\times10^{5}\,N/C)$

$$= \boxed{1.80\times10^{-14}\,N}$$

c) $W = \vec{F}\cdot\vec{s} = (F)(s)\cos\theta$

$$= (1.80\times10^{-14}\,N)\left[(5.33-2.90)\times10^{-3}\,m\right]\cos 0^\circ$$

$$= 4.38\times10^{-17}\,J$$

10. $\Delta y = V_{oy}t + \frac{1}{2}a_y t^2$

$$0 = V_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow a_y = \frac{-2V_{oy}}{t}$$

From Newton's 2nd law, $a_y = \dfrac{\Sigma F_y}{m} = \dfrac{-mg - qE}{m}$

$$\Rightarrow a_y = -\left(g + \frac{qE}{m}\right)$$

$$\Rightarrow \frac{-2V_{oy}}{t} = -\left(g + \frac{qE}{m}\right) \Rightarrow E = \left(\frac{m}{q}\right)\left(\frac{2V_{oy}}{t} - g\right)$$

10. $E = \left(\frac{2.00\text{ kg}}{5\times10^{-6}\text{ C}}\right)\left[\frac{2(20.1\text{ m/s})}{4.10\text{ s}} - 9.80\text{ m/s}^2\right]$

$= 1.95\times10^3\text{ N/C}$

Also, using $v_y^2 = v_{oy}^2 + 2a_y\Delta y$

Δy_{max} occurs when $v_y = 0$ (at top of path)

$\Rightarrow \Delta y_{max} = \frac{0 - v_{oy}^2}{2a_y} = \frac{-v_{oy}^2}{2(-2v_{oy}/t)} = \frac{v_{oy}t}{4}$

$= \frac{(20.1\text{ m/s})(4.10\text{ s})}{4} = 20.6\text{ m}$

Therefore, $\Delta V_{max} = (\Delta y_{max})E$

$= (20.6\text{ m})(1.95\times10^3\text{ N/C}) =$

$= 4.02\times10^4\text{ V} = \boxed{40.2\text{ kV}}$

13. $V = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2+r_2^2}}\right)$

$= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{8\times10^{-6}\text{ C}}{0.06\text{ m}} + \frac{4\times10^{-6}\text{ C}}{0.03\text{ m}} + \frac{2\times10^{-6}\text{ C}}{\sqrt{(0.06\text{ m})^2+(0.03\text{ m})^2}}\right)$

$\Rightarrow \boxed{V = 2.67\times10^6\text{ V}}$

16. The potential at distance $r = 0.300\,m$ from a charge $Q = +9.00 \times 10^{-9}\,C$ is

$$V = \frac{k_e Q}{r} = \frac{(8.99 \times 10^{9}\,N \cdot m/C^2)(9.00 \times 10^{-9}\,C)}{0.300\,m} = +270\,V$$

$$W = qV = (3.00 \times 10^{-9}\,C)(+270\,V) = \boxed{8.09 \times 10^{-7}\,J}$$

18. Outside the spherical charge distribution, the potential is the same as for a point charge at the center of the sphere:

$$V = \frac{k_e Q}{r}, \text{ where } Q = +1.00 \times 10^{-9}\,C$$

$$\Delta(PE_e) = q\Delta V = -e k_e Q \left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

From conservation of energy, $\Delta KE = -\Delta PE_e$

$$\Rightarrow \tfrac{1}{2} m_e v^2 - 0 = -\left[-e k_e Q \left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right]$$

$$\Rightarrow v = \sqrt{\frac{2 k_e Q e}{m_e}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$

$$\Rightarrow v = \sqrt{\frac{2(8.99 \times 10^{9}\,\frac{N \cdot m^2}{C^2})(1 \times 10^{-9}\,C)(1.6 \times 10^{-19}\,C)}{9.11 \times 10^{-31}\,kg}\left(\frac{1}{.02\,m} - \frac{1}{.03\,m}\right)}$$

$$\Rightarrow \boxed{v = 7.25 \times 10^{6}\,m/s}$$

25. a) $E = \frac{\Delta V}{d} = \frac{20.0\,V}{1.80\times 10^{-3}\,m} = 1.11\times 10^{4}\,V/m$

$= \boxed{11.1\,kV/m}$ directed toward the negative plate

b) $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times 10^{-12}\,C^2/N\cdot m^2)(7.60\times 10^{-4}\,m^2)}{1.80\times 10^{-3}\,m}$

$= 3.74\times 10^{-12}\,F = \boxed{3.74\,pF}$

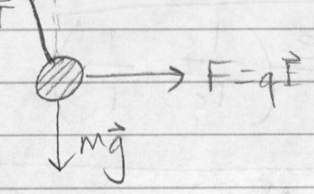
c) $Q = C\Delta V = (3.74\times 10^{-12}\,F)(20.0\,V)$

$= 7.47\times 10^{-11}\,C = \boxed{74.7\,pC}$ on one

plate and $\boxed{-74.7\,pC}$ on the other plate

28. $\Sigma F_y = 0 \Rightarrow T\cos(15.0^\circ) = mg$ or $T = \frac{mg}{\cos(15.0^\circ)}$

$\Sigma F_x = 0 \Rightarrow qE = T\sin(15.0^\circ) = mg\tan(15.0^\circ)$



$\Rightarrow E = \frac{mg\tan(15^\circ)}{q}$

$\Delta V = Ed = \frac{mgd(\tan 15^\circ)}{q}$

$$\Delta V = \frac{(350\times 10^{-6}\,kg)(9.80\,m/s^2)(0.04\,m)\tan 15^\circ}{30\times 10^{-9}\,C}$$

$= 1.23\times 10^{3}\,V = \boxed{1.23\,kV}$

32. $C_{parallel} = C_1 + C_2 = 9.00\,pF \Rightarrow C_1 = 9.00\,pF - C_2$

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\,pF$$

$$\Rightarrow C_{series} = \frac{(9.00\,pF - C_2)C_2}{(9.00\,pF - C_2) + C_2} = 2.00\,pF$$

$$\Rightarrow C_2^2 - (9\,pF)C_2 + 18(pF)^2 = 0$$

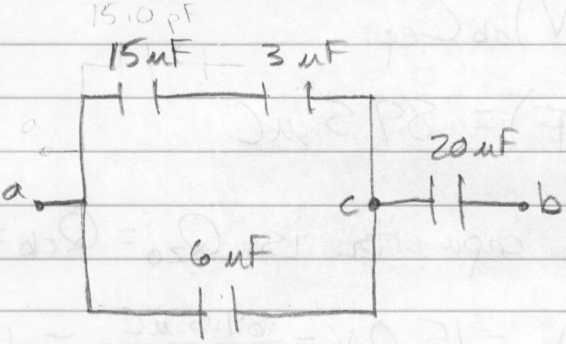
$$\Rightarrow (C_2 - 6\,pF)(C_2 - 3\,pF) = 0$$

Therefore, $C_2 = 6\,pF$ or $3\,pF$ and using

$C_1 = 9(pF) - C_2$, $C_1 = 3\,pF$ or $6\,pF$

$\boxed{\text{The two capacitances are } 3\,pF \text{ and } 6\,pF}$

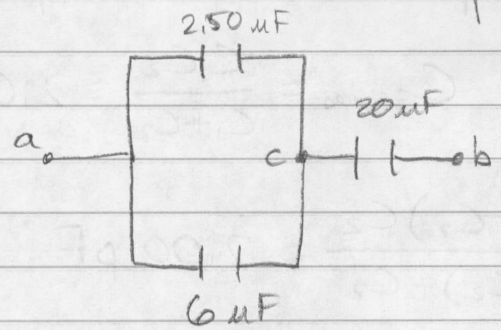
33.



a) The equivalent capacitance of the upper branch between a and c is

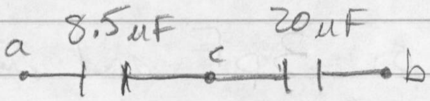
$$C_{upper} = \frac{(15\,\mu F)(3\,\mu F)}{15\,\mu F + 3\,\mu F} = 2.50\,\mu F$$

33. The circuit is then equivalent to



$\Rightarrow C_{ac} = 2.5\ \mu F + 6\ \mu F = 8.5\ \mu F$

Now the circuit is equivalent to



$\Rightarrow$ Total capacitance $= C_{eq} = \left(\frac{1}{8.5\,\mu F} + \frac{1}{20\,\mu F}\right)^{-1} = \boxed{5.96\ \mu F}$

b) $Q_{ab} = Q_{ac} = Q_{cb} = (\Delta V)_{ab} C_{eq}$

$= (15.0\ V)(5.96\ \mu F) = 89.5\ \mu C$

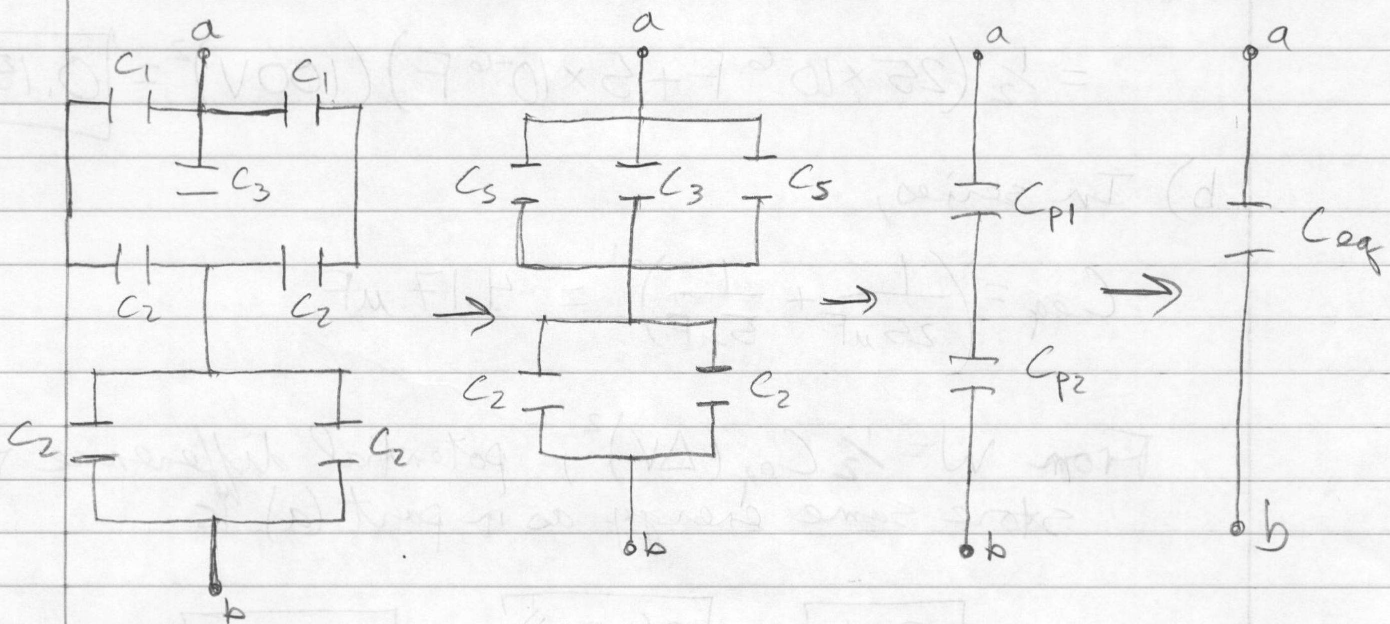
Charge on the 20 μC capacitor is $Q_{20} = Q_{cb} = \boxed{89.5\ \mu C}$

$(\Delta V)_{ac} = (\Delta V)_{ab} - (\Delta V)_{bc} = 15.0\ V - \frac{89.5\ \mu C}{20.0\ \mu F} = 10.53\ V$

Then, $Q_6 = (\Delta V)_{ac}(6\ \mu F) = \boxed{63.2\ \mu C}$

and $Q_{15} = Q_3 = (\Delta V)_{ac}(2.50\ \mu F) = \boxed{26.3\ \mu C}$

40. The circuit reduces to a single equivalent capacitor as follows...



$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5\,\mu F} + \frac{1}{10\,\mu F}\right)^{-1} = 3.33\,\mu F$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\,\mu F) + 2\,\mu F = 8.66\,\mu F$$

$$C_{p2} = C_2 + C_2 = 2(10\,\mu F) = 20\,\mu F$$

$$C_{eq} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\,\mu F} + \frac{1}{20\,\mu F}\right)^{-1}$$

$$= \boxed{6.04\,\mu F}$$

44. a) When connected in parallel, the energy stored is

$$W = \tfrac{1}{2} C_1 (\Delta V)^2 + \tfrac{1}{2} C_2 (\Delta V)^2$$

$$= \tfrac{1}{2}\left(25\times10^{-6}\,\text{F} + 5\times10^{-6}\,\text{F}\right)(100\,\text{V})^2 = \boxed{0.150\,\text{J}}$$

b) In series,

$$C_{eq} = \left(\frac{1}{25\,\mu\text{F}} + \frac{1}{5\,\mu\text{F}}\right)^{-1} = 4.17\,\mu\text{F}$$

From $W = \tfrac{1}{2} C_{eq} (\Delta V)^2$, potential difference to store same energy as in part (a) is

$$\Delta V = \sqrt{\frac{2W}{C_{eq}}} = \sqrt{\frac{2(0.150\,\text{J})}{4.17\times10^{-6}\,\text{F}}} = \boxed{268\,\text{V}}$$

47. With air between plates, initial capacitance is

$C_i = \dfrac{Q}{\Delta V_i}$ and final capacitance is $C_f = \dfrac{Q}{\Delta V_f}$

Q is the constant quantity of charge stored on the plates.

$$\Rightarrow \quad K = \frac{C_f}{C_i} = \frac{\Delta V_i}{\Delta V_f} = \frac{100\,\text{V}}{25\,\text{V}} = \boxed{4.0}$$

54. For parallel combination, $C_p = C_1 + C_2 \Rightarrow C_2 = C_p - C_1$

For series, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_2} = \frac{1}{C_s} - \frac{1}{C_1} = \frac{C_1 - C_s}{C_s C_1}$

$$\Rightarrow C_2 = \frac{C_s C_1}{C_1 - C_s}$$

Combining expressions for C_2 gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \Rightarrow C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

$$\Rightarrow C_1^2 - C_p C_1 + C_p C_s = 0$$

Using quadratic formula,

$$\boxed{C_1 = \tfrac{1}{2} C_p \pm \sqrt{\tfrac{1}{4} C_p^2 - C_p C_s}}$$

and using $C_2 = C_p - C_1$, $\boxed{C_2 = \tfrac{1}{2} C_p \mp \sqrt{\tfrac{1}{4} C_p^2 - C_p C_s}}$

59. $W = \tfrac{1}{2} C (\Delta V)^2$

$$\Rightarrow \Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(300\,\text{J})}{30 \times 10^{-6}\,\text{F}}} = 4.47 \times 10^3\ \text{V}$$

$$= \boxed{4.47\ \text{kV}}$$

61. Charges initially stored on capacitors:

$$Q_1 = C_1 (\Delta V)_i = (6\,\mu F)(250\,V) = 1.5\times10^3\,\mu C$$

$$Q_2 = C_2 (\Delta V)_i = (2\,\mu F)(250\,V) = 5.0\times10^2\,\mu C$$

When capacitors are connected in parallel, with the negative plate of one connected to the positive plate of the other, net charge is

$$Q = Q_1 - Q_2 = 1.5\times10^3\,\mu C - 5.0\times10^2\,\mu C$$

$$= 1.0\times10^3\,\mu C$$

In parallel, $C_{eq} = C_1 + C_2 = 8.0\,\mu F$

$$\Rightarrow (\Delta V)_f = \frac{Q}{C_{eq}} = \frac{1.0\times10^3\,\mu C}{8.0\,\mu F} = 125\,V$$

$$\Rightarrow Q_{1,f} = C_1 \Delta V_f = (6\,\mu F)(125\,V) = 750\,\mu C$$

$$= \boxed{0.75\,mC}$$

$$\Rightarrow Q_{2,f} = C_2 \Delta V_f = (2\,\mu F)(125\,V) = 250\,\mu C$$

$$= \boxed{0.25\,mC}$$