Problem Solutions

16.1 (a) The work done is $W = F \cdot s\cos\theta = (qE)\cdot s\cos\theta$, or

$$W = \left(1.60\times10^{-19}\ \text{C}\right)\left(200\ \text{N/C}\right)\left(2.00\times10^{-2}\ \text{m}\right)\cos 0^\circ = \boxed{6.40\times10^{-19}\ \text{J}}$$

(b) The change in the electrical potential energy is

$$\Delta PE_e = -W = \boxed{-6.40\times10^{-19}\ \text{J}}$$

(c) The change in the electrical potential is

$$\Delta V = \frac{\Delta PE_e}{q} = \frac{-6.40\times10^{-19}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = \boxed{-4.00\ \text{V}}$$

16.2 (a) We follow the path from (0,0) to (20 cm,0) to (20 cm,50 cm). The work done on the charge by the field is

$$W = W_1 + W_2 = (qE)\cdot s_1\cos\theta_1 + (qE)\cdot s_2\cos\theta_2$$

$$= (qE)\left[(0.20\ \text{m})\cos 0^\circ + (0.50\ \text{m})\cos 90^\circ\right]$$

$$= \left(12\times10^{-6}\ \text{C}\right)\left(250\ \text{V/m}\right)\left[(0.20\ \text{m}) + 0\right] = 6.0\times10^{-4}\ \text{J}$$

Thus, $\Delta PE_e = -W = \boxed{-6.0\times10^{-4}\ \text{J}}$

(b) $$\Delta V = \frac{\Delta PE_e}{q} = \frac{-6.0\times10^{-4}\ \text{J}}{12\times10^{-6}\ \text{C}} = -50\ \text{J/C} = \boxed{-50\ \text{V}}$$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{input} = -W_{field} = -\left(-\Delta PE_e\right) = +q(\Delta V)$$

$$= \left(1.60\times10^{-19}\ \text{C}\right)\left(+90\times10^{-3}\ \frac{\text{J}}{\text{C}}\right) = \boxed{1.4\times10^{-20}\ \text{J}}$$

16.4 $\Delta PE_e = q(\Delta V) = q\left(V_f - V_i\right)$, so $q = \dfrac{\Delta PE_e}{V_f - V_i} = \dfrac{-1.92\times10^{-17}\ \text{J}}{+60.0\ \text{J/C}} = \boxed{-3.20\times10^{-19}\ \text{C}}$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\,000 \text{ J/C}}{1.5\times10^{-2} \text{ m}} = \boxed{1.7\times10^{6} \text{ N/C}}$

16.6 Since potential difference is work per unit charge $\Delta V = \dfrac{W}{q}$, the work done is

$$W = q(\Delta V) = (3.6\times10^{5} \text{ C})(+12 \text{ J/C}) = \boxed{4.3\times10^{6} \text{ J}}$$

16.7 (a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600 \text{ J/C}}{5.33\times10^{-3} \text{ m}} = \boxed{1.13\times10^{5} \text{ N/C}}$

(b) $F = |q|E = (1.60\times10^{-19} \text{ C})(1.13\times10^{5} \text{ N/C}) = \boxed{1.80\times10^{-14} \text{ N}}$

(c) $W = F\cdot s\cos\theta$

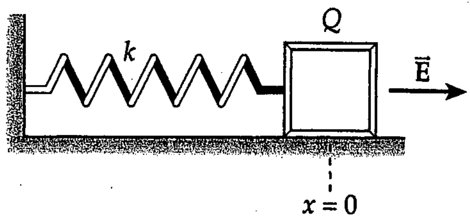
$$= (1.80\times10^{-14} \text{ N})\left[(5.33-2.90)\times10^{-3} \text{ m}\right]\cos 0° = \boxed{4.38\times10^{-17} \text{ J}}$$

16.8 From conservation of energy, $\frac{1}{2}mv_f^2 - 0 = |q(\Delta V)|$ or $v_f = \sqrt{\dfrac{2|q(\Delta V)|}{m}}$

(a) For the proton, $v_f = \sqrt{\dfrac{2|(1.60\times10^{-19} \text{ C})(-120 \text{ V})|}{1.67\times10^{-27} \text{ kg}}} = \boxed{1.52\times10^{5} \text{ m/s}}$

(b) For the electron, $v_f = \sqrt{\dfrac{2|(-1.60\times10^{-19} \text{ C})(+120 \text{ V})|}{9.11\times10^{-31} \text{ kg}}} = \boxed{6.49\times10^{6} \text{ m/s}}$

16.9 (a) Use conservation of energy



$$(KE + PE_s + PE_e)_f = (KE + PE_s + PE_e)_i$$

or $\Delta(KE) + \Delta(PE_s) + \Delta(PE_e) = 0$

$\Delta(KE) = 0$ since the block is at rest at both beginning and end.

$$\Delta(PE_s) = \frac{1}{2}kx_{\max}^2 - 0,$$

where $x_{\max}$ is the maximum stretch of the spring.

$$\Delta(PE_e) = -W = -(QE)x_{\max}$$

Thus, $0 + \frac{1}{2}kx_{\max}^2 - (QE)x_{\max} = 0$, giving

$$x_{\max} = \frac{2QE}{k} = \frac{2(50.0\times10^{-6}\text{ C})(5.00\times10^{5}\text{ V/m})}{100\text{ N/m}} = \boxed{0.500\text{ m}}$$

(b) At equilibrium, $\Sigma F = -F_s + F_e = 0$, or $-kx_{eq} + QE = 0$

Therefore, $x_{eq} = \frac{QE}{k} = \frac{1}{2}x_{max} = \boxed{0.250\text{ m}}$

Note that when the block is released from rest, it overshoots the equilibrium position and oscillates with simple harmonic motion in the electric field.

16.10 Using $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ for the full flight gives

$$0 = v_{0y}t + \frac{1}{2}a_y t^2, \text{ or } a_y = \frac{-2v_{0y}}{t}$$

Then, using $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ for the upward part of the flight gives

$$(\Delta y)_{\max} = \frac{0 - v_{0y}^2}{2a_y} = \frac{-v_{0y}^2}{2(-2v_{0y}/t)} = \frac{v_{0y}\,t}{4} = \frac{(20.1\text{ m/s})(4.10\text{ s})}{4} = 20.6\text{ m}$$

From Newton's second law, $a_y = \frac{\Sigma F_y}{m} = \frac{-mg - qE}{m} = -\left(g + \frac{qE}{m}\right)$. Equating this to the earlier result gives $a_y = -\left(g + \frac{qE}{m}\right) = \frac{-2v_{0y}}{t}$, so the electric field strength is

$$E = \left(\frac{m}{q}\right)\left[\frac{2v_{0y}}{t} - g\right] = \left(\frac{2.00 \text{ kg}}{5.00\times10^{-6} \text{ C}}\right)\left[\frac{2(20.1 \text{ m/s})}{4.10 \text{ s}} - 9.80 \text{ m/s}^2\right] = 1.95\times10^3 \text{ N/C}$$

Thus, $(\Delta V)_{max} = (\Delta y_{max})E = (20.6 \text{ m})(1.95\times10^3 \text{ N/C}) = 4.02\times10^4 \text{ V} = \boxed{40.2 \text{ kV}}$

16.11 (a) $V = \frac{k_e q}{r} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})}{1.00\times10^{-2} \text{ m}} = \boxed{1.44\times10^{-7} \text{ V}}$

(b) $\Delta V = V_2 - V_1 = \frac{k_e q}{r_2} - \frac{k_e q}{r_1} = (k_e q)\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$= (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})\left(\frac{1}{0.0200 \text{ m}} - \frac{1}{0.0100 \text{ m}}\right)$$

$$= \boxed{-7.19\times10^{-8} \text{ V}}$$

16.12 $V = V_1 + V_2 = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$ where $r_1 = 0.60 \text{ m} - 0 = 0.60 \text{ m}$, and $r_2 = 0.60 \text{ m} - 0.30 \text{ m} = 0.30 \text{ m}$. Thus,

$$V = \left(8.99\times10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{3.0\times10^{-9} \text{ C}}{0.60 \text{ m}} + \frac{6.0\times10^{-9} \text{ C}}{0.30 \text{ m}}\right) = \boxed{2.2\times10^2 \text{ V}}$$

16.13 (a) Calling the 2.00 μC charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{8.00\times10^{-6}\ \text{C}}{0.060\,0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{0.030\,0\ \text{m}} + \frac{2.00\times10^{-6}\ \text{C}}{\sqrt{(0.060\,0)^2 + (0.030\,0)^2}\ \text{m}}\right)$$

$$V = \boxed{2.67\times10^6\ \text{V}}$$

(b) Replacing 2.00×10^{-6} C by -2.00×10^{-6} C in part (a) yields

$$V = \boxed{2.13\times10^6\ \text{V}}$$

16.14 $W = q(\Delta V) = q(V_f - V_i)$, and

$V_f = 0$ since the 8.00 μC is infinite distance from other charges.

$$V_i = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.030\,0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{\sqrt{(0.030\,0)^2 + (0.060\,0)^2}\ \text{m}}\right)$$

$$= 1.135\times10^6\ \text{V}$$

Thus, $W = (8.00\times10^{-6}\ \text{C})(0 - 1.135\times10^6\ \text{V}) = \boxed{-9.08\ \text{J}}$

16.15 (a) $V = \sum_i \frac{k_e q_i}{r_i}$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.00\times10^{-9}\ \text{C}}{0.175\ \text{m}} - \frac{3.00\times10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$$

(b) $PE = \frac{k_e q_1 q_2}{r_{12}}$

$$= \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(5.00\times10^{-9}\ \text{C}\right)\left(-3.00\times10^{-9}\ \text{C}\right)}{0.350\ \text{m}} = \boxed{-3.85\times10^{-7}\ \text{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges (that is, bring them up to a state of zero potential energy).

16.16 The potential at distance $r = 0.300$ m from a charge $Q = +9.00\times10^{-9}$ C is

$$V = \frac{k_e Q}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(9.00\times10^{-9}\ \text{C}\right)}{0.300\ \text{m}} = +270\ \text{V}$$

Thus, the work required to carry a charge $q = 3.00\times10^{-9}$ C from infinity to this location is

$$W = qV = \left(3.00\times10^{-9}\ \text{C}\right)(+270\ \text{V}) = \boxed{8.09\times10^{-7}\ \text{J}}$$

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as

$$r_3 = \sqrt{(4.00\ \text{cm})^2 - (1.00\ \text{cm})^2} = \sqrt{15}\ \text{cm} = \sqrt{15}\times10^{-2}\ \text{m}$$

Then, the potential at the midpoint of the base is $V = \sum_i k_e q_i / r_i$, or

$$V = \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{\left(-7.00\times10^{-9}\ \text{C}\right)}{0.0100\ \text{m}} + \frac{\left(-7.00\times10^{-9}\ \text{C}\right)}{0.0100\ \text{m}} + \frac{\left(+7.00\times10^{-9}\ \text{C}\right)}{\sqrt{15}\times10^{-2}\ \text{m}}\right)$$

$$= -1.10\times10^{4}\ \text{V} = \boxed{-11.0\ \text{kV}}$$

16.18 Outside the spherical charge distribution, the potential is the same as for a point charge at the center of the sphere,

$$V = k_e Q/r \text{, where } Q = 1.00\times10^{-9}\ \text{C}$$

Thus, $\Delta(PE_e) = q(\Delta V) = -ek_e Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

and from conservation of energy $\Delta(KE) = -\Delta(PE_e)$,

or $\quad \frac{1}{2}m_e v^2 - 0 = -\left[-ek_e Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right]$ This gives $v = \sqrt{\frac{2k_e Qe}{m_e}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$, or

$$v = \sqrt{\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.00\times10^{-9}\ \text{C})(1.60\times10^{-19}\ \text{C})}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1}{0.020\,0\ \text{m}} - \frac{1}{0.030\,0\ \text{m}}\right)}$$

$$v = \boxed{7.25\times10^6\ \text{m/s}}$$

16.19 From conservation of energy, $(KE + PE_e)_f = (KE + PE_e)_i$, which gives

$$0 + \frac{k_e Qq}{r_f} = \frac{1}{2}m_\alpha v_i^2 + 0 \text{ or } r_f = \frac{2k_e Qq}{m_\alpha v_i^2} = \frac{2k_e(79e)(2e)}{m_\alpha v_i^2}$$

$$r_f = \frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(158)(1.60\times10^{-19}\ \text{C})^2}{(6.64\times10^{-27}\ \text{kg})(2.00\times10^7\ \text{m/s})^2} = \boxed{2.74\times10^{-14}\ \text{m}}$$

16.20 By definition, the work required to move a charge from one point to any other point on an equipotential surface is zero. From the definition of work, $W = (F\cos\theta)\cdot s$, the work is zero only if $s = 0$ or $F\cos\theta = 0$. The displacement s cannot be assumed to be zero in all cases. Thus, one must require that $F\cos\theta = 0$. The force F is given by $F = qE$ and neither the charge q nor the field strength E can be assumed to be zero in all cases. Therefore, the only way the work can be zero in all cases is if $\cos\theta = 0$. But if $\cos\theta = 0$, then $\theta = 90°$ or the force (and hence the electric field) must be perpendicular to the displacement s (which is tangent to the surface). That is, the field must be perpendicular to the equipotential surface at all points on that surface.

16.21 $V = \frac{k_e Q}{r}$ so

$$r = \frac{k_e Q}{V} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-9}\ \text{C})}{V} = \frac{71.9\ \text{V}\cdot\text{m}}{V}$$

For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.719\ \text{m},\ 1.44\ \text{m, and } 2.88\ \text{m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

16.22 (a) $Q = C(\Delta V) = (4.00\times10^{-6}\ \text{F})(12.0\ \text{V}) = 48.0\times10^{-6}\ \text{C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00\times10^{-6}\ \text{F})(1.50\ \text{V}) = 6.00\times10^{-6}\ \text{C} = \boxed{6.00\ \mu\text{C}}$

16.23 (a) $C = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{(1.0\times10^6\ \text{m}^2)}{(800\ \text{m})} = \boxed{1.1\times10^{-8}\ \text{F}}$

(b) $Q_{max} = C(\Delta V)_{max} = C(E_{max}d)$

$$= (1.11\times10^{-8}\ \text{F})(3.0\times10^6\ \text{N/C})(800\ \text{m}) = \boxed{27\ \text{C}}$$

16.24 For a parallel plate capacitor, $\Delta V = \frac{Q}{C} = \frac{Q}{\epsilon_0 (A/d)} = \frac{Qd}{\epsilon_0 A}$.

(a) Doubling d while holding Q and A constant doubles ΔV to $\boxed{800\ \text{V}}$.

(b) $Q = \frac{(\epsilon_0 A)\Delta V}{d}$ Thus, doubling d while holding ΔV and A constant will cut the charge in half, or $\boxed{Q_f = Q_i/2}$

16.25 (a) $E = \frac{\Delta V}{d} = \frac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = 1.11\times10^4\ \text{V/m} = \boxed{11.1\ \text{kV/m}}$ directed toward the negative plate

(b) $C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.60\times10^{-4}\ \text{m}^2)}{1.80\times10^{-3}\ \text{m}}$

$$= 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$$

(c) $Q = C(\Delta V) = (3.74\times10^{-12}\ \text{F})(20.0\ \text{V}) = 7.47\times10^{-11}\ \text{C} = \boxed{74.7\ \text{pC}}$ on one plate and $\boxed{-74.7\ \text{pC}}$ on the other plate.

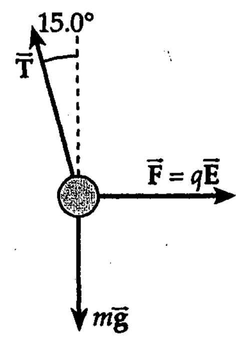
16.26 $C = \dfrac{\epsilon_0 A}{d}$, so $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(21.0\times10^{-12}\ \text{m}^2)}{60.0\times10^{-15}\ \text{F}} = 3.10\times10^{-9}\ \text{m}$

$$d = (3.10\times10^{-9}\ \text{m})\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = \boxed{31.0\ \text{Å}}$$

16.27 (a) $\Delta V = \dfrac{Q}{C} = \dfrac{Q}{\epsilon_0 A/d} = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(400\times10^{-12}\ \text{C})(1.00\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = \boxed{90.4\ \text{V}}$

(b) $E = \dfrac{|\Delta V|}{d} = \dfrac{90.4\ \text{V}}{1.00\times10^{-3}\ \text{m}} = \boxed{9.04\times10^4\ \text{V/m}}$

16.28 $\Sigma F_y = 0 \Rightarrow T\cos15.0° = mg$ or $T = \dfrac{mg}{\cos15.0°}$



$\Sigma F_x = 0 \Rightarrow qE = T\sin15.0° = mg\tan15.0°$

or $E = \dfrac{mg\tan15.0°}{q}$

$$\Delta V = Ed = \frac{mgd\tan15.0°}{q}$$

$$\Delta V = \frac{(350\times10^{-6}\ \text{kg})(9.80\ \text{m/s}^2)(0.040\,0\ \text{m})\tan15.0°}{30.0\times10^{-9}\ \text{C}} = 1.23\times10^3\ \text{V} = \boxed{1.23\ \text{kV}}$$

16.29 (a) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$

$$Q = C_{eq}(\Delta V) = \left(\frac{C_1C_2}{C_1+C_2}\right)\Delta V$$

$$= \left[\frac{(0.050\ \mu\text{F})(0.100\ \mu\text{F})}{0.050\ \mu\text{F}+0.100\ \mu\text{F}}\right](400\ \text{V}) = \boxed{13.3\ \mu\text{C on each}}$$

(b) $Q_1 = C_1(\Delta V) = (0.050\ \mu\text{F})(400\ \text{V}) = \boxed{20.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V) = (0.100\ \mu\text{F})(400\ \text{V}) = \boxed{40.0\ \mu\text{C}}$

16.30 (a) For parallel connection,

$$C_{eq} = C_1 + C_2 + C_3 = (5.00 + 4.00 + 9.00)\ \mu\text{F} = \boxed{18.0\ \mu\text{F}}$$

(b) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$\frac{1}{C_{eq}} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} + \frac{1}{9.00\ \mu\text{F}},\ \text{giving}\ C_{eq} = \boxed{1.78\ \mu\text{F}}$$

16.31 (a) Using the rules for combining capacitors in series and in parallel, the circuit is reduced in steps as shown below. The equivalent capacitor is shown to be a $\boxed{2.00\ \mu\text{F}}$ capacitor.

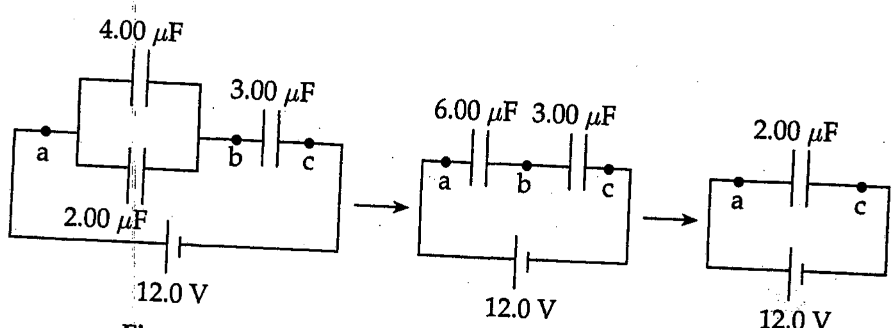


Figure 1 **Figure 2** **Figure 3**

(b) From Figure 3: $Q_{ac} = C_{ac}(\Delta V)_{ac} = (2.00\ \mu\text{F})(12.0\ \text{V}) = 24.0\ \mu\text{C}$

From Figure 2: $Q_{ab} = Q_{bc} = Q_{ac} = 24.0\ \mu\text{C}$

Thus, the charge on the $3.00\ \mu\text{F}$ capacitor is $Q_3 = \boxed{24.0\ \mu\text{C}}$

Continuing to use Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ab}}{C_{ab}} = \dfrac{24.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 4.00\ \text{V}$

and $(\Delta V)_3 = (\Delta V)_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{8.00\ \text{V}}$

From Figure 1, $(\Delta V)_4 = (\Delta V)_2 = (\Delta V)_{ab} = \boxed{4.00\ \text{V}}$

and $Q_4 = C_4(\Delta V)_4 = (4.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V)_2 = (2.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{8.00\ \mu\text{C}}$

16.32 $C_{parallel} = C_1 + C_2 = 9.00\ \text{pF} \Rightarrow C_1 = 9.00\ \text{pF} - C_2$ (1)

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\ \text{pF}$$

Thus, using equation (1), $C_{series} = \dfrac{(9.00\ \text{pF} - C_2)C_2}{(9.00\ \text{pF} - C_2) + C_2} = 2.00\ \text{pF}$ which reduces to

$$C_2^2 - (9.00\ \text{pF})C_2 + 18.0\ (\text{pF})^2 = 0,\ \text{or}\ (C_2 - 6.00\ \text{pF})(C_2 - 3.00\ \text{pF}) = 0$$

Therefore, either $C_2 = 6.00\ \text{pF}$ and, from equation (1), $C_1 = 3.00\ \text{pF}$

or $C_2 = 3.00\ \text{pF}$ and $C_1 = 6.00\ \text{pF}$.

We conclude that the two capacitances are $\boxed{3.00\ \text{pF and } 6.00\ \text{pF}}$.

16.33

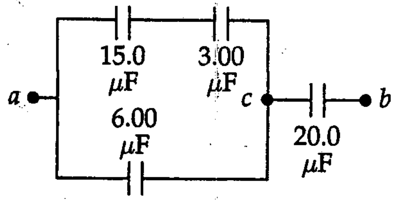


Figure 1

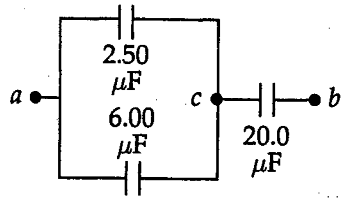


Figure 2

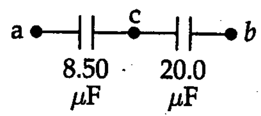


Figure 3

(a) The equivalent capacitance of the upper branch between points a and c in Figure 1 is

$$C_s = \frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F} + 3.00\ \mu\text{F}} = 2.50\ \mu\text{F}$$

Then, using Figure 2, the total capacitance between points a and c is

$$C_{ac} = 2.50\ \mu\text{F} + 6.00\ \mu\text{F} = 8.50\ \mu\text{F}$$

From Figure 3, the total capacitance is

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $$Q_{ab} = Q_{ac} = Q_{cb} = (\Delta V)_{ab} C_{eq}$$

$$= (15.0\ \text{V})(5.96\ \mu\text{F}) = 89.5\ \mu\text{C}$$

Thus, the charge on the $20.0\ \mu\text{C}$ is $Q_{20} = Q_{cb} = \boxed{89.5\ \mu\text{C}}$

$$(\Delta V)_{ac} = (\Delta V)_{ab} - (\Delta V)_{bc} = 15.0\ \text{V} - \left(\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}}\right) = 10.53\ \text{V}$$

Then, $Q_6 = (\Delta V)_{ac}(6.00\ \mu F) = \boxed{63.2\ \mu\text{C}}$ and

$$Q_{15} = Q_3 = (\Delta V)_{ac}(2.50\ \mu F) = \boxed{26.3\ \mu\text{C}}$$

16.34 (a) The combination reduces to an equivalent capacitance of $\boxed{12.0\ \mu\text{F}}$ in stages as shown below.

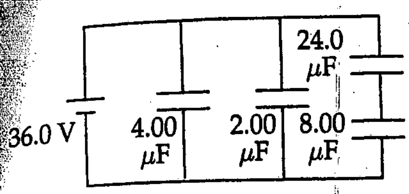


Figure 1

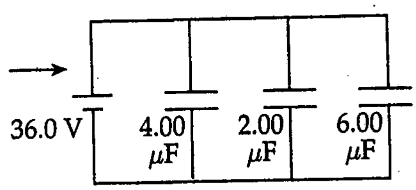


Figure 2

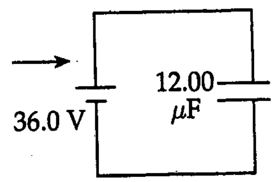


Figure 3

(b) From Figure 2, $Q_4 = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$

$Q_2 = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72.0\ \mu\text{C}}$

and $Q_6 = (6.00\ \mu\text{F})(36.0\ \text{V}) = 216\ \mu\text{C}$

Then, from Figure 1, $Q_{24} = Q_8 = Q_6 = \boxed{216\ \mu\text{C}}$

16.35

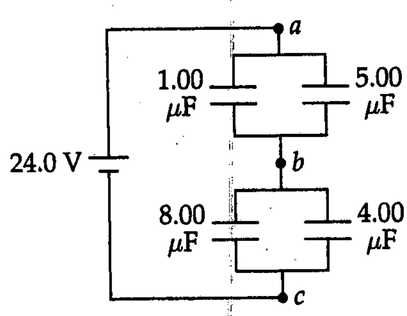


Figure 1

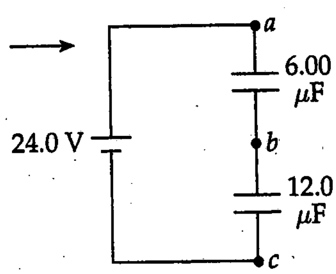


Figure 2

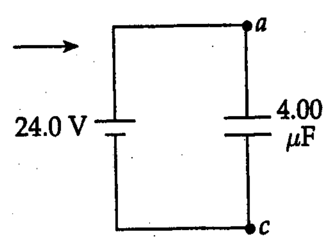


Figure 3

The circuit may be reduced in steps as shown above.

Using the Figure 3, $Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

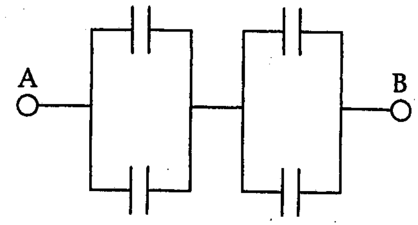
and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\ \text{V} - 16.0\ \text{V} = 8.00\ \text{V}$

Finally, using Figure 1, $Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}$, $Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}}$

and $Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$

16.36 The technician combines two of the capacitors in parallel making a capacitor of capacitance $200\ \mu\text{F}$. Then she does it again with two more of the capacitors. Then the two resulting $200\ \mu\text{F}$ capacitors are connected in series to yield an equivalent capacitance of $100\ \mu\text{F}$. Because of the symmetry of the solution, every capacitor in the combination has the same voltage across it,



$$\Delta V = (\Delta V)_{ab}/2 = (90.0\ \text{V})/2 = \boxed{45.0\ \text{V}}$$

16.37 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25\times10^3\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00\times10^3\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) When the two capacitors are connected in parallel, the equivalent capacitance is $C_{eq} = C_1 + C_2 = 25.0\ \mu\text{F} + 40.0\ \mu\text{F} = 65.0\ \mu\text{F}$.

Since the negative plate of one was connected to the positive plate of the other, the total charge stored in the parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00\times10^3\ \mu\text{C} - 1.25\times10^3\ \mu\text{C} = 750\ \mu\text{C}$$

The potential difference across each capacitor of the parallel combination is

$$\Delta V = \frac{Q}{C_{eq}} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

and the final charge stored in each capacitor is

$$Q'_{25} = C_1(\Delta V) = (25.0\ \mu\text{F})(11.5\ \text{V}) = \boxed{288\ \mu\text{C}}$$

and $Q'_{40} = Q - Q'_{25} = 750\ \mu\text{C} - 288\ \mu\text{C} = \boxed{462\ \mu\text{C}}$

16.38 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_{10} = (10.0\ \mu\text{F})(12.0\ \text{V}) = 120\ \mu\text{C} \text{ and } Q_x = C_x(0) = 0$$

After the capacitors are connected in parallel, the potential difference across each is $\Delta V' = 3.00\ \text{V}$, and the total charge of $Q = Q_{10} + Q_x = 120\ \mu\text{C}$ is divided between the two capacitors as

$$Q'_{10} = (10.0\ \mu\text{F})(3.00\ \text{V}) = 30.0\ \mu\text{C} \text{ and}$$

$$Q'_x = Q - Q'_{10} = 120\ \mu\text{C} - 30.0\ \mu\text{C} = 90.0\ \mu\text{C}$$

Thus, $C_x = \dfrac{Q'_x}{\Delta V'} = \dfrac{90.0\ \mu\text{C}}{3.00\ \text{V}} = \boxed{30.0\ \mu\text{F}}$

16.39 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_1 = (1.00\ \mu\text{F})(10.0\ \text{V}) = 10.0\ \mu\text{C} \text{ and } Q_2 = (2.00\ \mu\text{F})(0) = 0$$

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

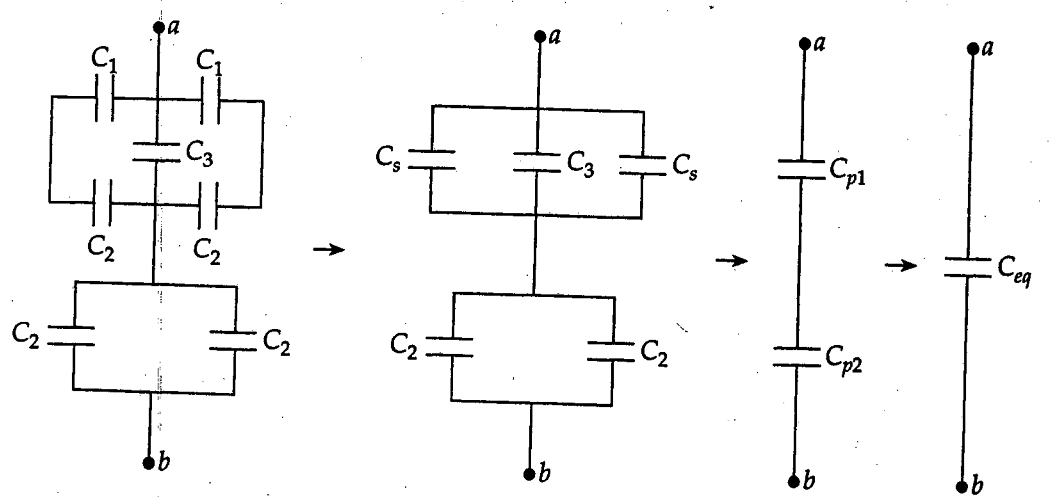
$$\Delta V = \frac{Q'_1}{1.00\ \mu\text{F}} = \frac{Q'_2}{2.00\ \mu\text{F}} \text{ or } Q'_2 = 2Q'_1 \qquad (1)$$

From conservation of charge, $Q'_1 + Q'_2 = Q_1 + Q_2 = 10.0\ \mu\text{C}$. Then, substituting from equation (1), this becomes

$Q'_1 + 2Q'_1 = 10.0\ \mu\text{C}$, giving $\qquad Q'_1 = \boxed{\dfrac{10}{3}\ \mu\text{C}}$

Finally, from equation (1), $\qquad Q'_2 = \boxed{\dfrac{20}{3}\ \mu\text{C}}$

16.40 The original circuit reduces to a single equivalent capacitor in the steps shown below.



$$C_s = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}}+\frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\ \mu\text{F}) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2(10.0\ \mu\text{F}) = 20.0\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{C_{p1}}+\frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}}+\frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

16.41 Refer to the solution of Problem 16.40 given above. The total charge stored between points a and b is

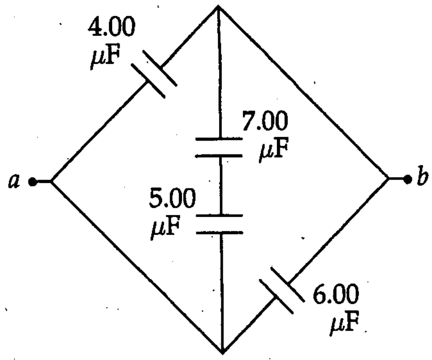
$$Q_{eq} = C_{eq}(\Delta V)_{ab} = (6.04\ \mu\text{F})(60.0\ \text{V}) = 362\ \mu\text{C}$$

Then, looking at the third figure, observe that the charges of the series capacitors of that figure are $Q_{p1} = Q_{p2} = Q_{eq} = 362\ \mu\text{C}$. Thus, the potential difference across the upper parallel combination shown in the second figure is

$$(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362\ \mu\text{C}}{8.66\ \mu\text{F}} = 41.8\ \text{V}$$

Finally, the charge on C_3 is

$$Q_3 = C_3(\Delta V)_{p1} = (2.00\ \mu\text{F})(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$



16.42 Recognize that the 7.00 μF and the 5.00 μF of the center branch are connected in series. The total capacitance of that branch is

$$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

Then recognize that this capacitor, the 4.00 μF capacitor, and the 6.00 μF capacitor are all connected in parallel between points *a* and *b*. Thus, the equivalent capacitance between points *a* and *b* is

$$C_{eq} = 4.00\ \mu\text{F} + 2.92\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.9\ \mu\text{F}}$$

16.43 The capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^{-4}\ \text{m}^2)}{5.00\times10^{-3}\ \text{m}} = 3.54\times10^{-13}\ \text{F}$$

and the stored energy is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.54\times10^{-13}\ \text{F})(12.0\ \text{V})^2 = \boxed{2.55\times10^{-11}\ \text{J}}$$

16.44 (a) When connected in parallel, the energy stored is

$$W = \frac{1}{2}C_1(\Delta V)^2 + \frac{1}{2}C_2(\Delta V)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V)^2$$

$$= \frac{1}{2}\left[(25.0 + 5.00)\times10^{-6}\ \text{F}\right](100\ \text{V})^2 = \boxed{0.150\ \text{J}}$$

(b) When connected in series, the equivalent capacitance is

$$C_{eq} = \left(\frac{1}{25.0} + \frac{1}{5.00}\right)^{-1}\ \mu\text{F} = 4.17\ \mu\text{F}$$

From $W = \frac{1}{2}C_{eq}(\Delta V)^2$, the potential difference required to store the same energy as in part (a) above is

$$\Delta V = \sqrt{\frac{2W}{C_{eq}}} = \sqrt{\frac{2(0.150\ \text{J})}{4.17\times10^{-6}\ \text{F}}} = \boxed{268\ \text{V}}$$

16.45 The capacitance of this parallel plate capacitor is

$$C = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}\right)\frac{\left(1.0\times10^6\ m^2\right)}{(800\ m)} = 1.1\times10^{-8}\ F$$

With an electric field strength of $E = 3.0\times10^6$ N/C and a plate separation of $d = 800$ m, the potential difference between plates is

$$\Delta V = Ed = \left(3.0\times10^6\ V/m\right)(800\ m) = 2.4\times10^9\ V$$

Thus, the energy available for release in a lightning strike is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(1.1\times10^{-8}\ F\right)\left(2.4\times10^9\ V\right)^2 = \boxed{3.2\times10^{10}\ J}$$

16.46 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ C)\left(1.00\times10^8\ V\right)}{200} = 2.50\times10^7\ J$$

Thus, if m is the mass of water boiled away,

$W = m\left[c(\Delta T) + L_v\right]$ becomes

$$2.50\times10^7\ J = m\left[\left(4186\ \frac{J}{kg\cdot{}^\circ C}\right)(100^\circ C - 30.0^\circ C) + 2.26\times10^6\ J/kg\right]$$

giving $m = \dfrac{2.50\times10^7\ J}{2.55\ J/kg} = \boxed{9.79\ kg}$

16.47 The initial capacitance (with air between the plates) is $C_i = Q/(\Delta V)_i$, and the final capacitance (with the dielectric inserted) is $C_f = Q/(\Delta V)_f$ where Q is the constant quantity of charge stored on the plates.

Thus, the dielectric constant is $\kappa = \dfrac{C_f}{C_i} = \dfrac{(\Delta V)_i}{(\Delta V)_f} = \dfrac{100\ V}{25\ V} = \boxed{4.0}$

16.48 (a) $E = \dfrac{\Delta V}{d} = \dfrac{6.00\ V}{2.00\times10^{-3}\ m} = \boxed{3.00\times10^3\ V/m}$

(b) With air between the plates, the capacitance is

$$C_{air} = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}\right)\frac{(2.00\times10^{-4}\ m^2)}{(2.00\times10^{-3}\ m)} = 8.85\times10^{-13}\ F$$

and with water $(\kappa = 80)$ between the plates, the capacitance is

$$C = \kappa C_{air} = (80)(8.85\times10^{-13}\ F) = 7.08\times10^{-11}\ F$$

The stored charge when water is between the plates is

$$Q = C(\Delta V) = (7.08\times10^{-11}\ F)(6.00\ V) = 4.25\times10^{-10}\ C = \boxed{42.5\ nC}$$

(c) When air is the dielectric between the plates, the stored charge is

$$Q_{air} = C_{air}(\Delta V) = (8.85\times10^{-13}\ F)(6.00\ V) = 5.31\times10^{-12}\ C = \boxed{5.31\ pC}$$

16.49 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{(2.1)(8.85\times10^{-12}\ C^2/N\cdot m^2)(175\times10^{-4}\ m^2)}{0.040\,0\times10^{-3}\ m}$$

$$C = 8.13\times10^{-9}\ F = \boxed{8.13\ nF}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60.0\times10^6\ V/m$, so the maximum voltage is

$$V_{max} = E_{max}d = (60.0\times10^6\ V/m)(0.040\,0\times10^{-3}\ m)$$

$$V_{max} = 2.40\times10^3\ V = \boxed{2.40\ kV}$$

16.50 Before the capacitor is rolled, the capacitance of this parallel plate capacitor is

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{\kappa\epsilon_0 (w\times L)}{d}$$

where A is the surface area of one side of a foil strip. Thus, the required length is

$$L = \frac{C\cdot d}{\kappa\epsilon_0 w} = \frac{(9.50\times10^{-8}\ F)(0.025\,0\times10^{-3}\ m)}{(3.70)(8.85\times10^{-12}\ C^2/N\cdot m^2)(7.00\times10^{-2}\ m)} = \boxed{1.04\ m}$$

16.51 (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = \dfrac{4\pi r^3}{3}$, the radius is $r = \left[\dfrac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \epsilon_0 A}{d}$

$$= \frac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^5}$$

16.52 Since the capacitors are in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 = \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d} + \frac{\epsilon_0 A_3}{d} = \frac{\epsilon_0 (A_1 + A_2 + A_3)}{d}$$

or $\quad C_{eq} = \boxed{\dfrac{\epsilon_0 A}{d} \text{ where } A = A_1 + A_2 + A_3}$

16.53 Since the capacitors are in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\epsilon_0 A} + \frac{d_2}{\epsilon_0 A} + \frac{d_3}{\epsilon_0 A} = \frac{d_1 + d_2 + d_3}{\epsilon_0 A}$$

or $\quad C_{eq} = \boxed{\dfrac{\epsilon_0 A}{d} \text{ where } d = d_1 + d_2 + d_3}$

16.54 For the parallel combination: $C_p = C_1 + C_2$ which gives $\qquad C_2 = C_p - C_1 \qquad (1)$

For the series combination: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ or $\dfrac{1}{C_2} = \dfrac{1}{C_s} - \dfrac{1}{C_1} = \dfrac{C_1 - C_s}{C_s C_1}$

Thus, we have $\quad C_2 = \dfrac{C_s C_1}{C_1 - C_s}$ and equating this to Equation (1) above gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \quad \text{or} \quad C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

We write this result as: $\qquad C_1^2 - C_p C_1 + C_p C_s = 0$

and use the quadratic formula to obtain $\qquad \boxed{C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

Then, Equation (1) gives $\qquad \boxed{C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

16.55 The charge stored on the capacitor by the battery is

$$Q = C(\Delta V)_1 = C(100\ \text{V})$$

This is also the total charge stored in the parallel combination when this charged capacitor is connected in parallel with an uncharged 10.0-μF capacitor. Thus, if $(\Delta V)_2$ is the resulting voltage across the parallel combination, $Q = C_p(\Delta V)_2$ gives

$$C(100\ \text{V}) = (C + 10.0\ \mu\text{F})(30.0\ \text{V}) \quad \text{or} \quad (70.0\ \text{V})C = (30.0\ \text{V})(10.0\ \mu\text{F})$$

and $\qquad C = \left(\dfrac{30.0\ \text{V}}{70.0\ \text{V}}\right)(10.0\ \mu\text{F}) = \boxed{4.29\ \mu\text{F}}$

16.56 (a) The 1.0-μC is located 0.50 m from point P, so its contribution to the potential at P is

$$V_1 = k_e\frac{q_1}{r_1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1.0\times10^{-6}\ \text{C}}{0.50\ \text{m}}\right) = \boxed{1.8\times10^4\ \text{V}}$$

(b) The potential at P due to the -2.0-μC charge located 0.50 m away is

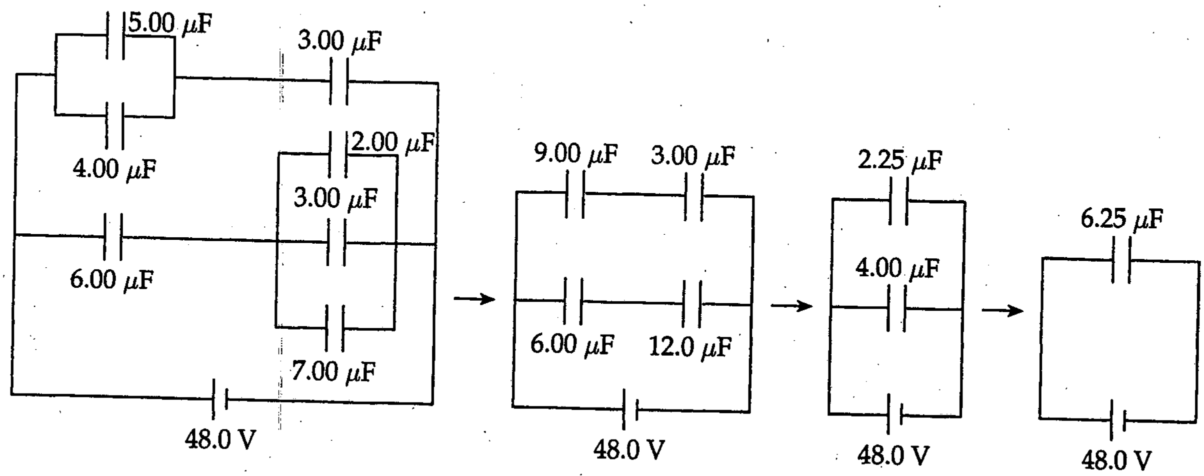
$$V_2 = k_e\frac{q_2}{r_2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-2.0\times10^{-6}\ \text{C}}{0.50\ \text{m}}\right) = \boxed{-3.6\times10^4\ \text{V}}$$

(c) The total potential at point P is $V_P = V_1 + V_2 = (+1.8 - 3.6)\times 10^4\ \text{V} = \boxed{-1.8\times 10^4\ \text{V}}$

(d) The work required to move a charge $q = 3.0\ \mu\text{C}$ to point P from infinity is

$$W = q\Delta V = q(V_P - V_\infty) = (3.0\times 10^{-6}\ \text{C})(-1.8\times 10^4\ \text{V} - 0) = \boxed{-5.4\times 10^{-2}\ \text{J}}$$

16.57 The stages for the reduction of this circuit are shown below.



Thus, $C_{eq} = \boxed{6.25\ \mu\text{F}}$

16.58 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by $V = \dfrac{k_e q}{r}$ where r is the distance from the center of that surface and q is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge $+Q$ and the outer shell having radius b and charge $-Q$, the total electric potential is given by

$$V = V_{\substack{\text{due to}\\ \text{inner shell}}} + V_{\substack{\text{due to}\\ \text{outer shell}}} = \frac{k_e Q}{r} + \frac{k_e(-Q)}{b} = k_e Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

The potential difference between the two shells is therefore,

$$\Delta V = V\big|_{r=a} - V\big|_{r=b} = k_e Q\left(\frac{1}{a}-\frac{1}{b}\right) - k_e Q\left(\frac{1}{b}-\frac{1}{b}\right) = k_e Q\left(\frac{b-a}{ab}\right)$$

The capacitance of this device is given by

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

(b) When $b >> a$, then $b - a \approx b$. Thus, in the limit as $b \to \infty$, the capacitance found above becomes

$$C \to \frac{ab}{k_e(b)} = \frac{a}{k_e} = \boxed{4\pi \epsilon_0 \, a}$$

16.59 The energy stored in a charged capacitor is $W = \frac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(300\text{ J})}{30.0\times 10^{-6}\text{ F}}} = 4.47\times 10^3\text{ V} = \boxed{4.47\text{ kV}}$$

16.60 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\Delta V}\right)\left(\frac{\Delta V}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$

16.61 The charges initially stored on the capacitors are

$$Q_1 = C_1(\Delta V)_i = (6.0\ \mu\text{F})(250\ \text{V}) = 1.5\times10^3\ \mu\text{C}$$

and $$Q_2 = C_2(\Delta V)_i = (2.0\ \mu\text{F})(250\ \text{V}) = 5.0\times10^2\ \mu\text{C}$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive plate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5\times10^3\ \mu\text{C} - 5.0\times10^2\ \mu\text{C} = 1.0\times10^3\ \mu\text{C}$$

The equivalent capacitance of the parallel combination is $C_{eq} = C_1 + C_2 = 8.0\ \mu\text{F}$. Thus, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q}{C_{eq}} = \frac{1.0\times10^3\ \mu\text{C}}{8.0\ \mu\text{F}} = 125\ \text{V}$$

and the final charge on each capacitor is

$$Q_1' = C_1(\Delta V)' = (6.0\ \mu\text{F})(125\ \text{V}) = 750\ \mu\text{C} = \boxed{0.75\ \text{mC}}$$

and $$Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(125\ \text{V}) = 250\ \mu\text{C} = \boxed{0.25\ \text{mC}}$$

16.62 When connected in series, the equivalent capacitance is

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{4.0\ \mu\text{F}} + \frac{1}{2.0\ \mu\text{F}}\right)^{-1} = \frac{4}{3}\ \mu\text{F}$$

and the charge stored on each capacitor is

$$Q_1 = Q_2 = Q_{eq} = C_{eq}(\Delta V)_i = \left(\frac{4}{3}\ \mu\text{F}\right)(100\ \text{V}) = \frac{400}{3}\ \mu\text{C}$$

When the capacitors are reconnected in parallel, with the positive plate of one connected to the positive plate of the other, the new equivalent capacitance is $C'_{eq} = C_1 + C_2 = 6.0\ \mu\text{F}$ and the net stored charge is $Q' = Q_1 + Q_2 = 800/3\ \mu\text{C}$. Therefore, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q'}{C'_{eq}} = \frac{800/3\ \mu\text{C}}{6.0\ \mu\text{F}} = 44.4\ \text{V}$$

The final charge on each of the capacitors is

$$Q_1' = C_1(\Delta V)' = (4.0\ \mu\text{F})(44.4\ \text{V}) = \boxed{1.8\times10^2\ \mu\text{C}}$$

and $\quad Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(44.4\ \text{V}) = \boxed{89\ \mu\text{C}}$

16.63 (a) $V = V_1 + V_2 + V_3 = \dfrac{k_eQ}{x+d} - \dfrac{2k_eQ}{x} + \dfrac{k_eQ}{x-d}$

$$= k_eQ\left[\frac{x(x-d) - 2(x^2-d^2) + x(x+d)}{x(x^2-d^2)}\right]$$

which simplifies to $V = \dfrac{2k_eQd^2}{x(x^2-d^2)} = \boxed{\dfrac{2k_eQd^2}{x^3 - xd^2}}$

(b) When $x >> d$, then $x^2 - d^2 \approx x^2$

and $V = \dfrac{2k_eQd^2}{x(x^2-d^2)}$ becomes $V \approx \boxed{\dfrac{2k_eQd^2}{x^3}}$

16.64 The energy required to melt the lead sample is

$$\begin{aligned} W &= m\left[c_{Pb}(\Delta T) + L_f\right] \\ &= (6.00\times10^{-6}\ \text{kg})\left[(128\ \text{J/kg}\cdot{}^\circ\text{C})(327.3^\circ\text{C} - 20.0^\circ\text{C}) + 24.5\times10^3\ \text{J/kg}\right] \\ &= 0.383\ \text{J} \end{aligned}$$

The energy stored in a capacitor is $W = \frac{1}{2}C(\Delta V)^2$, so the required potential difference is

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(0.383\ \text{J})}{52.0\times10^{-6}\ \text{F}}} = \boxed{121\ \text{V}}$$

16.65 The capacitance of a parallel plate capacitor is $C = \dfrac{\kappa \epsilon_0 A}{d}$

Thus, $\kappa \epsilon_0 A = C \cdot d$, and the given force equation may be rewritten as

$$F = \frac{Q^2}{2\kappa \epsilon_0 A} = \frac{Q^2}{2C \cdot d} = \frac{(Q/C)^2 C}{2d} = \frac{C(\Delta V)^2}{2d}$$

With the given data values, the force is

$$F = \frac{C(\Delta V)^2}{2d} = \frac{(20\times10^{-6}\ \text{F})(100\ \text{V})^2}{2(2.0\times10^{-3}\ \text{m})} = \boxed{50\ \text{N}}$$

16.66 The electric field between the plates is directed downward with magnitude

$$|E_y| = \frac{\Delta V}{d} = \frac{100\ \text{V}}{2.00\times10^{-3}\ \text{m}} = 5.00\times10^4\ \text{N/m}$$

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

$$a_y = \frac{F_y}{m_e} = \frac{qE_y}{m_e} = \frac{(-1.60\times10^{-19}\ \text{C})(-5.00\times10^4\ \text{N/m})}{9.11\times10^{-31}\ \text{kg}} = +8.78\times10^{15}\ \text{m/s}^2$$

(a) At the closest approach to the bottom plate, $v_y = 0$. Thus, the vertical displacement from point O is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$\Delta y = \frac{0-(v_0 \sin\theta_0)^2}{2a_y} = \frac{-\left[-(5.6\times10^6\ \text{m/s})\sin 45°\right]^2}{2(8.78\times10^{15}\ \text{m/s}^2)} = -0.89\ \text{mm}$$

The minimum distance above the bottom plate is then

$$d = \frac{D}{2} + \Delta y = 1.00\ \text{mm} - 0.89\ \text{mm} = \boxed{0.11\ \text{mm}}$$

(b) The time for the electron to go from point O to the upper plate is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$+1.00\times10^{-3}\text{ m} = \left[-\left(5.6\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\sin 45°\right]t + \frac{1}{2}\left(8.78\times10^{15}\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

Solving for t gives a positive solution of $t = 1.11\times10^{-9}$ s. The horizontal displacement from point O at this time is

$$\Delta x = v_{0x}t = \left[\left(5.6\times10^{6}\text{ m/s}\right)\cos 45°\right]\left(1.11\times10^{-9}\text{ s}\right) = \boxed{4.4\text{ mm}}$$