Problem Solutions

18.1 From $\Delta V = I(R+r)$, the internal resistance is

$$r = \frac{\Delta V}{I} - R = \frac{9.00\text{ V}}{0.117\text{ A}} - 72.0\ \Omega = \boxed{4.92\ \Omega}$$

18.2 (a) $R_{eq} = R_1 + R_2 + R_3 = +4.0\ \Omega + 8.0\ \Omega + 12\ \Omega = \boxed{24\ \Omega}$

(b) The same current exists in all resistors in a series combination.

$$I = \frac{\Delta V}{R_{eq}} = \frac{24\text{ V}}{24\ \Omega} = \boxed{1.0\text{ A}}$$

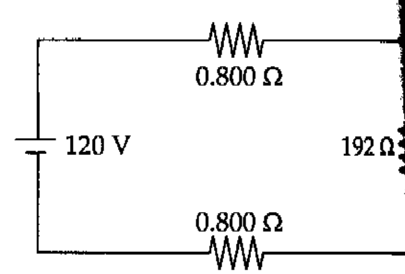
(c) If the three resistors were connected in parallel,

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{4.0\ \Omega} + \frac{1}{8.0\ \Omega} + \frac{1}{12\ \Omega}\right)^{-1} = \boxed{2.18\ \Omega}$$

Resistors in parallel have the same potential difference across them, so

$$I_4 = \frac{\Delta V}{R_4} = \frac{24\text{ V}}{4.0\ \Omega} = \boxed{6.0\text{ A}},\ I_8 = \frac{24\text{ V}}{8.0\ \Omega} = \boxed{3.0\text{ A}},\text{ and } I_{12} = \frac{24\text{ V}}{12\ \Omega} = \boxed{2.0\text{ A}}$$

18.3 For the bulb in use as intended,



$$R_{bulb} = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{75.0\text{ W}} = 192\ \Omega$$

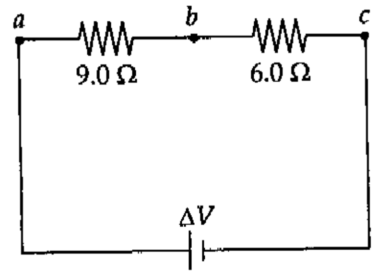
Now, presuming the bulb resistance is unchanged, the current in the circuit shown is

$$I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{0.800\ \Omega + 192\ \Omega + 0.800\ \Omega} = 0.620\text{ A}$$

and the actual power dissipated in the bulb is

$$\mathcal{P} = I^2 R_{bulb} = (0.620\text{ A})^2(192\ \Omega) = \boxed{73.8\text{ W}}$$

(a) The current through this series combination is



$$I = \frac{(\Delta V)_{bc}}{R_{bc}} = \frac{12\text{ V}}{6.0\ \Omega} = 2.0\text{ A}$$

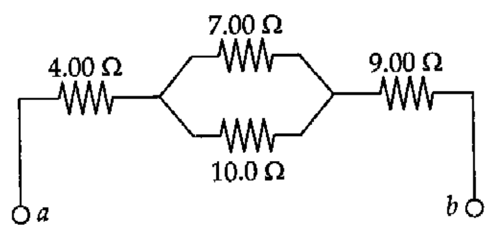
Therefore, the terminal potential difference of the power supply is

$$\Delta V = IR_{eq} = (2.0\text{ A})(9.0\ \Omega + 6.0\ \Omega) = \boxed{30\text{ V}}$$

(b) When connected in parallel, the potential difference across either resistor is the voltage setting of the power supply. Thus,

$$\Delta V = I_9 R_9 = (0.25\text{ A})(9.0\ \Omega) = \boxed{2.3\text{ V}}$$

3.5 (a) The equivalent resistance of the two parallel resistors is



$$R_p = \left(\frac{1}{7.00\ \Omega} + \frac{1}{10.0\ \Omega}\right)^{-1} = 4.12\ \Omega$$

Thus,

$$R_{ab} = R_4 + R_p + R_9 = (4.00 + 4.12 + 9.00)\ \Omega = \boxed{17.1\ \Omega}$$

(b) $$I_{ab} = \frac{(\Delta V)_{ab}}{R_{ab}} = \frac{34.0\text{ V}}{17.1\ \Omega} = 1.99\text{ A, so } \boxed{I_4 = I_9 = 1.99\text{ A}}$$

Also, $$(\Delta V)_p = I_{ab}R_p = (1.99\text{ A})(4.12\ \Omega) = 8.18\text{ V}$$

Then, $$I_7 = \frac{(\Delta V)_p}{R_7} = \frac{8.18\text{ V}}{7.00\ \Omega} = \boxed{1.17\text{ A}}$$

and $$I_{10} = \frac{(\Delta V)_p}{R_{10}} = \frac{8.18\text{ V}}{10.0\ \Omega} = \boxed{0.818\text{ A}}$$

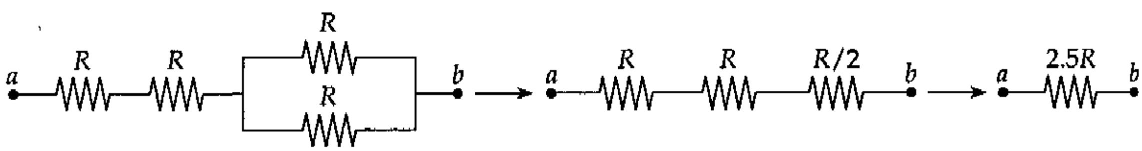
18.6 The equivalent resistance of the parallel combination of three resistors is

$$R_p = \left(\frac{1}{18\ \Omega} + \frac{1}{9.0\ \Omega} + \frac{1}{6.0\ \Omega}\right)^{-1} = 3.0\ \Omega$$

Hence, the equivalent resistance of the circuit connected to the 30 V source is

$$R_{eq} = R_{12} + R_p = 12\ \Omega + 3.0\ \Omega = \boxed{15\ \Omega}$$

18.7 If a potential difference is applied between points *a* and *b*, the vertical resistor with a free end is not part of any closed current path. Hence, it has no effect on the circuit and can be ignored. The remaining four resistors between *a* and *b* reduce to a single equivalent resistor, $R_{eq} = \boxed{2.5R}$, as shown below:



18.8 (a) The rules for combining resistors in series and parallel are used to reduce the circuit to an equivalent resistor in the stages shown below. The result is $R_{eq} = \boxed{5.13\ \Omega}$.

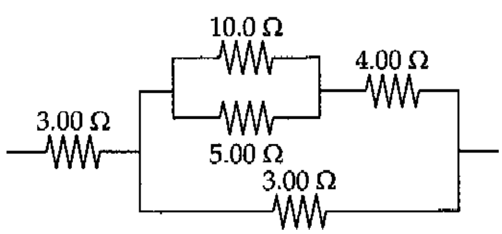


Figure 1

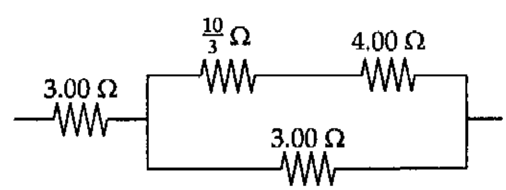


Figure 2

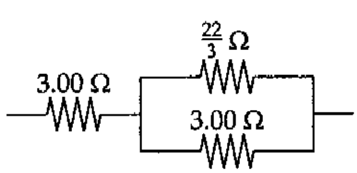


Figure 3

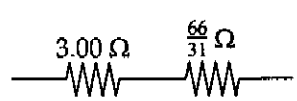


Figure 4

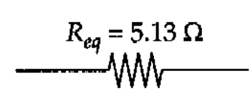


Figure 5

(b) From $\mathcal{P} = (\Delta V)^2 / R_{eq}$, the emf of the power source is

$$\Delta V = \sqrt{\mathcal{P} \cdot R_{eq}} = \sqrt{(4.00\ \text{W})(5.13\ \Omega)} = \boxed{4.53\ \text{V}}$$

Turn the circuit given in Figure P18.9 90° counterclockwise to observe that it is equivalent to that shown in Figure 1 below. This reduces, in stages, as shown in the following figures.

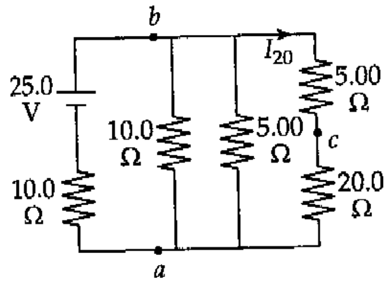


Figure 1

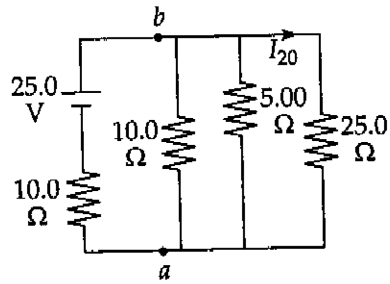


Figure 2

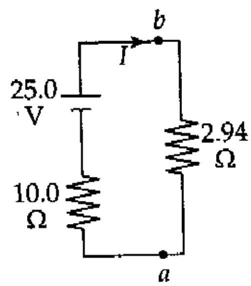


Figure 3

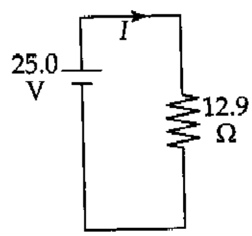


Figure 4

From Figure 4,

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.9\ \Omega} = 1.93\ \text{A}$$

(b) From Figure 3,

$$(\Delta V)_{ba} = I R_{ba}$$

$$= (1.93\ \text{A})(2.94\ \Omega) = \boxed{5.68\ \text{V}}$$

(a) From Figures 1 and 2, the current through the 20.0 Ω resistor is

$$I_{20} = \frac{(\Delta V)_{ba}}{R_{bca}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = \boxed{0.227\ \text{A}}$$

18.10 First, consider the parallel case. The resistance of resistor B is

$$R_B = \frac{(\Delta V)_B}{I_B} = \frac{6.0\text{ V}}{2.0\text{ A}} = \boxed{3.0\ \Omega}$$

In the series combination, the potential difference across B is given by

$$(\Delta V)_B = (\Delta V)_{battery} - (\Delta V)_A = 6.0\text{ V} - 4.0\text{ V} = 2.0\text{ V}$$

The current through the series combination is then

$$I_s = \frac{(\Delta V)_B}{R_B} = \frac{2.0\text{ V}}{3.0\ \Omega} = \frac{2}{3}\text{A}$$

and the resistance of resistor A is $R_A = \frac{(\Delta V)_A}{I_s} = \frac{4.0\text{ V}}{2/3\text{ A}} = \boxed{6.0\ \Omega}$

18.11 The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches;

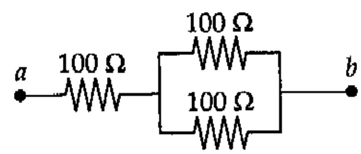
$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40\ \Omega} + \frac{1}{R + 5.0\ \Omega}\right)^{-1} = \left(\frac{1}{30\ \Omega} + \frac{1}{R + 5.0\ \Omega}\right)^{-1} = \frac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega}$$

Thus, $75\ \Omega = R + \frac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega} = \frac{R^2 + (65\ \Omega)R + 150\ \Omega^2}{R + 35\ \Omega}$

which reduces to $R^2 - (10\ \Omega)R - 2\,475\ \Omega^2 = 0$ or $(R - 55\ \Omega)(R + 45\ \Omega) = 0$.

Only the positive solution is physically acceptable, so $R = \boxed{55\ \Omega}$

(a) The total current from a to b is equal to the maximum current allowed in the 100 Ω series resistor adjacent to point a. This current has a value of



$$I_{max} = \sqrt{\frac{\mathcal{P}_{max}}{R}} = \sqrt{\frac{25.0\ \text{W}}{100\ \Omega}} = 0.500\ \text{A}$$

The total resistance is

$$R_{ab} = 100\ \Omega + \left(\frac{1}{100\ \Omega} + \frac{1}{100\ \Omega}\right)^{-1} = 100\ \Omega + 50.0\ \Omega = 150\ \Omega$$

Thus, $(\Delta V)_{max} = I_{max} R_{ab} = (0.500\ \text{A})(150\ \Omega) = \boxed{75.0\ \text{V}}$

(b) The power dissipated in the series resistor is $\mathcal{P}_1 = I_{max}^2 R = \boxed{25.0\ \text{W}}$, and the power dissipated in each of the identical parallel resistors is

$$\mathcal{P}_2 = \mathcal{P}_3 = (I_{max}/2)^2 R = (0.250\ \text{A})^2 (100\ \Omega) = \boxed{6.25\ \text{W}}$$

The total power delivered is

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 = (25.0 + 6.25 + 6.25)\ \text{W} = \boxed{37.5\ \text{W}}$$

18.13 The resistors in the circuit can be combined in the stages shown below to yield an equivalent resistance of $R_{ad} = (63/11)\ \Omega$.

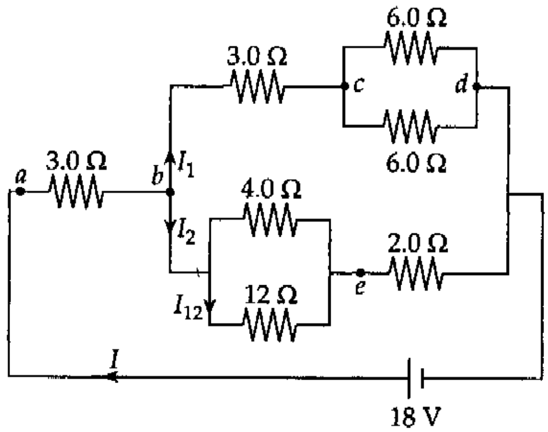


Figure 1

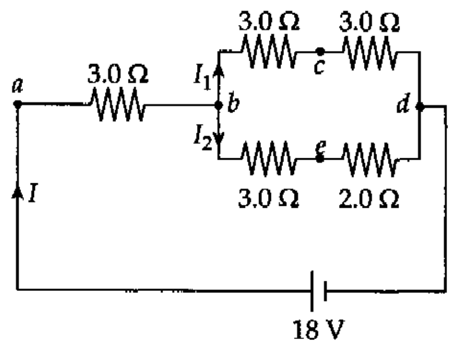


Figure 2

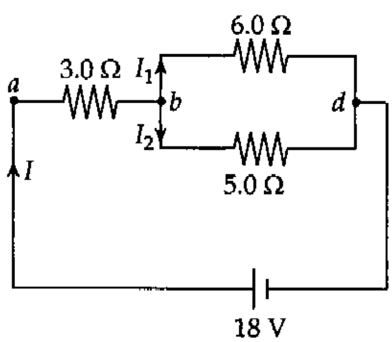


Figure 3

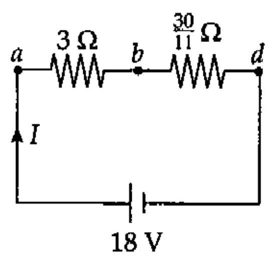


Figure 4

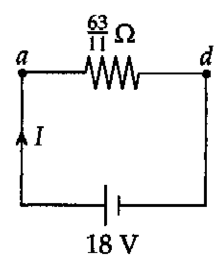


Figure 5

From Figure 5, $I = \dfrac{(\Delta V)_{ad}}{R_{ad}} = \dfrac{18\ \text{V}}{(63/11)\ \Omega} = 3.14\ \text{A}$

Then, from Figure 4, $(\Delta V)_{bd} = I R_{bd} = (3.14\ \text{A})(30/11\ \Omega) = 8.57\ \text{V}$

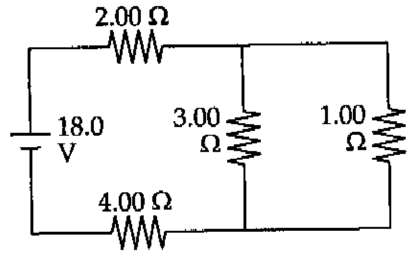
Now, look at Figure 2 and observe that

$$I_2 = \frac{(\Delta V)_{bd}}{3.0\ \Omega + 2.0\ \Omega} = \frac{8.57\ \text{V}}{5.0\ \Omega} = 1.71\ \text{A}$$

so $(\Delta V)_{be} = I_2 R_{be} = (1.71\ \text{A})(3.0\ \Omega) = 5.14\ \text{V}$

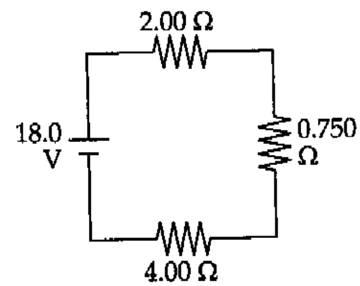
Finally, from Figure 1, $I_{12} = \dfrac{(\Delta V)_{be}}{R_{12}} = \dfrac{5.14\ \text{V}}{12\ \Omega} = \boxed{0.43\ \text{A}}$

14 The resistance of the parallel combination of the $3.00\ \Omega$ and $1.00\ \Omega$ resistors is



$$R_p = \left(\frac{1}{3.00\ \Omega} + \frac{1}{1.00\ \Omega}\right)^{-1} = 0.750\ \Omega$$

The equivalent resistance of the circuit connected to the battery is



$$R_{eq} = 2.00\ \Omega + R_p + 4.00\ \Omega = 6.75\ \Omega$$

and the current supplied by the battery is

$$I = \frac{\Delta V}{R_{eq}} = \frac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$$

The power dissipated in the $2.00\text{-}\Omega$ resistor is

$$\mathcal{P}_2 = I^2 R_2 = (2.67\ \text{A})^2 (2.00\ \Omega) = \boxed{14.2\ \text{W}}$$

and that dissipated in the $4.00\text{-}\Omega$ resistor is

$$\mathcal{P}_4 = I^2 R_4 = (2.67\ \text{A})^2 (4.00\ \Omega) = \boxed{28.4\ \text{W}}$$

The potential difference across the parallel combination of the $3.00\ \Omega$ and $1.00\ \Omega$ resistors is

$$(\Delta V)_p = I R_p = (2.67\ \text{A})(0.750\ \Omega) = 2.00\ \text{V}$$

Thus, the power dissipation in these resistors is given by

$$\mathcal{P}_3 = \frac{(\Delta V)_p^2}{R_3} = \frac{(2.00\ \text{V})^2}{3.00\ \Omega} = 1.33\ \text{W}$$

and $$\mathcal{P}_1 = \frac{(\Delta V)_p^2}{R_1} = \frac{(2.00\ \text{V})^2}{1.00\ \Omega} = 4.00\ \text{W}$$

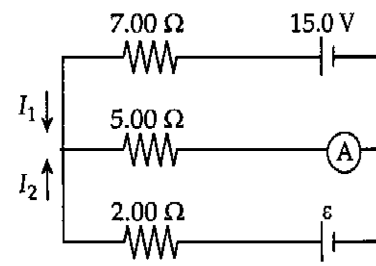
18.15 (a) Connect $\boxed{\text{two 50-}\Omega\text{ resistors in parallel}}$ to get $25\ \Omega$. Then connect that parallel combination $\boxed{\text{in series with a 20-}\Omega\text{ resistor}}$ for a total resistance of $45\ \Omega$.

(b) Connect $\boxed{\text{two 50-}\Omega\text{ resistors in parallel}}$ to get $25\ \Omega$.

Also, $\boxed{\text{connect two 20-}\Omega\text{ resistors in parallel}}$ to get $10\ \Omega$.

Then, $\boxed{\text{connect these two parallel combinations in series}}$ to obtain $35\ \Omega$.

18.16 Going counterclockwise around the upper loop, applying Kirchhoff's loop rule, gives



$$+15.0\ \text{V} - (7.00)I_1 - (5.00)(2.00\ \text{A}) = 0$$

or $$I_1 = \frac{15.0\ \text{V} - 10.0\ \text{V}}{7.00\ \Omega} = \boxed{0.714\ \text{A}}$$

From Kirchhoff's junction rule, $I_1 + I_2 - 2.00\ \text{A} = 0$

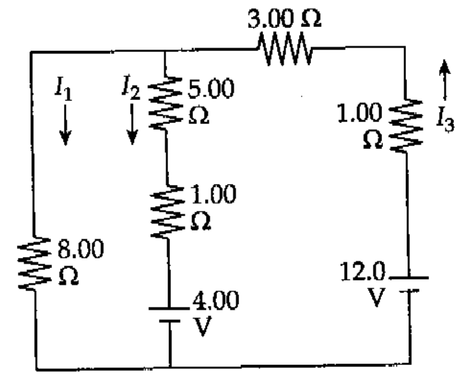
so $$I_2 = 2.00\ \text{A} - I_1 = 2.00\ \text{A} - 0.714\ \text{A} = \boxed{1.29\ \text{A}}$$

Going around the lower loop in a clockwise direction gives

$$+\mathcal{E} - (2.00)I_2 - (5.00)(2.00\ \text{A}) = 0$$

or $$\mathcal{E} = (2.00\ \Omega)(1.29\ \text{A}) + (5.00\ \Omega)(2.00\ \text{A}) = \boxed{12.6\ \text{V}}$$

17 We name the currents I_1, I_2, and I_3 as shown. Using Kirchhoff's loop rule on the rightmost loop gives



$$+12.0\text{ V}-(1.00+3.00)I_3 -(5.00+1.00)I_2-4.00\text{ V}=0$$

or $(2.00)I_3+(3.00)I_2=4.00\text{ V}$ (1)

Applying the loop rule to the leftmost loop yields

$$+4.00\text{ V}+(1.00+5.00)I_2-(8.00)I_1=0$$

or $(4.00)I_1-(3.00)I_2=2.00\text{ V}$ (2)

From Kirchhoff's junction rule, $I_1+I_2=I_3$ (3)

Solving equations (1), (2) and (3) simultaneously gives

$$\boxed{I_1=0.846\text{ A},\ I_2=0.462\text{ A, and } I_3=1.31\text{ A}}$$

All currents are in the directions indicated by the arrows in the circuit diagram.

18.18 Observe that the center branch of this circuit, that is the branch containing points *a* and *b*, is not a continuous conducting path, so no current can flow in this branch. The only current in the circuit flows counterclockwise around the perimeter of this circuit. Going counterclockwise around the this outer loop and applying Kirchhoff's loop rule gives

$$-8.0\text{ V}-(2.0\ \Omega)I-(3.0\ \Omega)I+12\text{ V}-(10\ \Omega)I-(5.0\ \Omega)I=0$$

or $I=\dfrac{12\text{ V}-8.0\text{ V}}{20\ \Omega}=0.20\text{ A}$

Now, we start at point *b* and go around the upper panel of the circuit to point *a*, keeping track of changes in potential as they occur. This gives

$$\Delta V_{ab}=V_a-V_b=-4.0\text{ V}+(6.0\ \Omega)(0)-(3.0\ \Omega)(0.20\text{ A})+12\text{ V}-(10\ \Omega)(0.20\text{ A})=+5.4\text{ V}$$

Since $\Delta V_{ab}>0$, $\boxed{\text{point } a \text{ is 5.4 V higher in potential than point } b}$

18.19 (a) Applying Kirchhoff's loop rule, as you go clockwise around the loop, gives

$$+20.0 \text{ V} - (2\,000)I - 30.0 \text{ V} - (2\,500)I + 25.0 \text{ V} - (500)I = 0,$$

or $I = 3.00\times10^{-3} \text{ A} = \boxed{3.00 \text{ mA}}$

(b) Start at the grounded point and move up the left side, recording changes in potential as you go, to obtain

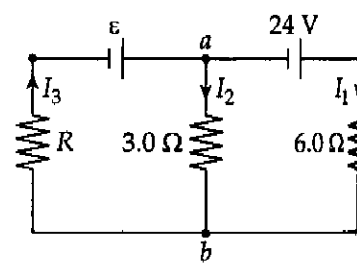
$$V_A = +20.0 \text{ V} - (2\,000\,\Omega)(3.00\times10^{-3} \text{ A}) - 30.0 \text{ V} - (1\,000\,\Omega)(3.00\times10^{-3} \text{ A})$$

or $V_A = \boxed{-19.0 \text{ V}}$

(c) $(\Delta V)_{1\,500} = (1\,500\,\Omega)(3.00\times10^{-3} \text{ A}) = \boxed{4.50 \text{ V}}$

(The upper end is at the higher potential.)

18.20 Following the path of I_1 from a to b, and recording changes in potential gives



$$V_b - V_a = +24 \text{ V} - (6.0\,\Omega)(3.0 \text{ A}) = +6.0 \text{ V}$$

Now, following the path of I_2 from a to b, and recording changes in potential gives

$$V_b - V_a = -(3.0\,\Omega)I_2 = +6.0 \text{ V}, \text{ or } I_2 = \boxed{-2.0 \text{ A}}$$

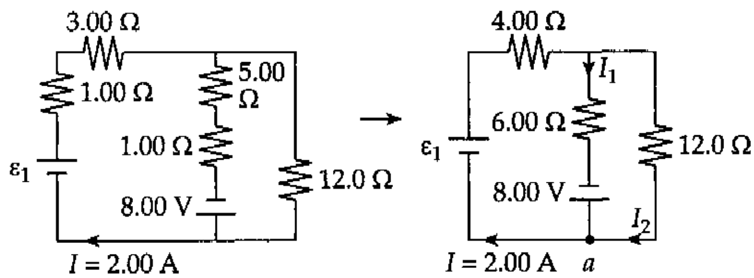
Thus, I_2 is directed from $\boxed{\text{from } b \text{ toward } a}$ and has magnitude of 2.0 A.

Applying Kirchhoff's junction rule at point a gives

$$I_3 = I_1 + I_2 = 3.0 \text{ A} + (-2.0 \text{ A}) = \boxed{1.0 \text{ A}}$$

st simplify the circuit combining the series istors. Then, apply rchhoff's junction rule point *a* to find

$$I_1 + I_2 = 2.00\text{ A}$$



Next, we apply Kirchhoff's loop rule to the rightmost loop to obtain

$$-8.00\text{ V} + (6.00)I_1 - (12.0)I_2 = 0$$

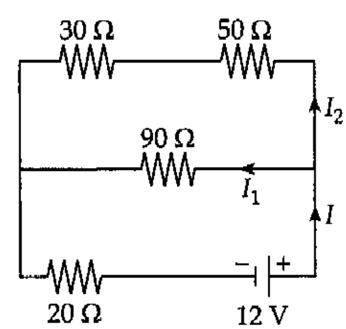
or $-8.00\text{ V} + (6.00)I_1 - (12.0)(2.00\text{ A} - I_1) = 0$ This yields $I_1 = 1.78\text{ A}$

Finally, apply Kirchhoff's loop rule to the leftmost loop to obtain

$$+\varepsilon_1 - (4.00)(2.00\text{ A}) - (6.00)I_1 + 8.00\text{ V} = 0$$

or $\varepsilon_1 = (4.00)(2.00\text{ A}) + (6.00)(1.78\text{ A}) - 8.00\text{ V} = \boxed{10.7\text{ V}}$

From Kirchhoff's point rule, note that $I_1 = I - I_2$ in the circuit shown at the right. Going counterclockwise around the upper pane of the circuit, Kirchhoff's loop rule gives



$$-(50\ \Omega)I_2 - (30\ \Omega)I_2 + (90\ \Omega)(I - I_2) = 0$$

or $I = \left(\dfrac{170\ \Omega}{90\ \Omega}\right)I_2$ (1)

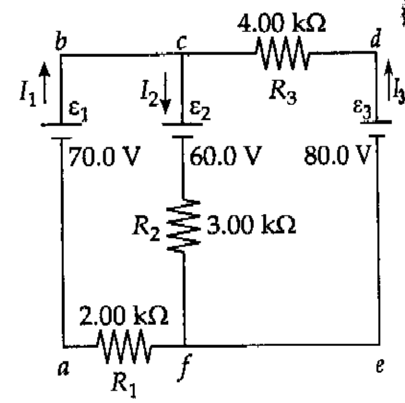
Now, going counterclockwise around the outer perimeter of the circuit, Kirchhoff's loop rule gives

$+12\text{ V} - (50\ \Omega)I_2 - (30\ \Omega)I_2 - (20\ \Omega)I = 0$ or, using Equation (1)

$\left[50\ \Omega + 30\ \Omega + (20\ \Omega)\left(\dfrac{170\ \Omega}{90\ \Omega}\right)\right]I_2 = 12\text{ V}$ which gives $I_2 = 0.10\text{ A}$

The power delivered to the 50-Ω resistor is $\mathcal{P} = I_2^2(50\ \Omega) = (0.10\text{ A})^2(50\ \Omega) = \boxed{0.50\text{ W}}$

18.23 (a) We name the currents I_1, I_2, and I_3 as shown.



Applying Kirchhoff's loop rule to loop *abcfa*, gives $+\varepsilon_1 - \varepsilon_2 - R_2 I_2 - R_1 I_1 = 0$

or $3I_2 + 2I_1 = 10.0 \text{ mA}$ (1)

Applying the loop rule to loop *edcfe* yields

$$+\varepsilon_3 - R_3 I_3 - \varepsilon_2 - R_2 I_2 = 0$$

or $3I_2 + 4I_3 = 20.0 \text{ mA}$ (2)

Finally, applying Kirchhoff's junction rule at junction *c* gives

$$I_2 = I_1 + I_3 \qquad (3)$$

Solving equations (1), (2), and (3) simultaneously yields

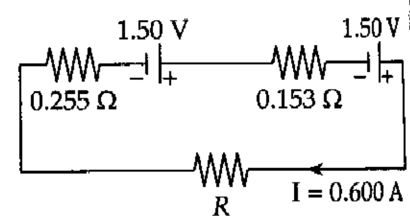
$$\boxed{I_1 = 0.385 \text{ mA}, I_2 = 3.08 \text{ mA, and } I_3 = 2.69 \text{ mA}}$$

(b) Start at point *c* and go to point *f*, recording changes in potential to obtain

$$V_f - V_c = -\varepsilon_2 - R_2 I_2 = -60.0 \text{ V} - \left(3.00\times10^3 \ \Omega\right)\left(3.08\times10^{-3} \text{ A}\right) = -69.2 \text{ V}$$

or $|\Delta V|_{cf} =$ $\boxed{69.2 \text{ V and point } c \text{ is at the higher potential}}$

18.24 (a) Applying Kirchhoff's loop rule to the circuit gives



$$+3.00 \text{ V} - (0.255 \ \Omega + 0.153 \ \Omega + R)(0.600 \text{ A}) = 0$$

or $R = \dfrac{3.00 \text{ V}}{0.600 \text{ A}} - (0.255 \ \Omega + 0.153 \ \Omega) = \boxed{4.59 \ \Omega}$

The total power input to the circuit is

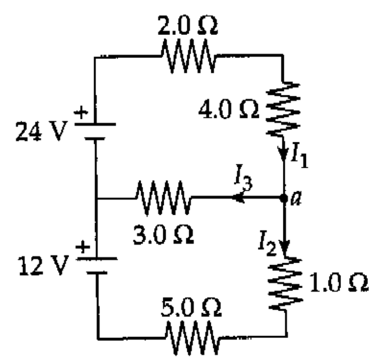
$$\mathcal{P}_{input} = (\varepsilon_1 + \varepsilon_2)I = (1.50\text{ V} + 1.50\text{ V})(0.600\text{ A}) = 1.80\text{ W}$$

$$\mathcal{P}_{loss} = I^2(r_1 + r_2) = (0.600\text{ A})^2(0.255\ \Omega + 0.153\ \Omega) = 0.147\text{ W}$$

Thus, the fraction of the power input that is dissipated internally is

$$\frac{\mathcal{P}_{loss}}{\mathcal{P}_{input}} = \frac{0.147\text{ W}}{1.80\text{ W}} = \boxed{0.0816\text{ or }8.16\%}$$

Applying Kirchhoff's junction rule at junction *a* gives



$$I_1 = I_2 + I_3 \qquad (1)$$

Using Kirchhoff's loop rule on the upper loop yields

$$+24\text{ V} - (2.0 + 4.0)I_1 - (3.0)I_3 = 0$$

or $$2I_1 + I_3 = 8.0\text{ A} \qquad (2)$$

and for the lower loop,

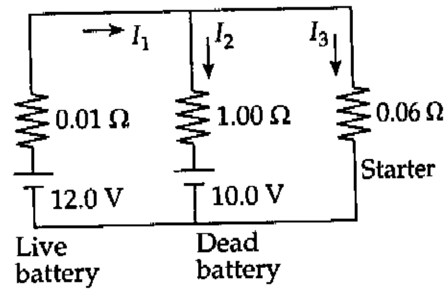
$$+12\text{ V} + (3.0)I_3 - (1.0 + 5.0)I_2 = 0$$

or $$2I_2 - I_3 = 4.0\text{ A} \qquad (3)$$

Solving equations (1), (2), and (3) simultaneously gives

$$\boxed{I_1 = 3.5\text{ A}, I_2 = 2.5\text{ A, and } I_3 = 1.0\text{ A}}$$

18.26 Using Kirchhoff's loop rule on the outer perimeter of the circuit gives



$$+12\text{ V}-(0.01)I_1-(0.06)I_3=0$$

or $I_1+6I_3=1.2\times10^3\text{ A}$ (1)

For the rightmost loop, the loop rule gives

$$+10\text{ V}+(1.00)I_2-(0.06)I_3=0$$

or $I_2-0.06I_3=-10\text{ A}$ (2)

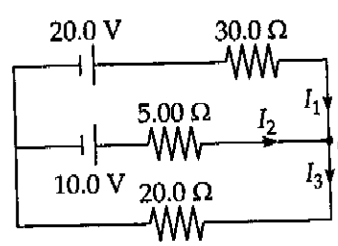
Applying Kirchhoff's junction rule at either junction gives

$$I_1=I_2+I_3 \quad (3)$$

Solving equations (1), (2), and (3) simultaneously yields

$$\boxed{I_2=0.28\text{ A (in dead battery) and }I_3=1.7\times10^2\text{ A (in starter)}}$$

18.27 Assume currents I_1, I_2, and I_3 in the directions shown. Then, using Kirchhoff's junction rule at junction *a* gives



$$I_3=I_1+I_2 \quad (1)$$

Applying Kirchhoff's loop rule on the upper loop,

$$+20.0\text{ V}-(30.0)I_1+(5.00)I_2-10.0\text{ V}=0$$

or $6I_1-I_2=2.00\text{ A}$ (2)

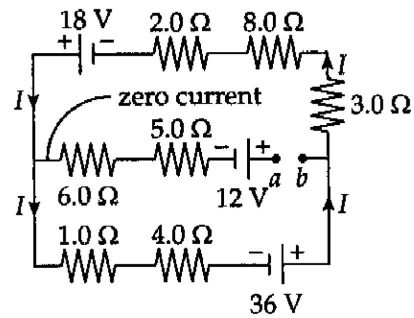
and for the lower loop, $+10.0\text{ V}-(5.00)I_2-(20.0)I_3=0$

or $I_2+4I_3=2.00\text{ A}$ (3)

Solving equations (1), (2), and (3) simultaneously yields

$$\boxed{I_1=0.353\text{ A, }I_2=0.118\text{ A, and }I_3=0.471\text{ A}}$$

18.28 (a) Since there is not a continuous path in the center branch, no current exists in that part of the circuit. Then, applying Kirchhoff's loop rule to the outer perimeter gives



$$+18\text{ V}+36\text{ V}-\left[(1.0+4.0+3.0+8.0+2.0)\ \Omega\right]I=0$$

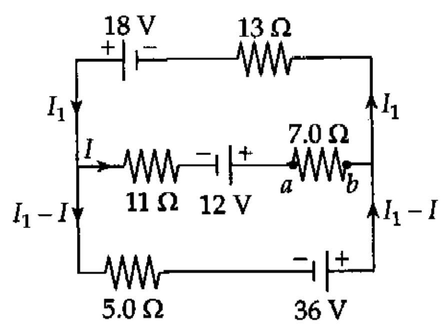
or $I=\dfrac{54\text{ V}}{18\ \Omega}=3.0\text{ A}$

Now, start at point b and go around the lower loop to point a, recording changes in potential to obtain

$$V_a-V_b=-36\text{ V}+(4.0\ \Omega+1.0\ \Omega)(3.0\text{ A})+(6.0\ \Omega+5.0\ \Omega)(0)+12\text{ V}=-9.0\text{ V}$$

or $|\Delta V|_{ab}=$ $\boxed{\text{9.0 V with point } b \text{ at a higher potential than } a}$

(b) Assume currents as shown in the modified circuit. Applying Kirchhoff's loop rule to the upper loop gives



$$-(11)I+12\text{ V}-(7.0)I-(13)I_1+18\text{ V}=0$$

or $18I+13I_1=30\text{ A}$ (1)

For the lower loop, the loop rule yields

$$-(5.0)(I_1-I)+36\text{ V}+(7.0)I-12\text{ V}+(11)I=0$$

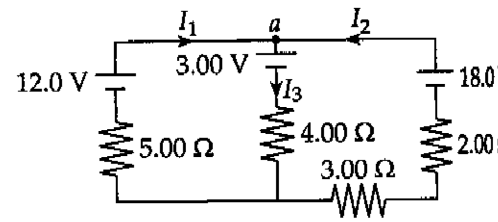
or $23I-5I_1=-24\text{ A}$ (2)

Solving equations (1) and (2) simultaneously gives $I_1=2.9\text{ A}$, and

$$I'=-0.42\text{ A}$$

Thus, the current in the 7.0-Ω resistor is $\boxed{\text{0.42 A flowing from } b \text{ to } a}$.

18.29 Applying Kirchhoff's junction rule at junction *a* gives



$$I_3 = I_1 + I_2 \qquad (1)$$

Using Kirchhoff's loop rule on the leftmost loop yields

$$-3.00\text{ V} - (4.00)I_3 - (5.00)I_1 + 12.0\text{ V} = 0$$

or $$5I_1 + 4I_3 = 9.00\text{ A} \qquad (2)$$

and for the rightmost loop,

$$-3.00\text{ V} - (4.00)I_3 - (3.00 + 2.00)I_2 + 18.0\text{ V} = 0$$

or $$5I_2 + 4I_3 = 15.0\text{ A} \qquad (3)$$

Solving equations (1), (2), and (3) simultaneously gives

$$I_1 = 0.323\text{ A},\ I_2 = 1.523\text{ A, and } I_3 = 1.846\text{ A}$$

Therefore, the potential differences across the resistors are

$$\Delta V_2 = I_2(2.00\ \Omega) = \boxed{3.05\text{ V}},\ \Delta V_3 = I_2(3.00\ \Omega) = \boxed{4.57\text{ V}}$$

$$\Delta V_4 = I_3(4.00\ \Omega) = \boxed{7.38\text{ V}},\text{ and } \Delta V_5 = I_1(5.00\ \Omega) = \boxed{1.62\text{ V}}$$

18.30 The time constant is $\tau = RC$. Considering units, we find

$$RC \to (\text{Ohms})(\text{Farads}) = \left(\frac{\text{Volts}}{\text{Amperes}}\right)\left(\frac{\text{Coulombs}}{\text{Volts}}\right) = \left(\frac{\text{Coulombs}}{\text{Amperes}}\right)$$

$$= \left(\frac{\text{Coulombs}}{\text{Coulombs/Second}}\right) = \text{Second}$$

or $\tau = RC$ has units of time.

18.31 (a) $\tau = RC = (2.0\times10^6\ \Omega)(6.0\times10^{-6}\text{ F}) = \boxed{12\text{ s}}$

(b) $Q_{max} = C\mathcal{E} = (6.0\times10^{-6}\text{ F})(20\text{ V}) = \boxed{1.2\times10^{-4}\text{ C}}$

18.32 (a) $\tau = RC = (100\ \Omega)(20.0\times10^{-6}\ \text{F}) = 2.00\times10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$

(b) $Q_{\text{max}} = C\mathcal{E} = (20.0\times10^{-6}\ \text{F})(9.00\ \text{V}) = 1.80\times10^{-4}\ \text{C} = \boxed{180\ \mu\text{C}}$

(c) $Q = Q_{\text{max}}\left(1-e^{-t/\tau}\right) = Q_{\text{max}}\left(1-e^{-\tau/\tau}\right) = Q_{\text{max}}\left(1-\dfrac{1}{e}\right) = \boxed{114\ \mu\text{C}}$

18.33 $Q_{\text{max}} = C\mathcal{E} = (5.0\times10^{-6}\ \text{F})(30\ \text{V}) = 1.5\times10^{-4}\ \text{C}$, and

$$\tau = RC = (1.0\times10^{6}\ \Omega)(5.0\times10^{-6}\ \text{F}) = 5.0\ \text{s}$$

Thus, at $t = 10\ \text{s} = 2\tau$

$$Q = Q_{\text{max}}\left(1-e^{-t/\tau}\right) = (1.5\times10^{-4}\ \text{C})\left(1-e^{-2}\right) = \boxed{1.3\times10^{-4}\ \text{C}}$$

18.34 The charge on the capacitor at time t is $Q = Q_{\text{max}}\left(1-e^{-t/\tau}\right)$, where

$$Q = C(\Delta V) \text{ and } Q_{\text{max}} = C\mathcal{E}\text{. Thus, } \Delta V = \mathcal{E}\left(1-e^{-t/\tau}\right) \text{ or } e^{-t/\tau} = 1-(\Delta V)/\mathcal{E}$$

We are given that, $\mathcal{E} = 12\ \text{V}$, and at $t = 1.0\ \text{s}$, $\Delta V = 10\ \text{V}$

Therefore, $e^{-1.0\ \text{s}/\tau} = 1-\dfrac{10}{12} = \dfrac{12-10}{12} = \dfrac{1}{6.0}$ or $e^{+1.0\ \text{s}/\tau} = 6.0$

Taking the natural logarithm of each side of the equation gives

$$\frac{1.0\ \text{s}}{\tau} = \ln(6.0) \qquad \text{or} \qquad \tau = \frac{1.0\ \text{s}}{\ln(6.0)} = 0.56\ \text{s}$$

Since the time constant is $\tau = RC$, we have

$$C = \frac{\tau}{R} = \frac{0.56\ \text{s}}{12\times10^{3}\ \Omega} = 4.7\times10^{-5}\ \text{F} = \boxed{47\ \mu\text{F}}$$

18.35 From $Q = Q_{max}\left(1 - e^{-t/\tau}\right)$, we have at $t = 0.900\text{ s}$,

$$\frac{Q}{Q_{max}} = 1 - e^{-0.900\text{ s}/\tau} = 0.600$$

Thus, $e^{-0.900\text{ s}/\tau} = 0.400$, or $-\dfrac{0.900\text{ s}}{\tau} = \ln(0.400)$

giving the time constant as $\tau = -\dfrac{0.900\text{ s}}{\ln(0.400)} = \boxed{0.982\text{ s}}$

18.36 (a) $I_{max} = \dfrac{\mathcal{E}}{R}$, so the resistance is

$$R = \frac{\mathcal{E}}{I_{max}} = \frac{48.0\text{ V}}{0.500\times10^{-3}\text{ A}} = 9.60\times10^{4}\ \Omega$$

The time constant is $\tau = RC$, so the capacitance is found to be

$$C = \frac{\tau}{R} = \frac{0.960\text{ s}}{9.60\times10^{4}\ \Omega} = 1.00\times10^{-5}\text{ F} = \boxed{10.0\ \mu\text{F}}$$

(b) $Q_{max} = C\mathcal{E} = (10.0\ \mu\text{F})(48.0\text{ V}) = 480\ \mu\text{C}$, so the charge stored in the capacitor at $t = 1.92\text{ s}$ is

$$Q = Q_{max}\left(1 - e^{-t/\tau}\right) = (480\ \mu\text{C})\left(1 - e^{-\frac{1.92\text{ s}}{0.960\text{ s}}}\right) = (480\ \mu\text{C})\left(1 - e^{-2}\right) = \boxed{415\ \mu\text{C}}$$

18.37 (a) The current drawn by each appliance is

Heater: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,300\text{ W}}{120\text{ V}} = \boxed{10.8\text{ A}}$

Toaster: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$

Grill: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$

(b) If the three appliances are operated simultaneously, they will draw a total current of $I_{total} = (10.8 + 8.33 + 12.5)\text{ A} = 31.7\text{ A}$. Therefore, a 30 ampere circuit breaker is $\boxed{\text{insufficient to handle the load}}$.

18.38 (a) The equivalent resistance of the parallel combination is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{150\ \Omega} + \frac{1}{25\ \Omega} + \frac{1}{50\ \Omega}\right)^{-1} = 15\ \Omega$$

so the total current supplied to the circuit is

$$I_{total} = \frac{\Delta V}{R_{eq}} = \frac{120\ \text{V}}{15\ \Omega} = \boxed{8.0\ \text{A}}$$

(b) Since the appliances are connected in parallel,

the voltage across each one is $\Delta V = \boxed{120\ \text{V}}$.

(c) $I_{lamp} = \dfrac{\Delta V}{R_{lamp}} = \dfrac{120\ \text{V}}{150\ \Omega} = \boxed{0.80\ \text{A}}$

(d) $\mathcal{P}_{heater} = \dfrac{(\Delta V)^2}{R_{heater}} = \dfrac{(120\ \text{V})^2}{25\ \Omega} = \boxed{5.8 \times 10^2\ \text{W}}$

18.39 From $\mathcal{P} = (\Delta V)^2/R$, the resistance of the element is

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(240\ \text{V})^2}{3\,000\ \text{W}} = 19.2\ \Omega$$

When the element is connected to a 120-V source, we find that

(a) $I = \dfrac{\Delta V}{R} = \dfrac{120\ \text{V}}{19.2\ \Omega} = \boxed{6.25\ \text{A}}$, and

(b) $\mathcal{P} = (\Delta V)I = (120\ \text{V})(6.25\ \text{A}) = \boxed{750\ \text{W}}$

18.40 The maximum power available from this line is

$$\mathcal{P}_{\max} = (\Delta V)I_{\max} = (120\ \text{V})(15\ \text{A}) = 1\,800\ \text{W}$$

Thus, the combined power requirements (2 400 W) exceeds the available power, and you $\boxed{\text{cannot operate the two appliances together}}$.

18.41 (a) The area of each surface of this axon membrane is

$$A = \ell(2\pi r) = (0.10\text{ m})\left[2\pi\left(10\times10^{-6}\text{ m}\right)\right] = 2\pi\times10^{-6}\text{ m}^2$$

and the capacitance is

$$C = \kappa\epsilon_0\frac{A}{d} = 3.0\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(\frac{2\pi\times10^{-6}\text{ m}^2}{1.0\times10^{-8}\text{ m}}\right) = 1.67\times10^{-8}\text{ F}$$

In the resting state, the charge on the outer surface of the membrane is

$$Q_i = C(\Delta V)_i = \left(1.67\times10^{-8}\text{ F}\right)\left(70\times10^{-3}\text{ V}\right) = 1.17\times10^{-9}\text{ C} \rightarrow \boxed{1.2\times10^{-9}\text{ C}}$$

The number of potassium ions required to produce this charge is

$$N_{K^+} = \frac{Q_i}{e} = \frac{1.17\times10^{-9}\text{ C}}{1.6\times10^{-19}\text{ C}} = \boxed{7.3\times10^{9}\text{ K}^+\text{ ions}}$$

and the charge per unit area on this surface is

$$\sigma = \frac{Q_i}{A} = \frac{1.17\times10^{-9}\text{ C}}{2\pi\times10^{-6}\text{ m}^2}\left(\frac{1\,e}{1.6\times10^{-19}\text{ C}}\right)\left(\frac{10^{-20}\text{ m}^2}{1\text{ Å}^2}\right) = \frac{1\,e}{8.6\times10^{4}\text{ Å}^2} = \boxed{\frac{1\,e}{\left(290\text{ Å}\right)^2}}$$

This corresponds to a low charge density of one electronic charge per square of side 290 Å, compared to a normal atomic spacing of one atom per several Å^2.

(b) In the resting state, the net charge on the inner surface of the membrane is $-Q_i = -1.17\times10^{-9}\text{ C}$, and the net positive charge on this surface in the excited state is

$$Q_f = C(\Delta V)_f = \left(1.67\times10^{-8}\text{ F}\right)\left(+30\times10^{-3}\text{ V}\right) = +5.0\times10^{-10}\text{ C}$$

The total positive charge which must pass through the membrane to produce the excited state is therefore

$$\Delta Q = Q_f - Q_i$$

$$= +5.0\times10^{-10}\text{ C} - \left(-1.17\times10^{-9}\text{ C}\right) = 1.67\times10^{-9}\text{ C} \rightarrow \boxed{1.7\times10^{-9}\text{ C}}$$

corresponding to

$$N_{Na^+} = \frac{\Delta Q}{e} = \frac{1.67\times10^{-9}\text{ C}}{1.6\times10^{-19}\text{ C/Na}^+\text{ ion}} = \boxed{1.0\times10^{10}\text{ Na}^+\text{ ions}}$$

(c) If the sodium ions enter the axon in a time of $\Delta t = 2.0\text{ ms}$, the average current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67\times10^{-9}\text{ C}}{2.0\times10^{-3}\text{ s}} = 8.3\times10^{-7}\text{ A} = \boxed{0.83\ \mu\text{A}}$$

(d) When the membrane becomes permeable to sodium ions, the initial influx of sodium ions neutralizes the capacitor with no required energy input. The energy input required to charge the now neutral capacitor to the potential difference of the excited state is

$$W = \frac{1}{2}C(\Delta V)_f^2 = \frac{1}{2}\left(1.67\times10^{-8}\text{ F}\right)\left(30\times10^{-3}\text{ V}\right)^2 = \boxed{7.5\times10^{-12}\text{ J}}$$

18.42 The capacitance of the 10 cm length of axon was found to be $C = 1.67\times10^{-8}\text{ F}$ in the solution of Problem 18.41.

(a) When the membrane becomes permeable to potassium ions, these ions flow out of the axon with no energy input required until the capacitor is neutralized. To maintain this outflow of potassium ions and charge the now neutral capacitor to the resting action potential requires an energy input of

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(1.67\times10^{-8}\text{ F}\right)\left(70\times10^{-3}\text{ V}\right)^2 = \boxed{4.1\times10^{-11}\text{ J}}.$$

(b) As found in the solution of Problem 18.41, the charge on the inner surface of the membrane in the resting state is $-1.17\times10^{-9}\text{ C}$ and the charge on this surface in the excited state is $+5.0\times10^{-10}\text{ C}$. Thus, the positive charge which must flow out of the axon as it goes from the excited state to the resting state is

$$\Delta Q = 5.0\times10^{-10}\text{ C} + 1.17\times10^{-9}\text{ C} = 1.67\times10^{-9}\text{ C},$$

and the average current during the 3.0 ms required to return to the resting state is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67\times10^{-9}\text{ C}}{3.0\times10^{-3}\text{ s}} = 5.6\times10^{-7}\text{ A} = \boxed{0.56\ \mu\text{A}}$$

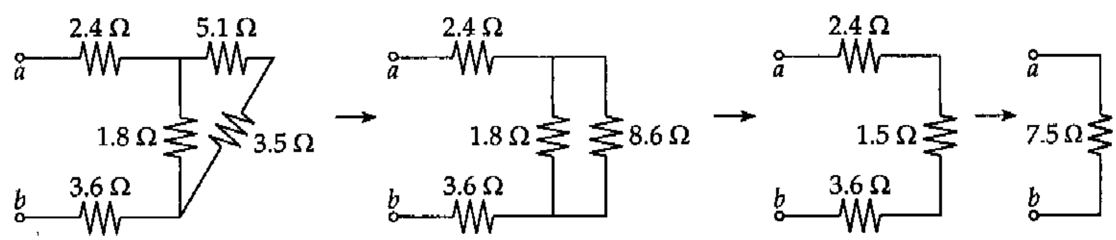
18.43 From Figure 18.28, the duration of an action potential pulse is 4.5 ms. From the solution Problem 18.41, the energy input required to reach the excited state is $W_1 = 7.5\times10^{-12}\text{ J}$. The energy input required during the return to the resting state is found in Problem 18.42 to be $W_2 = 4.1\times10^{-11}\text{ J}$. Therefore, the average power input required during an action potential pulse is

$$\wp = \frac{W_{total}}{\Delta t} = \frac{W_1 + W_2}{\Delta t} = \frac{7.5\times10^{-12}\text{ J} + 4.1\times10^{-11}\text{ J}}{4.5\times10^{-3}\text{ s}} = 1.1\times10^{-8}\text{ W} = \boxed{11\text{ nW}}$$

18.44 From $Q = Q_{max}\left(1 - e^{-t/\tau}\right)$, the ratio Q/Q_{max} at $t = 2\tau$ is found to be

$$\frac{Q}{Q_{max}} = 1 - e^{-2\tau/\tau} = 1 - \frac{1}{e^2} = \boxed{0.865}, \text{ or } Q \text{ is } \boxed{86.5\%} \text{ of } Q_{max} \text{ at } t = 2\tau$$

18.45 The resistive network between *a* an *b* reduces, in the stages shown below, to an equivalent resistance of $R_{eq} = \boxed{7.5\ \Omega}$.



18.46 (a) The circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{14\ \Omega}$.

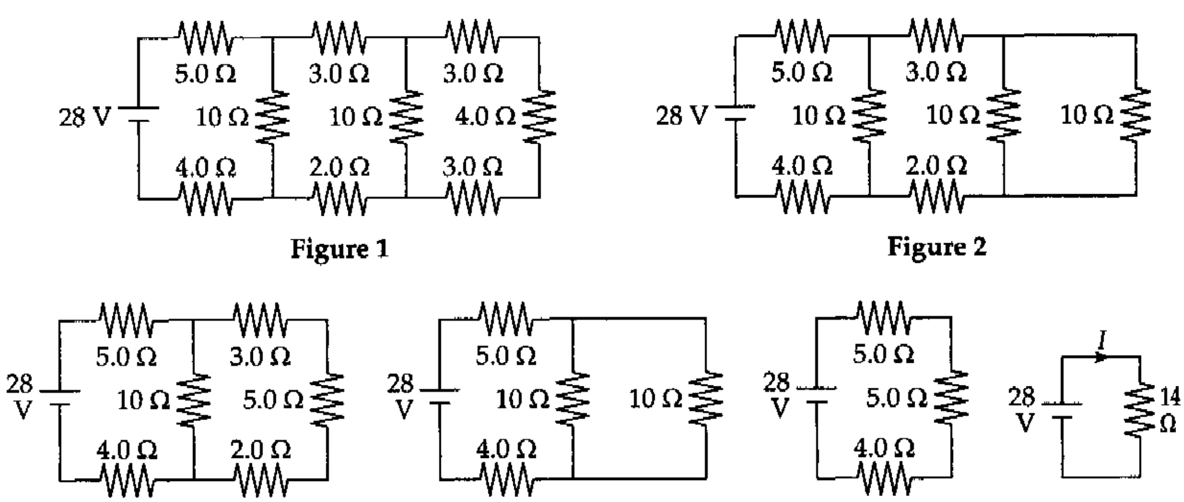


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5 Figure 6

(b) The power dissipated in the circuit is $\mathcal{P} = \dfrac{(\Delta V)^2}{R_{eq}} = \dfrac{(28\ \text{V})^2}{14\ \Omega} = \boxed{56\ \text{W}}$

(c) The current in the original 5.0-Ω resistor (in Figure 1) is the total current supplied by the battery. From Figure 6, this is

$$I = \frac{\Delta V}{R_{eq}} = \frac{28\ \text{V}}{14\ \Omega} = \boxed{2.0\ \text{A}}$$

47 (a) The resistors combine to an equivalent resistance of $R_{eq} = 15\ \Omega$ as shown.

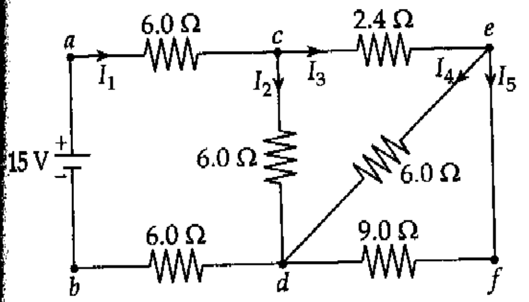


Figure 1

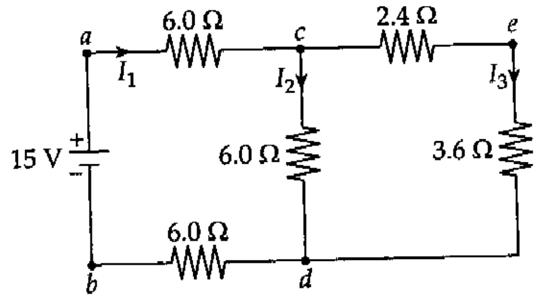


Figure 2

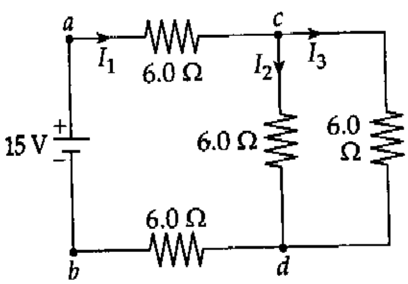


Figure 3

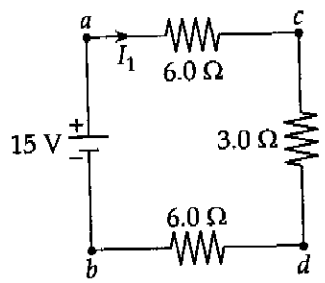


Figure 4

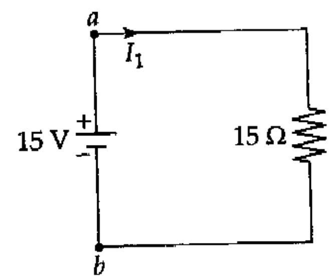


Figure 5

(b) From Figure 5, $I_1 = \dfrac{\Delta V_{ab}}{R_{eq}} = \dfrac{15\ \text{V}}{15\ \Omega} = \boxed{1.0\ \text{A}}$

Then, from Figure 4,

$$\Delta V_{ac} = \Delta V_{db} = I_1(6.0\ \Omega) = 6.0\ \text{V} \text{ and } \Delta V_{cd} = I_1(3.0\ \Omega) = 3.0\ \text{V}$$

From Figure 3, $\quad I_2 = I_3 = \dfrac{\Delta V_{cd}}{6.0\ \Omega} = \dfrac{3.0\ \text{V}}{6.0\ \Omega} = \boxed{0.50\ \text{A}}$

From Figure 2, $\quad \Delta V_{ed} = I_3(3.6\ \Omega) = 1.8\ \text{V}$

Then, from Figure 1, $\quad I_4 = \dfrac{\Delta V_{ed}}{6.0\ \Omega} = \dfrac{1.8\ \text{V}}{6.0\ \Omega} = \boxed{0.30\ \text{A}}$

and $\quad I_5 = \dfrac{\Delta V_{fd}}{9.0\ \Omega} = \dfrac{\Delta V_{ed}}{9.0\ \Omega} = \dfrac{1.8\ \text{V}}{9.0\ \Omega} = \boxed{0.20\ \text{A}}$

(c) From Figure 2, $\Delta V_{ce} = I_3(2.4\ \Omega) = \boxed{1.2\ \text{V}}$. All the other needed potential differences were calculated above in part (b). The results were

$$\Delta V_{ac} = \Delta V_{db} = \boxed{6.0\ \text{V}};\ \Delta V_{cd} = \boxed{3.0\ \text{V}};\ \text{and}\ \Delta V_{fd} = \Delta V_{ed} = \boxed{1.8\ \text{V}}$$

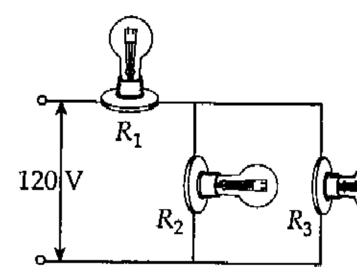
(d) The power dissipated in each resistor is found from $\mathcal{P} = (\Delta V)^2/R$ with the following results:

$$\mathcal{P}_{ac} = \frac{(\Delta V)_{ac}^2}{R_{ac}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}} \qquad \mathcal{P}_{ce} = \frac{(\Delta V)_{ce}^2}{R_{ce}} = \frac{(1.2\text{ V})^2}{2.4\ \Omega} = \boxed{0.60\text{ W}}$$

$$\mathcal{P}_{ed} = \frac{(\Delta V)_{ed}^2}{R_{ed}} = \frac{(1.8\text{ V})^2}{6.0\ \Omega} = \boxed{0.54\text{ W}} \qquad \mathcal{P}_{fd} = \frac{(\Delta V)_{fd}^2}{R_{fd}} = \frac{(1.8\text{ V})^2}{9.0\ \Omega} = \boxed{0.36\text{ W}}$$

$$\mathcal{P}_{cd} = \frac{(\Delta V)_{cd}^2}{R_{cd}} = \frac{(3.0\text{ V})^2}{6.0\ \Omega} = \boxed{1.5\text{ W}} \qquad \mathcal{P}_{db} = \frac{(\Delta V)_{db}^2}{R_{db}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}}$$

18.48 (a) From $\mathcal{P} = (\Delta V)^2/R$, the resistance of each of the three bulbs is given by



$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

As connected, the parallel combination of R_2 and R_3 is in series with R_1. Thus, the equivalent resistance of the circuit is

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 240\ \Omega + \left(\frac{1}{240\ \Omega} + \frac{1}{240\ \Omega}\right)^{-1} = 360\ \Omega$$

The total power delivered to the circuit is

$$\mathcal{P} = \frac{(\Delta V)^2}{R_{eq}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$$

(b) The current supplied by the source is $I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{360\ \Omega} = \frac{1}{3}\text{ A}$. Thus, the potential difference across R_1 is

$$(\Delta V)_1 = IR_1 = \left(\frac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}$$

The potential difference across the parallel combination of R_2 and R_3 is then

$$(\Delta V)_2 = (\Delta V)_3 = (\Delta V)_{source} - (\Delta V)_1 = 120\text{ V} - 80.0\text{ V} = \boxed{40.0\text{ V}}$$

(a) From $\mathcal{E} = I(r + R_{load})$, the current supplied when the headlights are the entire load is

$$I = \frac{\mathcal{E}}{r + R_{load}} = \frac{12.6 \text{ V}}{(0.080+5.00)\ \Omega} = 2.48 \text{ A}$$

The potential difference across the headlights is then

$$\Delta V = I R_{load} = (2.48 \text{ A})(5.00\ \Omega) = \boxed{12.4 \text{ V}}$$

(b) The starter motor connects in parallel with the headlights. If I_{hl} is the current supplied to the headlights, the total current delivered by the battery is $I = I_{hl} + 35.0 \text{ A}$

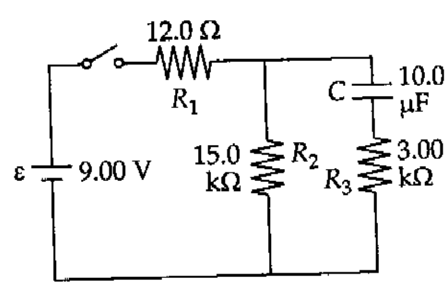
The terminal potential difference of the battery is $\Delta V = \mathcal{E} - Ir$, so the total current is $I = (\mathcal{E} - \Delta V)/r$ while the current to the headlights is $I_{hl} = \Delta V/5.00\ \Omega$. Thus, $I = I_{hl} + 35.0 \text{ A}$ becomes

$$\frac{\mathcal{E} - \Delta V}{r} = \frac{\Delta V}{5.00\ \Omega} + 35.0 \text{ A}$$

which yields

$$\Delta V = \frac{\mathcal{E} - (35.0 \text{ A})r}{1 + r/(5.00\ \Omega)} = \frac{12.6 \text{ V} - (35.0 \text{ A})(0.080\ \Omega)}{1 + (0.080\ \Omega)/(5.00\ \Omega)} = \boxed{9.65 \text{ V}}$$

50 (a) After steady-state conditions have been reached, there is no current in the branch containing the capacitor.



Thus, for R_3: $\boxed{I_{R_3} = 0 \text{ (steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12.0-kΩ and 15.0-kΩ resistors in series:

For R_1 and R_2:

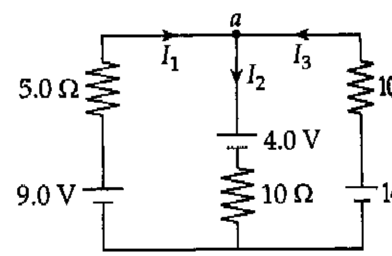
$$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1 + R_2} = \frac{9.00 \text{ V}}{(12.0 \text{ k}\Omega + 15.0 \text{ k}\Omega)} = \boxed{333\ \mu\text{A (steady-state)}}$$

(b) When the steady-state has been reached, the potential difference across C is the same as the potential difference across R_2 because there is no change in potentia across R_3. Therefore, the charge on the capacitor is

$$Q = C(\Delta V)_{R_2}$$

$$= C(IR_2) = (10.0\ \mu\text{F})(333\times10^{-6}\ \text{A})(15.0\times10^{3}\ \Omega) = \boxed{50.0\ \mu\text{C}}$$

18.51 Applying Kirchhoff's junction rule at junction *a* gives



$$I_2 = I_1 + I_3 \qquad (1)$$

Applying Kirchhoff's loop rule on the leftmost loop yields

$$+9.0\ \text{V} - (5.0)I_1 - 4.0\ \text{V} - (10)I_2 = 0$$

or $$I_1 + 2I_2 = 1.0\ \text{A} \qquad (2)$$

For the rightmost loop,

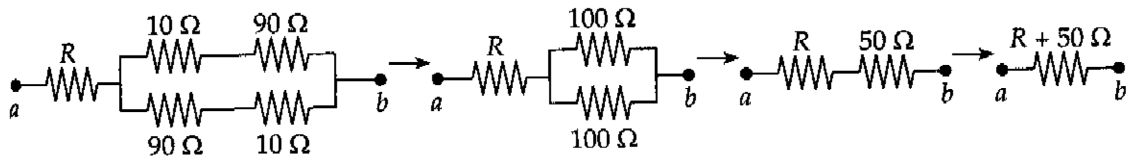
$$+(10)I_2 + 4.0\ \text{V} + (10)I_3 - 14\ \text{V} = 0$$

or $$I_2 + I_3 = 1.0\ \text{A} \qquad (3)$$

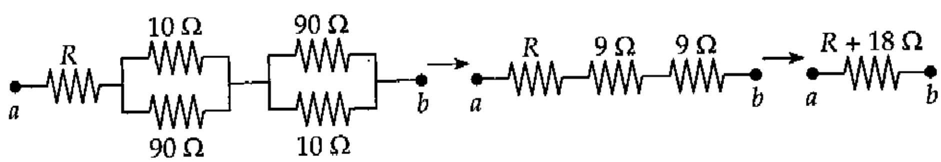
Solving equations (1), (2) and (3) simultaneously gives

$$\boxed{I_1 = 0,\ I_2 = I_3 = 0.50\ \text{A}}$$

With the switch open, the circuit may be reduced as follows:



With the switch closed, the circuit reduces as shown below:



Since the equivalent resistance with the switch closed is one-half that when the switch is open, we have

$$R+18\ \Omega=\frac{1}{2}(R+50\ \Omega)\text{, which yields } R=\boxed{14\ \Omega}$$

18.53 When a generator with emf $\mathcal{E}$ and internal resistance r supplies current I, its terminal voltage is $\Delta V=\mathcal{E}-Ir$.

If $\Delta V=110\text{ V}$ when $I=10.0\text{ A}$, then $\qquad 110\text{ V}=\mathcal{E}-(10.0\text{ A})r \qquad (1)$

Given that $\Delta V=106\text{ V}$ when $I=30.0\text{ A}$, yields $\qquad 106\text{ V}=\mathcal{E}-(30.0\text{ A})r \qquad (2)$

Solving equations (1) and (2) simultaneously gives

$$\boxed{\mathcal{E}=112\text{ V} \quad \text{and} \quad r=0.200\ \Omega}$$

18.54 At time t, the charge on the capacitor will be $Q=Q_{max}\left(1-e^{-t/\tau}\right)$ where

$$\tau=RC=\left(2.0\times10^{6}\ \Omega\right)\left(3.0\times10^{-6}\text{ F}\right)=6.0\text{ s}$$

When $Q=0.90Q_{max}$, this gives $0.90=1-e^{-t/\tau}$

or $\qquad e^{-t/\tau}=0.10 \qquad$ Thus, $-\dfrac{t}{\tau}=\ln(0.10)$

giving $t=-(6.0\text{ s})\ln(0.10)=\boxed{14\text{ s}}$

18.55 (a) For the first measurement, the equivalent circuit is as shown in Figure 1. From this,

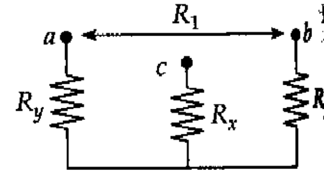


Figure 1

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

$$\text{so } R_y = \frac{1}{2}R_1 \qquad (1)$$

For the second measurement, the equivalent circuit is shown in Figure 2. This gives

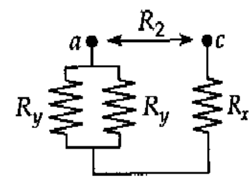


Figure 2

$$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad (2)$$

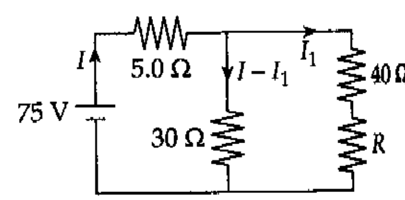
Substitute (1) into (2) to obtain

$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x \text{, or } \boxed{R_x = R_2 - \frac{1}{4}R_1}$$

(b) If $R_1 = 13\ \Omega$ and $R_2 = 6.0\ \Omega$, then $\boxed{R_x = 2.8\ \Omega}$

Since this exceeds the limit of $2.0\ \Omega$, the antenna is $\boxed{\text{inadequately grounded}}$.

18.56 Assume a set of currents as shown in the circuit diagram at the right. Applying Kirchhoff's loop rule to the leftmost loop gives



$$+75 - (5.0)I - (30)(I - I_1) = 0$$

or $\qquad 7I - 6I_1 = 15$

For the rightmost loop, the loop rule gives

$$-(40 + R)I_1 + (30)(I - I_1) = 0\text{, or } I = \left(\frac{7}{3} + \frac{R}{30}\right)I_1$$

Substituting equation (2) into (1) and simplifying gives

$$310I_1 + 7(I_1R) = 450$$

Also, it is known that $\mathcal{P}_R = I_1^2 R = 20\ \text{W}$, so $I_1R = \dfrac{20\ \text{W}}{I_1}$

Substitution of equation (4) into (3) yields

$$310 I_1 + \frac{140}{I_1} = 450 \text{ or } 310 I_1^2 - 450 I_1 + 140 = 0$$

Solving this quadratic equation gives two possible values for the current I_1. These are $I_1 = 1.0\text{ A}$ and $I_1 = 0.452\text{ A}$. Then, from $R = \frac{20\text{ W}}{I_1^2}$, we find two possible values for the resistance R. These are

$$\boxed{R = 20\ \Omega \quad \text{or} \quad R = 98\ \Omega}$$

18.57 When connected in series, the equivalent resistance is $R_{eq} = R_1 + R_2 + \cdots + R_n = nR$. Thus, the current is $I_s = (\Delta V)/R_{eq} = (\Delta V)/nR$, and the power consumed by the series configuration is

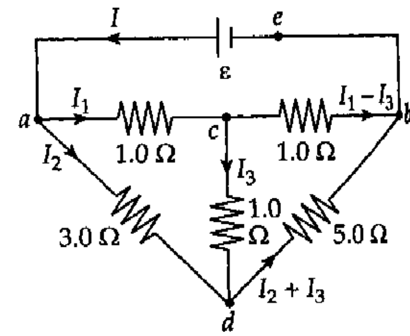
$$\mathcal{P}_s = I_s^2 R_{eq} = \frac{(\Delta V)^2}{(nR)^2}(nR) = \frac{(\Delta V)^2}{nR}$$

For the parallel connection, the power consumed by each individual resistor is $\mathcal{P}_1 = \frac{(\Delta V)^2}{R}$, and the total power consumption is

$$\mathcal{P}_p = n\mathcal{P}_1 = \frac{n(\Delta V)^2}{R}$$

Therefore, $\frac{\mathcal{P}_s}{\mathcal{P}_p} = \frac{(\Delta V)^2}{nR} \cdot \frac{R}{n(\Delta V)^2} = \frac{1}{n^2}$ or $\boxed{\mathcal{P}_s = \frac{1}{n^2}\mathcal{P}_p}$

18.58 Consider a battery of emf $\mathcal{E}$ connected between points a and b as shown. Applying Kirchhoff's loop rule to loop $acbea$ gives



$$-(1.0)I_1 - (1.0)(I_1 - I_3) + \mathcal{E} = 0$$

or $$2I_1 - I_3 = \mathcal{E} \quad (1)$$

Applying the loop rule to loop $adbea$ gives

$$-(3.0)I_2 - (5.0)(I_2 + I_3) + \mathcal{E} = 0$$

or $$8I_2 + 5I_3 = \mathcal{E} \quad (2)$$

For loop $adca$, the loop rule yields

$$-(3.0)I_2 + (1.0)I_3 + (1.0)I_1 = 0 \text{ or } I_1 + I_3 = 3I_2 \quad (3)$$

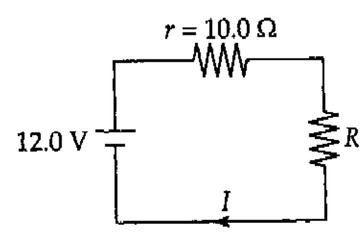
Solving equations (1), (2) and (3) simultaneously gives

$$I_1 = \frac{13}{27}\mathcal{E},\ I_2 = \frac{4}{27}\mathcal{E},\ \text{and } I_3 = -\frac{1}{27}\mathcal{E}$$

Then, applying Kirchhoff's junction rule at junction a gives

$$I = I_1 + I_2 = \frac{13}{27}\mathcal{E} + \frac{4}{27}\mathcal{E} = \frac{17}{27}\mathcal{E}\text{. Therefore, } R_{ab} = \frac{\mathcal{E}}{I} = \frac{\mathcal{E}}{(17\mathcal{E}/27)} = \boxed{\frac{27}{17}\ \Omega}$$

18.59 (a) and (b) - With R the value of the load resistor, the current in a series circuit composed of a 12.0 V battery, an internal resistance of $10.0\ \Omega$, and a load resistor is



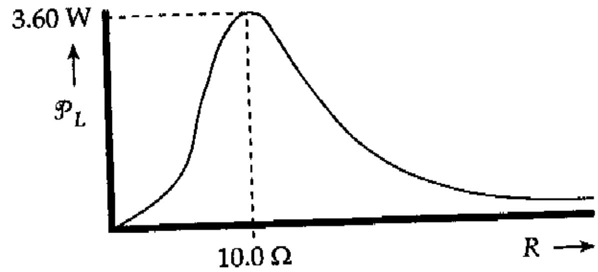
$$I = \frac{12.0\text{ V}}{R + 10.0\ \Omega}$$

and the power delivered to the load resistor is

$$\mathcal{P}_L = I^2 R = \boxed{\frac{(144\text{ V}^2)R}{(R + 10.0\ \Omega)^2}}$$

Some typical data values for the graph are

$R\ (\Omega)$	$\mathcal{P}_L$ (W)
1.00	1.19
5.00	3.20
10.0	3.60
15.0	3.46
20.0	3.20
25.0	2.94
30.0	2.70



The curve peaks at $\mathcal{P}_L = 3.60$ W at a load resistance of $R = 10.0\ \Omega$.

.60 The total resistance in the circuit is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{2.0\ \text{k}\Omega} + \frac{1}{3.0\ \text{k}\Omega}\right)^{-1} = 1.2\ \text{k}\Omega$$

and the total capacitance is $C = C_1 + C_2 = 2.0\ \mu\text{F} + 3.0\ \mu\text{F} = 5.0\ \mu\text{F}$

Thus, $Q_{max} = C\mathcal{E} = (5.0\ \mu\text{F})(120\ \text{V}) = 600\ \mu\text{C}$

and $\tau = RC = (1.2\times10^3\ \Omega)(5.0\times10^{-6}\ \text{F}) = 6.0\times10^{-3}\ \text{s} = \dfrac{6.0\ \text{s}}{1\,000}$

The total stored charge at any time t is then

$$Q = Q_1 + Q_2 = Q_{max}\left(1 - e^{-t/\tau}\right) \text{ or } Q_1 + Q_2 = (600\ \mu\text{C})\left(1 - e^{-1\,000t/6.0\ \text{s}}\right) \qquad (1)$$

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

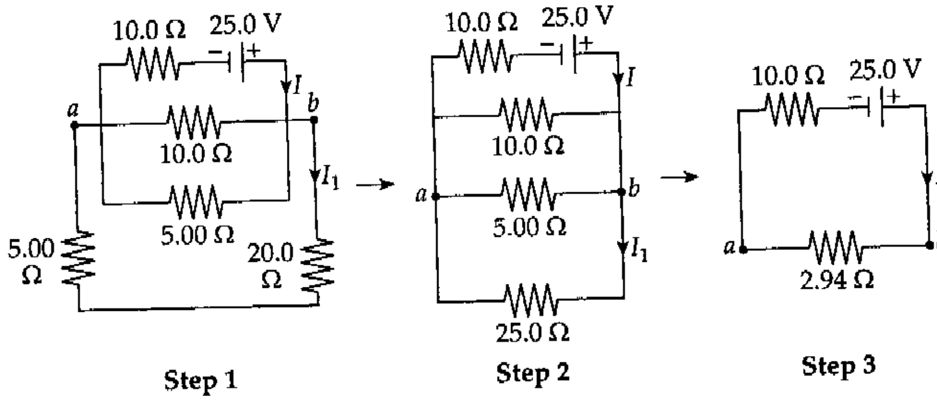
Therefore, $$(\Delta V)_C = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}, \text{ or } Q_2 = \left(\frac{C_2}{C_1}\right)Q_1 = 1.5Q_1 \qquad (2)$$

Solving equations (1) and (2) simultaneously gives

$$Q_1 = \boxed{(240\ \mu\text{C})\left(1 - e^{-1\,000t/6.0\ \text{s}}\right)}$$

and $$Q_2 = \boxed{(360\ \mu\text{C})\left(1 - e^{-1\,000t/6.0\ \text{s}}\right)}$$

18.61 (a) Using the rules for combining resistors in series and parallel, the circuit reduces as shown below:

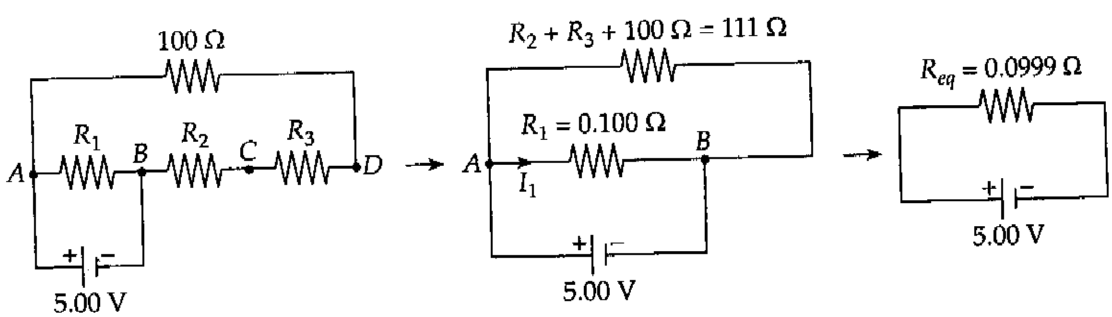


From the figure of Step 3, observe that

$$I = \frac{25.0 \text{ V}}{12.94 \text{ A}} = 1.93 \text{ A} \quad \text{and} \quad \Delta V_{ab} = I(2.94\ \Omega) = (1.93 \text{ A})(2.94\ \Omega) = \boxed{5.68 \text{ V}}$$

(b) From the figure of Step 1, observe that $I_1 = \frac{\Delta V_{ab}}{25.0\ \Omega} = \frac{5.68 \text{ V}}{25.0\ \Omega} = \boxed{0.227 \text{ A}}$

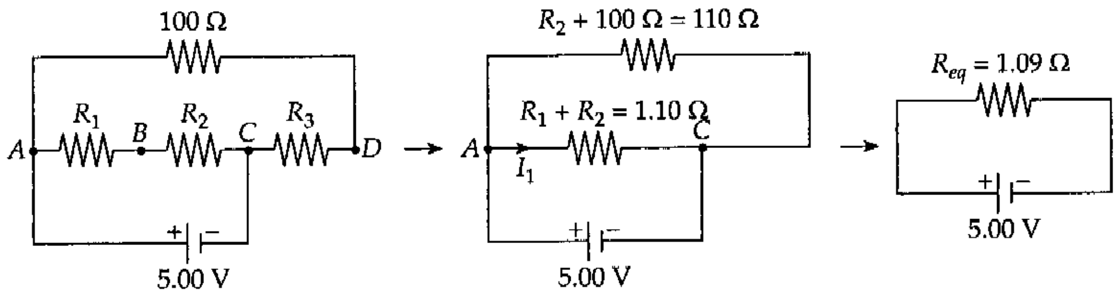
18.62 (a) When the power supply is connected to points A and B, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{0.099\,9\ \Omega}$.



From the center figure above, observe that $I_{R_1} = I_1 = \frac{5.00 \text{ V}}{0.100\ \Omega} = \boxed{50.0 \text{ A}}$

and $I_{R_2} = I_{R_3} = I_{100} = \frac{5.00 \text{ V}}{111\ \Omega} = 0.045\,0 \text{ A} = \boxed{45.0 \text{ mA}}$

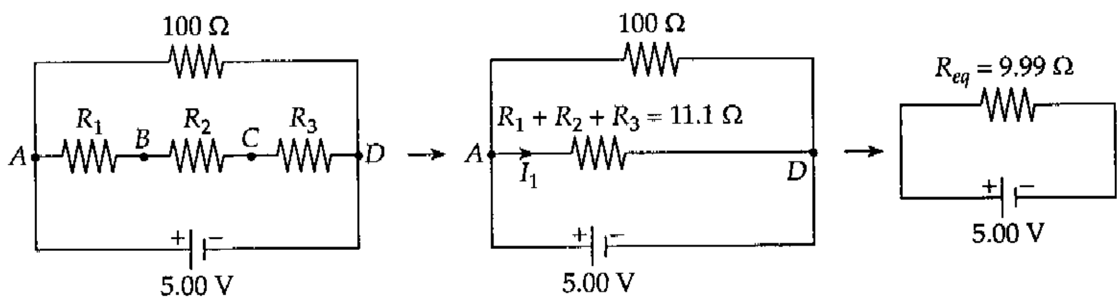
(b) When the power supply is connected to points A and C, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{1.09\ \Omega}$.



From the center figure above, observe that $I_{R_1} = I_{R_2} = I_1 = \dfrac{5.00\ \text{V}}{1.10\ \Omega} = \boxed{4.55\ \text{A}}$

and $\qquad I_{R_3} = I_{100} = \dfrac{5.00\ \text{V}}{110\ \Omega} = 0.045\,5\ \text{A} = \boxed{45.5\ \text{mA}}$

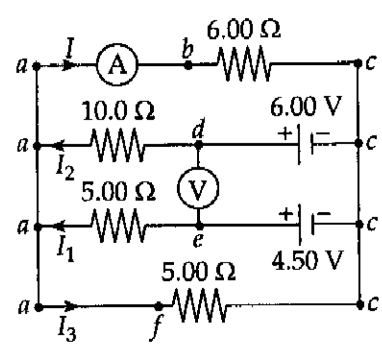
(c) When the power supply is connected to points A and D, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{9.99\ \Omega}$.



From the center figure above, observe that $I_{R_1} = I_{R_2} = I_{R_3} = I_1 = \dfrac{5.00\ \text{V}}{11.1\ \Omega} = \boxed{0.450\ \text{A}}$

and $\qquad I_{100} = \dfrac{5.00\ \text{V}}{100\ \Omega} = 0.050\,0\ \text{A} = \boxed{50.0\ \text{mA}}$

18.63 In the circuit diagram at the right, note that all points labeled a are at the same potential and equivalent to each other. Also, all points labeled c are equivalent.



To determine the voltmeter reading, go from point e to point d along the path ecd, keeping track of all changes in potential to find:

$$\Delta V_{ed} = V_d - V_e = -4.50\ \text{V} + 6.00\ \text{V} = \boxed{+1.50\ \text{V}}$$

Apply Kirchhoff's loop rule around loop *abcfa* to find

$$-(6.00\ \Omega)I+(6.00\ \Omega)I_3=0 \qquad \text{or} \qquad I_3=I$$

Apply Kirchhoff's loop rule around loop *abcda* to find

$$-(6.00\ \Omega)I+6.00\ \text{V}-(10.0\ \Omega)I_2=0 \qquad \text{or} \qquad I_2=0.600\ \text{A}-0.600I$$

Apply Kirchhoff's loop rule around loop *abcea* to find

$$-(6.00\ \Omega)I+4.50\ \text{V}-(5.00\ \Omega)I_1=0 \qquad \text{or} \qquad I_1=0.900\ \text{A}-1.20I$$

Finally, apply Kirchhoff's junction rule at either point *a* or point *c* to obtain

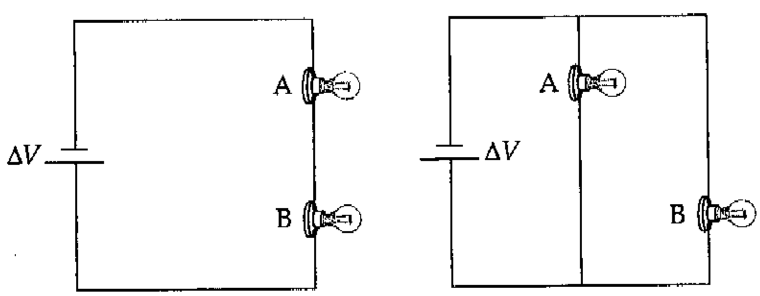
$$I+I_3=I_1+I_2$$

Substitute equations (1), (2), and (3) into equation (4) to obtain the current through the ammeter. This gives

$$I+I=0.900\ \text{A}-1.20I+0.600\ \text{A}-0.600I$$

or $3.80I=1.50\ \text{A}$ and $I=1.50\ \text{A}/3.80=\boxed{0.395\ \text{A}}$

18.64 In the figure given below, note that all bulbs have the same resistance, R.



Case 1 **Case 2**

(a) In the series situation, Case 1, the same current I_1 flows through both bulbs. Thus, the same power, $\mathcal{P}_1=I_1^2R$, is supplied to each bulb. Since the brightness of a bulb is proportional to the power supplied to it, they will have the same brightness. We conclude that the $\boxed{\text{bulbs have the same current, power supplied, and brightness}}$.

(b) In the parallel case, Case 2, the same potential difference ΔV is maintained across each of the bulbs. Thus, the $\boxed{\text{same current}}$ $I_2=\Delta V/R$ will flow in each branch of this parallel circuit. This means that, again, the $\boxed{\text{same power}}$ $\mathcal{P}_2=I_2^2R$ is supplied to each bulb, and the two bulbs will have $\boxed{\text{equal brightness}}$.

(c) The total resistance of the single branch of the series circuit (Case 1) is 2R. Thus, the current in this case is $I_1 = \Delta V/2R$. Note that this is one half of the current I_2 that flows through each bulb in the parallel circuit (Case 2). Since the power supplied is proportional to the square of the current, the power supplied to each bulb in Case 2 is four times that supplied to each bulb in Case 1. Thus, the bulbs in $\boxed{\text{Case 2}}$ are much brighter than those in Case 1.

(d) If either bulb goes out in Case 1, the only conducting path of the circuit is broken and all current ceases. Thus, in the series case, $\boxed{\text{the other bulb must also go out}}$. If one bulb goes out in Case 2, there is still a continuous conducting path through the other bulb. Neglecting any internal resistance of the battery, the battery continues to maintain the same potential difference ΔV across this bulb as was present when both bulbs were lit. Thus, in the parallel case, $\boxed{\text{the second bulb remains lit}}$ with $\boxed{\text{unchanged current and brightness}}$ when one bulb fails.