

## Problem Solutions

15.1 Since the charges have opposite signs, the force is one of **attraction**.

Its magnitude is

$$F = \frac{k_e |q_1 q_2|}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(4.5 \times 10^{-9} \text{ C})(2.8 \times 10^{-9} \text{ C})}{(3.2 \text{ m})^2} = \boxed{1.1 \times 10^{-8} \text{ N}}$$

15.2 The electrical force would need to have the same magnitude as the current gravitational force, or

$$k_e \frac{q^2}{r^2} = G \frac{M_E m_{\text{moon}}}{r^2} \quad \text{giving} \quad q = \sqrt{\frac{GM_E m_{\text{moon}}}{k_e}}$$

This yields

$$q = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.36 \times 10^{22} \text{ kg})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = \boxed{5.71 \times 10^{13} \text{ C}}$$

15.3  $F = \frac{k_e (2e)(79e)}{r^2}$

$$= \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(158)(1.60 \times 10^{-19} \text{ C})^2}{(2.0 \times 10^{-14} \text{ m})^2} = \boxed{91 \text{ N (repulsion)}}$$

- 15.4 The attractive forces exerted on the positive charge by the negative charges are shown in the sketch and have magnitudes

$$F_1 = F_2 = \frac{k_e q^2}{a^2} \quad \text{and} \quad F_3 = \frac{k_e q^2}{(a\sqrt{2})^2} = \frac{k_e q^2}{2a^2}$$

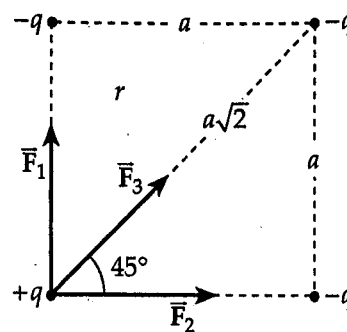
$$\Sigma F_x = F_2 + F_3 \cos 45^\circ = \frac{k_e q^2}{a^2} + \frac{k_e q^2}{2a^2} (0.707) = 1.35 \left( \frac{k_e q^2}{a^2} \right)$$

and  $\Sigma F_y = F_1 + F_3 \sin 45^\circ = \frac{k_e q^2}{a^2} + \frac{k_e q^2}{2a^2} (0.707) = 1.35 \left( \frac{k_e q^2}{a^2} \right)$

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 1.91 \frac{k_e q^2}{a^2} \quad \text{and} \quad \theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1}(1) = 45^\circ$$

so

$$\vec{F}_R = 1.91 \left( \frac{k_e q^2}{a^2} \right) \text{ along the diagonal toward the negative charge}$$



15.5 (a)  $F = \frac{k_e (2e)^2}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{4(1.60 \times 10^{-19})^2}{(5.00 \times 10^{-15} \text{ m})^2} = \boxed{36.8 \text{ N}}$

- (b) The mass of an alpha particle is  $m = 4.0026 \text{ u}$ , where  $\text{u} = 1.66 \times 10^{-27} \text{ kg}$  is the unified mass unit. The acceleration of either alpha particle is then

$$a = \frac{F}{m} = \frac{36.8 \text{ N}}{4.0026 (1.66 \times 10^{-27} \text{ kg})} = \boxed{5.54 \times 10^{27} \text{ m/s}^2}$$

- 15.6 The attractive force between the charged ends tends to compress the molecule. Its magnitude is

$$F = \frac{k_e (1e)^2}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(2.17 \times 10^{-6} \text{ m})^2} = 4.89 \times 10^{-17} \text{ N}.$$

The compression of the "spring" is

$$x = (0.0100)r = (0.0100)(2.17 \times 10^{-6} \text{ m}) = 2.17 \times 10^{-8} \text{ m},$$

so the spring constant is  $k = \frac{F}{x} = \frac{4.89 \times 10^{-17} \text{ N}}{2.17 \times 10^{-8} \text{ m}} = \boxed{2.25 \times 10^{-9} \text{ N/m}}$

- 15.7 1.00 g of hydrogen contains Avogadro's number of atoms, each containing one proton and one electron. Thus, each charge has magnitude  $|q| = N_A e$ . The distance separating these charges is  $r = 2R_E$ , where  $R_E$  is Earth's radius. Thus,

$$F = \frac{k_e (N_A e)^2}{(2R_E)^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{[(6.02 \times 10^{23})(1.60 \times 10^{-19} \text{ C})]^2}{4(6.38 \times 10^6 \text{ m})^2} = \boxed{5.12 \times 10^5 \text{ N}}$$

- 15.8 The magnitude of the repulsive force between electrons must equal the weight of an electron, Thus,  $k_e e^2 / r^2 = m_e g$

or  $r = \sqrt{\frac{k_e e^2}{m_e g}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(9.80 \text{ m/s}^2)}} = \boxed{5.08 \text{ m}}$

- 15.9 (a) The spherically symmetric charge distributions behave as if all charge was located at the centers of the spheres. Therefore, the magnitude of the attractive force is

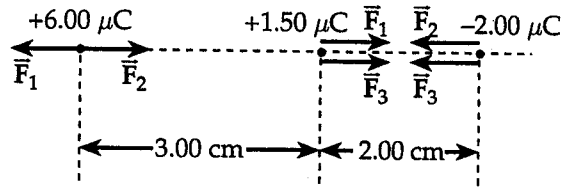
$$F = \frac{k_e q_1 |q_2|}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(12 \times 10^{-9} \text{ C})(18 \times 10^{-9} \text{ C})}{(0.30 \text{ m})^2} = \boxed{2.2 \times 10^{-5} \text{ N}}$$

- (b) When the spheres are connected by a conducting wire, the net charge  $q_{net} = q_1 + q_2 = -6.0 \times 10^{-9} \text{ C}$  will divide equally between the two identical spheres. Thus, the force is now

$$F = \frac{k_e (q_{net}/2)^2}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(-6.0 \times 10^{-9} \text{ C})^2}{4(0.30 \text{ m})^2}$$

or  $F = \boxed{9.0 \times 10^{-7} \text{ N (repulsion)}}$

- 15.10 The forces are as shown in the sketch at the right.



$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(6.00 \times 10^{-6} \text{ C})(1.50 \times 10^{-6} \text{ C})}{(3.00 \times 10^{-2} \text{ m})^2} = 89.9 \text{ N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(6.00 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{(5.00 \times 10^{-2} \text{ m})^2} = 43.2 \text{ N}$$

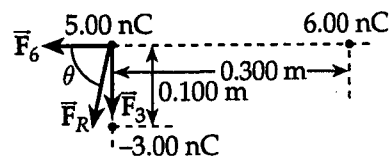
$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(1.50 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2} = 67.4 \text{ N}$$

The net force on the  $6 \mu\text{C}$  charge is  $F_6 = F_1 - F_2 = \boxed{46.7 \text{ N (to the left)}}$

The net force on the  $1.5 \mu\text{C}$  charge is  $F_{1.5} = F_1 + F_3 = \boxed{157 \text{ N (to the right)}}$

The net force on the  $-2 \mu\text{C}$  charge is  $F_{-2} = F_2 + F_3 = \boxed{111 \text{ N (to the left)}}$

- 15.11 In the sketch at the right,  $F_R$  is the resultant of the forces  $F_6$  and  $F_3$  that are exerted on the charge at the origin by the 6.00 nC and the -3.00 nC charges respectively.



$$F_6 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(6.00 \times 10^{-9} \text{ C})(5.00 \times 10^{-9} \text{ C})}{(0.300 \text{ m})^2}$$

$$= 3.00 \times 10^{-6} \text{ N}$$

$$F_3 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(3.00 \times 10^{-9} \text{ C})(5.00 \times 10^{-9} \text{ C})}{(0.100 \text{ m})^2} = 1.35 \times 10^{-5} \text{ N}$$

The resultant is  $F_R = \sqrt{(F_6)^2 + (F_3)^2} = 1.38 \times 10^{-5} \text{ N}$  at  $\theta = \tan^{-1} \left( \frac{F_3}{F_6} \right) = 77.5^\circ$

or  $\vec{F}_R = \boxed{1.38 \times 10^{-5} \text{ N at } 77.5^\circ \text{ below } -x \text{ axis}}$

- 15.12 Consider the arrangement of charges shown in the sketch at the right. The distance  $r$  is

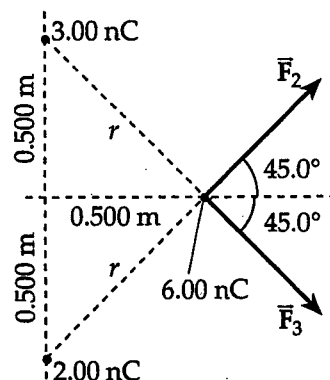
$$r = \sqrt{(0.500 \text{ m})^2 + (0.500 \text{ m})^2} = 0.707 \text{ m}$$

The forces exerted on the 6.00 nC charge are

$$F_2 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(6.00 \times 10^{-9} \text{ C})(2.00 \times 10^{-9} \text{ C})}{(0.707 \text{ m})^2}$$

$$= 2.16 \times 10^{-7} \text{ N}$$

and  $F_3 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(6.00 \times 10^{-9} \text{ C})(3.00 \times 10^{-9} \text{ C})}{(0.707 \text{ m})^2} = 3.24 \times 10^{-7} \text{ N}$



$$\text{Thus, } \Sigma F_x = (F_2 + F_3) \cos 45.0^\circ = 3.81 \times 10^{-7} \text{ N}$$

$$\text{and } \Sigma F_y = (F_2 - F_3) \sin 45.0^\circ = -7.63 \times 10^{-8} \text{ N}$$

The resultant force on the 6.00 nC charge is then

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 3.89 \times 10^{-7} \text{ N at } \theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = -11.3^\circ$$

$$\text{or } \vec{F}_R = \boxed{3.89 \times 10^{-7} \text{ N at } 11.3^\circ \text{ below } +x \text{ axis}}$$

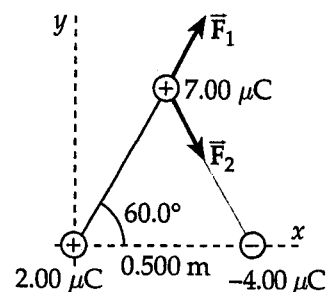
15.13 The forces on the 7.00  $\mu\text{C}$  charge are shown at the right.

$$F_1 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(7.00 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{(0.500 \text{ m})^2}$$

$$= 0.503 \text{ N}$$

$$F_2 = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(7.00 \times 10^{-6} \text{ C})(4.00 \times 10^{-6} \text{ C})}{(0.500 \text{ m})^2}$$

$$= 1.01 \text{ N}$$



$$\text{Thus, } \Sigma F_x = (F_1 + F_2) \cos 60.0^\circ = 0.755 \text{ N}$$

$$\text{and } \Sigma F_y = (F_1 - F_2) \sin 60.0^\circ = -0.436 \text{ N}$$

The resultant force on the 7.00  $\mu\text{C}$  charge is

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 0.872 \text{ N at } \theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = -30.0^\circ$$

$$\text{or } \vec{F}_R = \boxed{0.872 \text{ N at } 30.0^\circ \text{ below the } +x \text{ axis}}$$

- 15.14 Assume that the third bead has charge  $Q$  and is located at  $0 < x < d$ . Then the forces exerted on it by the  $+3q$  charge and by the  $+1q$  charge have magnitudes

$$F_3 = \frac{k_e Q(3q)}{x^2} \text{ and } F_1 = \frac{k_e Q(q)}{(d-x)^2} \text{ respectively}$$

These forces are in opposite directions, so charge  $Q$  is in equilibrium if  $F_3 = F_1$ . This gives  $3(d-x)^2 = x^2$ , and solving for  $x$ , the equilibrium position is seen to be

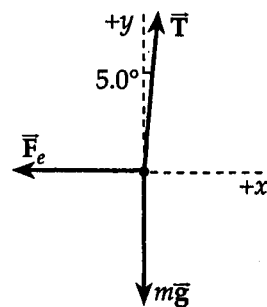
$$x = \frac{d}{1+1/\sqrt{3}} = \boxed{0.634d}$$

This is a position of stable equilibrium if  $Q > 0$ . In that case, a small displacement from the equilibrium position produces a net force directed so as to move  $Q$  back toward the equilibrium position.

- 15.15 Consider the free-body diagram of one of the spheres given at the right. Here,  $T$  is the tension in the string and  $F_e$  is the repulsive electrical force exerted by the other sphere.

$$\Sigma F_y = 0 \Rightarrow T \cos 5.0^\circ = mg, \text{ or } T = \frac{mg}{\cos 5.0^\circ}$$

$$\Sigma F_x = 0 \Rightarrow F_e = T \sin 5.0^\circ = mg \tan 5.0^\circ$$

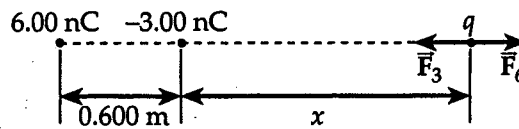


At equilibrium, the distance separating the two spheres is  $r = 2L \sin 5.0^\circ$ .

Thus,  $F_e = mg \tan 5.0^\circ$  becomes  $\frac{k_e q^2}{(2L \sin 5.0^\circ)^2} = mg \tan 5.0^\circ$  and yields

$$\begin{aligned} q &= (2L \sin 5.0^\circ) \sqrt{\frac{mg \tan 5.0^\circ}{k_e}} \\ &= [2(0.300 \text{ m}) \sin 5.0^\circ] \sqrt{\frac{(0.20 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan 5.0^\circ}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = \boxed{7.2 \text{ nC}} \end{aligned}$$

- 15.16 The required position is shown in the sketch at the right. Note that this places  $q$  closer to the smaller charge, which will allow the two forces to cancel. Requiring that



$$F_6 = F_3 \text{ gives}$$

$$\frac{k_e (6.00 \text{ nC}) q}{(x + 0.600 \text{ m})^2} = \frac{k_e (3.00 \text{ nC}) q}{x^2}, \text{ or } 2x^2 = (x + 0.600 \text{ m})^2$$

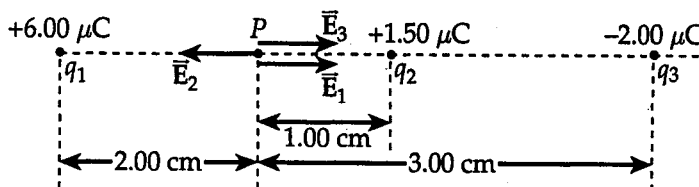
Solving for  $x$  gives the equilibrium position as

$$x = \frac{0.600 \text{ m}}{\sqrt{2} - 1} = \boxed{1.45 \text{ m beyond the } -3.00 \text{ nC charge}}$$

- 15.17 For the object to "float" it is necessary that the electrical force support the weight, or

$$qE = mg \quad \text{or} \quad m = \frac{qE}{g} = \frac{(24 \times 10^{-6} \text{ C})(610 \text{ N/C})}{9.8 \text{ m/s}^2} = \boxed{1.5 \times 10^{-3} \text{ kg}}$$

- 15.18 (a) Taking to the right as positive, the resultant electric field at point  $P$  is given by



$$E_R = E_1 + E_3 - E_2$$

$$= \frac{k_e |q_1|}{r_1^2} + \frac{k_e |q_3|}{r_3^2} - \frac{k_e |q_2|}{r_2^2}$$

$$= \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left[ \frac{6.00 \times 10^{-6} \text{ C}}{(0.0200 \text{ m})^2} + \frac{2.00 \times 10^{-6} \text{ C}}{(0.0300 \text{ m})^2} - \frac{1.50 \times 10^{-6} \text{ C}}{(0.0100 \text{ m})^2} \right]$$

This gives  $E_R = +2.00 \times 10^7 \text{ N/C}$

$$\text{or } \vec{E}_R = \boxed{2.00 \times 10^7 \text{ N/C to the right}}$$

- (b)  $\vec{F} = q\vec{E}_R = (-2.00 \times 10^{-6} \text{ C})(2.00 \times 10^7 \text{ N/C}) = -40.0 \text{ N}$

$$\text{or } \vec{F} = \boxed{40.0 \text{ N to the left}}$$



- 15.19 We shall treat the concentrations as point charges. Then, the resultant field consists of two contributions, one due to each concentration.

The contribution due to the positive charge at 3 000 m altitude is

$$E_+ = k_e \frac{|q|}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(40.0 \text{ C})}{(1000 \text{ m})^2} = 3.60 \times 10^5 \text{ N/C (downward)}$$

The contribution due to the negative charge at 1 000 m altitude is

$$E_- = k_e \frac{|q|}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(40.0 \text{ C})}{(1000 \text{ m})^2} = 3.60 \times 10^5 \text{ N/C (downward)}$$

The resultant field is then

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \boxed{7.20 \times 10^5 \text{ N/C (downward)}}$$

- 15.20 (a) The magnitude of the force on the electron is  $F = |q|E = eE$ , and the acceleration is

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{(1.60 \times 10^{-19} \text{ C})(300 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = \boxed{5.27 \times 10^{13} \text{ m/s}^2}$$

(b)  $v = v_0 + at = 0 + (5.27 \times 10^{13} \text{ m/s}^2)(1.00 \times 10^{-8} \text{ s}) = \boxed{5.27 \times 10^5 \text{ m/s}}$

- 15.21 If the electric force counterbalances the weight of the ball, then

$$qE = mg \quad \text{or} \quad E = \frac{mg}{q} = \frac{(5.0 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)}{4.0 \times 10^{-6} \text{ C}} = \boxed{1.2 \times 10^4 \text{ N/C}}$$

- 15.22 The force an electric field exerts on a positive charge is in the direction of the field. Since this force must serve as a retarding force and bring the proton to rest, the force and hence the field must be in the direction opposite to the proton's velocity.

The work-energy theorem,  $W_{\text{net}} = KE_f - KE_i$ , gives the magnitude of the field as

$$-(qE)\Delta x = 0 - KE_i \quad \text{or} \quad E = \frac{KE_i}{q(\Delta x)} = \frac{3.25 \times 10^{-15} \text{ J}}{(1.60 \times 10^{-19} \text{ C})(1.25 \text{ m})} = \boxed{1.63 \times 10^4 \text{ N/C}}$$

$$15.23 \quad (a) \quad a = \frac{F}{m} = \frac{qE}{m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(640 \text{ N/C})}{1.673 \times 10^{-27} \text{ kg}} = \boxed{6.12 \times 10^{10} \text{ m/s}^2}$$

$$(b) \quad t = \frac{\Delta v}{a} = \frac{1.20 \times 10^6 \text{ m/s}}{6.12 \times 10^{10} \text{ m/s}^2} = 1.96 \times 10^{-5} \text{ s} = \boxed{19.6 \mu\text{s}}$$

$$(c) \quad \Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{(1.20 \times 10^6 \text{ m/s})^2 - 0}{2(6.12 \times 10^{10} \text{ m/s}^2)} = \boxed{11.8 \text{ m}}$$

$$(d) \quad KE_f = \frac{1}{2} m_p v_f^2 = \frac{1}{2} (1.673 \times 10^{-27} \text{ kg})(1.20 \times 10^6 \text{ m/s})^2 = \boxed{1.20 \times 10^{-15} \text{ J}}$$

15.24 The altitude of the triangle is

$$h = (0.500 \text{ m}) \sin 60.0^\circ = 0.433 \text{ m}$$

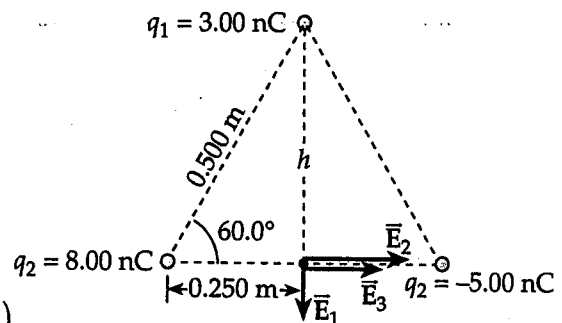
and the magnitudes of the fields due to each of the charges are

$$E_1 = \frac{k_e q_1}{h^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-9} \text{ C})}{(0.433 \text{ m})^2}$$

$$= 144 \text{ N/C}$$

$$E_2 = \frac{k_e q_2}{r_2^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.00 \times 10^{-9} \text{ C})}{(0.250 \text{ m})^2} = 1.15 \times 10^3 \text{ N/C}$$

$$\text{and} \quad E_3 = \frac{k_e |q_3|}{r_3^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-9} \text{ C})}{(0.250 \text{ m})^2} = 719 \text{ N/C}$$



Thus,  $\Sigma E_x = E_2 + E_3 = 1.87 \times 10^3 \text{ N/C}$  and  $\Sigma E_y = -E_1 = -144 \text{ N/C}$   
giving

$$E_R = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2} = 1.88 \times 10^3 \text{ N/C}$$

and

$$\theta = \tan^{-1}(\Sigma E_y / \Sigma E_x) = \tan^{-1}(-0.0769) = -4.40^\circ$$

Hence  $\boxed{\vec{E}_R = 1.88 \times 10^3 \text{ N/C at } 4.40^\circ \text{ below the } +x \text{ axis}}$

15.25 From the symmetry of the charge distribution, students should recognize that the resultant electric field at the center is

$$\boxed{\vec{E}_R = 0}$$

If one does not recognize this intuitively, consider:

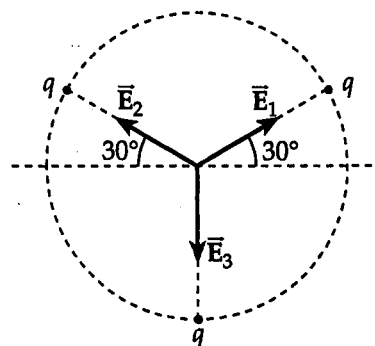
$$\vec{E}_R = \vec{E}_1 + \vec{E}_2 + \vec{E}_3, \text{ so}$$

$$E_x = E_{1x} - E_{2x} = \frac{k_e |q|}{r^2} \cos 30^\circ - \frac{k_e |q|}{r^2} \cos 30^\circ = 0$$

and

$$E_y = E_{1y} + E_{2y} - E_3 = \frac{k_e |q|}{r^2} \sin 30^\circ + \frac{k_e |q|}{r^2} \sin 30^\circ - \frac{k_e |q|}{r^2} = 0$$

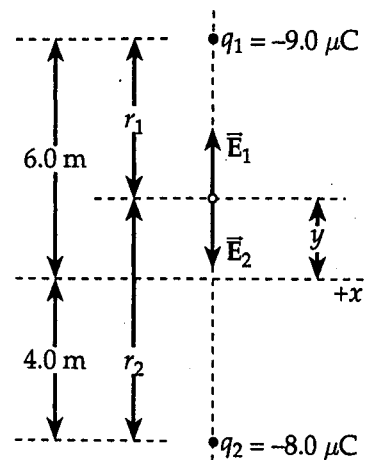
Thus,  $E_R = \sqrt{E_x^2 + E_y^2} = 0$



- 15.26 If the resultant field is to be zero, the contributions of the two charges must be equal in magnitude and must have opposite directions. This is only possible at a point on the line between the two negative charges.

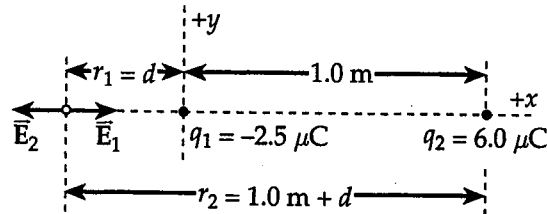
Assume the point of interest is located on the  $y$ -axis at  $-4.0 \text{ m} < y < 6.0 \text{ m}$ . Then, for equal magnitudes,

$$\frac{k_e |q_1|}{r_1^2} = \frac{k_e |q_2|}{r_2^2} \text{ or } \frac{9.0 \mu\text{C}}{(6.0 \text{ m} - y)^2} = \frac{8.0 \mu\text{C}}{(y + 4.0 \text{ m})^2}$$



Solving for  $y$  gives  $y + 4.0 \text{ m} = \sqrt{\frac{8}{9}}(6.0 \text{ m} - y)$ , or  $y = \boxed{+0.85 \text{ m}}$

- 15.27 If the resultant field is zero, the contributions from the two charges must be in opposite directions and also have equal magnitudes. Choose the line connecting the charges as the  $x$ -axis, with the origin at the  $-2.5 \mu\text{C}$  charge. Then, the two contributions will have opposite directions only in the regions  $x < 0$  and  $x > 1.0 \text{ m}$ . For the magnitudes to be equal, the point must be nearer the smaller charge. Thus, the point of zero resultant field is on the  $x$ -axis at  $x < 0$ .



Requiring equal magnitudes gives  $\frac{k_e |q_1|}{r_1^2} = \frac{k_e |q_2|}{r_2^2}$  or  $\frac{2.5 \mu\text{C}}{d^2} = \frac{6.0 \mu\text{C}}{(1.0 \text{ m} + d)^2}$

$$\text{Thus, } (1.0 \text{ m} + d)\sqrt{\frac{2.5}{6.0}} = d$$

Solving for  $d$  yields

$$d = 1.8 \text{ m, or } \boxed{1.8 \text{ m to the left of the } -2.5 \mu\text{C charge}}$$

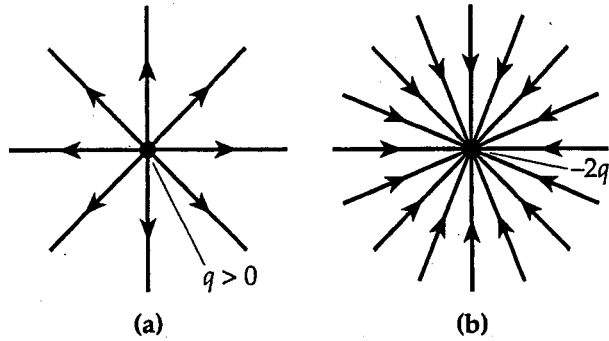
- 15.28 The magnitude of  $q_2$  is three times the magnitude of  $q_1$  because 3 times as many lines emerge from  $q_2$  as enter  $q_1$ .  $|q_2| = 3|q_1|$

(a) Then,  $\boxed{q_1/q_2 = -1/3}$

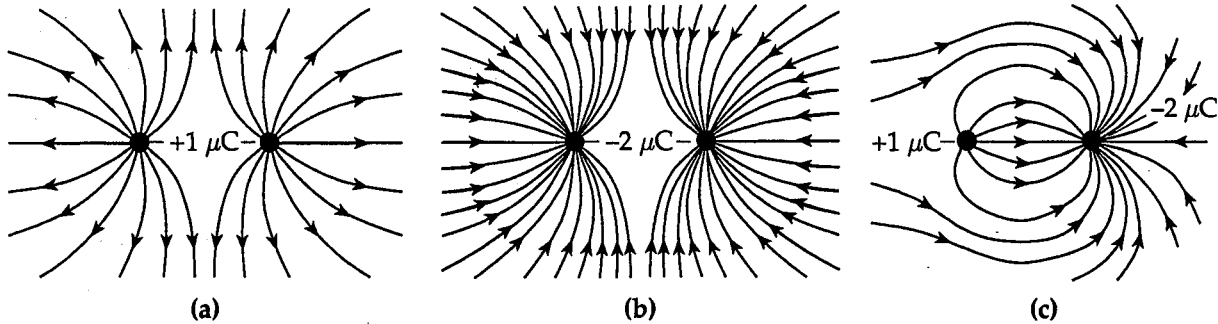
(b)  $q_2 > 0$  because lines emerge from it,

and  $q_1 < 0$  because lines terminate on it.

15.29 Note in the sketches at the right that electric field lines originate on positive charges and terminate on negative charges. The density of lines is twice as great for the  $-2q$  charge in (b) as it is for the  $1q$  charge in (a).

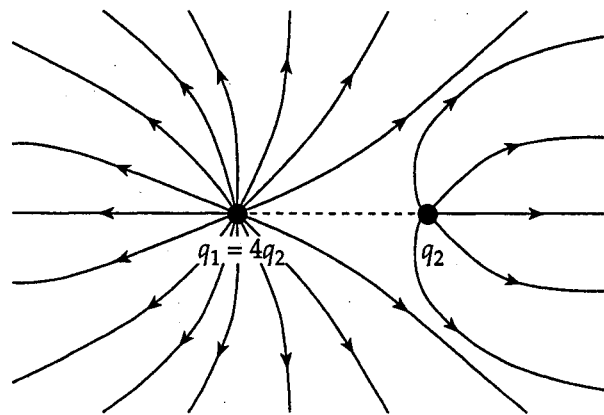


15.30 Rough sketches for these charge configurations are shown below.

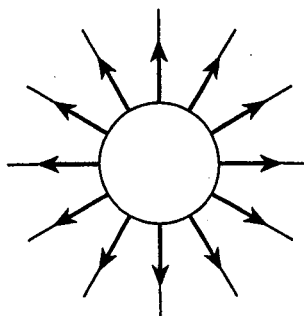


15.31 (a) The sketch for (a) is shown at the right. Note that four times as many lines should leave  $q_1$  as emerge from  $q_2$  although, for clarity, this is not shown in this sketch.

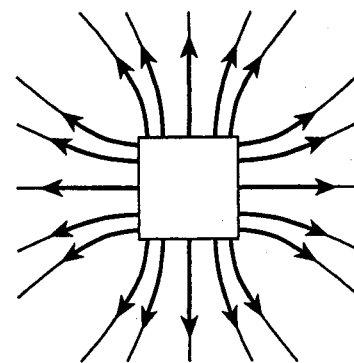
(b) The field pattern looks the same here as that shown for (a) with the exception that the arrows are reversed on the field lines.



- 15.32 (a) In the sketch for (a) at the right, note that there are no lines inside the sphere. On the outside of the sphere, the field lines are uniformly spaced and radially outward.



(a)



(b)

- (b) In the sketch for (b) above, note that the lines are perpendicular to the surface at the points where they emerge. They should also be symmetrical about the symmetry axes of the cube. The field is zero inside the cube.

- 15.33 (a) **Zero** net charge on each surface of the sphere.
- (b) The negative charge lowered into the sphere repels  **$-5 \mu\text{C}$  to the outside** surface, and leaves  **$+5 \mu\text{C}$  on the inside** surface of the sphere.
- (c) The negative charge lowered inside the sphere neutralizes the inner surface, leaving **zero charge on the inside**. This leaves  **$-5 \mu\text{C}$  on the outside** surface of the sphere.
- (d) When the object is removed, the sphere is left with  **$-5.00 \mu\text{C}$  on the outside** surface and **zero charge on the inside**.
- 15.34 (a) The dome is a closed conducting surface. Therefore, the electric field is zero everywhere inside it.

At the surface and outside of this spherically symmetric charge distribution, the field is as if all the charge were concentrated at the center of the sphere.

- (b) At the surface,

$$E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.0 \times 10^{-4} \text{ C})}{(1.0 \text{ m})^2} = \boxed{1.8 \times 10^6 \text{ N/C}}$$

(c) Outside the spherical dome,  $E = \frac{k_e q}{r^2}$ . Thus, at  $r = 4.0 \text{ m}$ ,

$$E = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.0 \times 10^{-4} \text{ C})}{(4.0 \text{ m})^2} = \boxed{1.1 \times 10^5 \text{ N/C}}$$

15.35 For a uniformly charged sphere, the field is strongest at the surface.

$$\text{Thus, } E_{\text{max}} = \frac{k_e q_{\text{max}}}{R^2},$$

$$\text{or } q_{\text{max}} = \frac{R^2 E_{\text{max}}}{k_e} = \frac{(2.0 \text{ m})^2 (3.0 \times 10^6 \text{ N/C})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = \boxed{1.3 \times 10^{-3} \text{ C}}$$

15.36 If the weight of the drop is balanced by the electric force, then  $mg = |q|E = eE$  or the mass of the drop must be

$$m = \frac{eE}{g} = \frac{(1.6 \times 10^{-19} \text{ C})(3 \times 10^4 \text{ N/C})}{9.8 \text{ m/s}^2} \approx 5 \times 10^{-16} \text{ kg}$$

$$\text{But, } m = \rho V = \rho \left( \frac{4}{3} \pi r^3 \right) \text{ and the radius of the drop is } r = \left[ \frac{3m}{4\pi\rho} \right]^{1/3}$$

$$r = \left[ \frac{3(5 \times 10^{-16} \text{ kg})}{4\pi(858 \text{ kg/m}^3)} \right]^{1/3} = 5.2 \times 10^{-7} \text{ m} \quad \text{or} \quad r \sim \boxed{1 \mu\text{m}}$$

$$15.37 \text{ (a) } F = qE = (1.60 \times 10^{-19} \text{ C})(3.0 \times 10^4 \text{ N/C}) = \boxed{4.8 \times 10^{-15} \text{ N}}$$

$$\text{(b) } a = \frac{F}{m_p} = \frac{4.8 \times 10^{-15} \text{ N}}{1.673 \times 10^{-27} \text{ kg}} = \boxed{2.9 \times 10^{12} \text{ m/s}^2}$$

15.38 The flux through an area is  $\Phi_E = EA \cos \theta$ , where  $\theta$  is the angle between the direction of the field  $E$  and the line perpendicular to the area  $A$ .

$$\text{(a) } \Phi_E = EA \cos \theta = (6.2 \times 10^5 \text{ N/C})(3.2 \text{ m}^2) \cos 0^\circ = \boxed{2.0 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$\text{(b) In this case, } \theta = 90^\circ \text{ and } \Phi_E = \boxed{0}$$

15.39 The area of the rectangular plane is  $A = (0.350 \text{ m})(0.700 \text{ m}) = 0.245 \text{ m}^2$ .

(a) When the plane is parallel to the  $yz$  plane,  $\theta = 0^\circ$ , and the flux is

$$\Phi_E = EA \cos \theta = (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2) \cos 0^\circ = \boxed{858 \text{ N} \cdot \text{m}^2/\text{C}}$$

(b) When the plane is parallel to the  $x$ -axis,  $\theta = 90^\circ$  and  $\Phi_E = \boxed{0}$

(c)  $\Phi_E = EA \cos \theta = (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2) \cos 40.0^\circ = \boxed{657 \text{ N} \cdot \text{m}^2/\text{C}}$

15.40 In this problem, we consider part (b) first.

(b) Since the field is radial everywhere, the charge distribution generating it must be spherically symmetric. Also, since the field is radially inward, the net charge inside the sphere is negative charge.

(a) Outside a spherically symmetric charge distribution, the field is  $E = \frac{k_e Q}{r^2}$ . Thus, just outside the surface where  $r = R$ , the magnitude of the field is  $E = k_e |Q|/R^2$ , so

$$|Q| = \frac{R^2 E}{k_e} = \frac{(0.750 \text{ m})^2 (890 \text{ N/C})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 5.57 \times 10^{-8} \text{ C} = 55.7 \text{ nC}$$

Since we have determined that  $Q < 0$ , we now have  $Q = \boxed{-55.7 \text{ nC}}$

15.41  $\Phi_E = EA \cos \theta$  and  $\Phi_E = \Phi_{E, \max}$  when  $\theta = 0^\circ$

$$\text{Thus, } E = \frac{\Phi_{E, \max}}{A} = \frac{\Phi_{E, \max}}{\pi d^2/4} = \frac{4(5.2 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C})}{\pi(0.40 \text{ m})^2} = \boxed{4.1 \times 10^6 \text{ N/C}}$$

15.42  $\Phi_E = EA \cos \theta = \left(\frac{k_e q}{R^2}\right)(4\pi R^2) \cos 0^\circ = 4\pi k_e q$

$$\Phi_E = 4\pi(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C}) = \boxed{5.65 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}$$



15.43 We choose a spherical gaussian surface, concentric with the charged spherical shell and of radius  $r$ . Then,  $\Sigma EA \cos \theta = E(4\pi r^2) \cos 0^\circ = 4\pi r^2 E$ .

(a) For  $r > a$  (that is, outside the shell), the total charge enclosed by the gaussian surface is  $Q = +q - q = 0$ . Thus, Gauss's law gives  $4\pi r^2 E = 0$ , or  $E = 0$ .

(b) Inside the shell,  $r < a$ , and the enclosed charge is  $Q = +q$ .

$$\text{Therefore, from Gauss's law, } 4\pi r^2 E = \frac{q}{\epsilon_0}, \text{ or } E = \frac{q}{4\pi \epsilon_0 r^2} = \frac{k_e q}{r^2}$$

The field for  $r < a$  is  $\boxed{\vec{E} = \frac{k_e q}{r^2} \text{ directed radially outward}}$ .

15.44 Construct a gaussian surface just barely inside the surface of the conductor, where  $E = 0$ .

Since  $E = 0$  inside, Gauss' law says  $\frac{Q}{\epsilon_0} = 0$  inside. Thus, any excess charge residing on the conductor must be outside our gaussian surface (that is, on the surface of the conductor).

15.45  $E = 0$  at all points inside the conductor, and  $\cos \theta = \cos 90^\circ = 0$  on the cylindrical surface. Thus, the only flux through the gaussian surface is on the outside end cap and Gauss's law reduces to  $\Sigma EA \cos \theta = EA_{\text{cap}} = \frac{Q}{\epsilon_0}$ .

The charge enclosed by the gaussian surface is  $Q = \sigma A$ , where  $A$  is the cross-sectional area of the cylinder and also the area of the end cap, so Gauss's law becomes

$$EA = \frac{\sigma A}{\epsilon_0}, \text{ or } E = \boxed{\frac{\sigma}{\epsilon_0}}$$

15.46 Choose a very small cylindrical gaussian surface with one end inside the conductor. Position the other end parallel to and just outside the surface of the conductor.

Since, in static conditions,  $E = 0$  at all points inside a conductor, there is no flux through the inside end cap of the gaussian surface. At all points outside, but very close to, a conductor the electric field is perpendicular to the conducting surface. Thus, it is parallel to the cylindrical side of the gaussian surface and no flux passes through this cylindrical side. The total flux through the gaussian surface is then  $\Phi = EA$ , where  $A$  is the cross-sectional area of the cylinder as well as the area of the end cap.

The total charge enclosed by the cylindrical gaussian surface is  $Q = \sigma A$ , where  $\sigma$  is the charge density on the conducting surface. Hence, Gauss's law gives

$$EA = \frac{\sigma A}{\epsilon_0} \text{ or } E = \boxed{\frac{\sigma}{\epsilon_0}}$$

$$15.47 \quad F = \frac{k_e |q_1| |q_2|}{r^2} = \frac{k_e e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(2.00 \times 10^{-15} \text{ m})^2} = \boxed{57.5 \text{ N}}$$

$$15.48 \quad (a) \quad F = \frac{k_e |q_1| |q_2|}{r^2} = \frac{k_e e^2}{r^2} \\ = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(0.53 \times 10^{-10} \text{ m})^2} = \boxed{8.2 \times 10^{-8} \text{ N}}$$

$$(b) \quad F = m_e a_c = m_e (v^2/r), \text{ so}$$

$$v = \sqrt{\frac{r \cdot F}{m_e}} = \sqrt{\frac{(0.53 \times 10^{-10} \text{ m})(8.2 \times 10^{-8} \text{ N})}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{2.2 \times 10^6 \text{ m/s}}$$

15.49 The three contributions to the resultant electric field at the point of interest are shown in the sketch at the right.

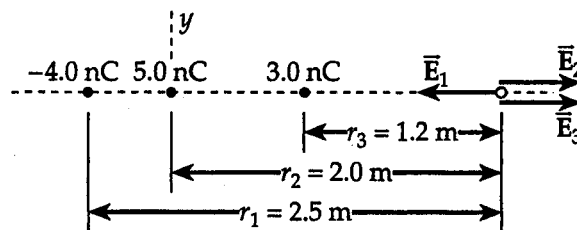
The magnitude of the resultant field is

$$E_R = -E_1 + E_2 + E_3$$

$$E_R = -\frac{k_e |q_1|}{r_1^2} + \frac{k_e |q_2|}{r_2^2} + \frac{k_e |q_3|}{r_3^2} = k_e \left[ -\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} + \frac{|q_3|}{r_3^2} \right]$$

$$E_R = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left[ -\frac{4.0 \times 10^{-9} \text{ C}}{(2.5 \text{ m})^2} + \frac{5.0 \times 10^{-9} \text{ C}}{(2.0 \text{ m})^2} + \frac{3.0 \times 10^{-9} \text{ C}}{(1.2 \text{ m})^2} \right]$$

$$E_R = +24 \text{ N/C}, \text{ or } \vec{E}_R = \boxed{24 \text{ N/C in the } +x \text{ direction}}$$



15.50 Consider the free-body diagram shown at the right.

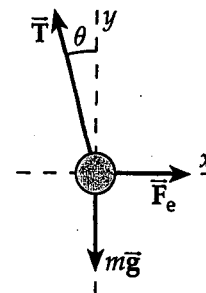
$$\Sigma F_y = 0 \Rightarrow T \cos \theta = mg \quad \text{or} \quad T = \frac{mg}{\cos \theta}$$

$$\Sigma F_x = 0 \Rightarrow F_e = T \sin \theta = mg \tan \theta$$

Since  $F_e = qE$ , we have

$$qE = mg \tan \theta, \quad \text{or} \quad q = \frac{mg \tan \theta}{E}$$

$$q = \frac{(2.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan 15.0^\circ}{1.00 \times 10^3 \text{ N/C}} = 5.25 \times 10^{-6} \text{ C} = \boxed{5.25 \mu\text{C}}$$



15.51 (a) At a point on the  $x$ -axis, the contributions by the two charges to the resultant field have equal magnitudes given by  $E_1 = E_2 = \frac{k_e q}{r^2}$ .

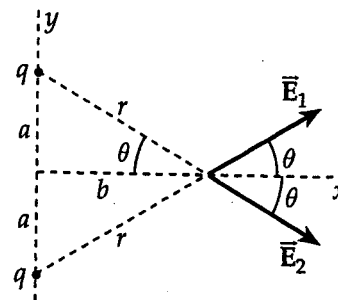
The components of the resultant field are

$$E_y = E_{1y} - E_{2y} = \left( \frac{k_e q}{r^2} \right) \sin \theta - \left( \frac{k_e q}{r^2} \right) \sin \theta = 0$$

and 
$$E_x = E_{1x} + E_{2x} = \left( \frac{k_e q}{r^2} \right) \cos \theta + \left( \frac{k_e q}{r^2} \right) \cos \theta = \left[ \frac{k_e (2q)}{r^2} \right] \cos \theta$$

Since  $\frac{\cos \theta}{r^2} = \frac{b/r}{r^2} = \frac{b}{r^3} = \frac{b}{(a^2 + b^2)^{3/2}}$ , the resultant field is

$$\vec{E}_R = \boxed{\frac{k_e (2q) b}{(a^2 + b^2)^{3/2}} \text{ in the } +x \text{ direction}}$$



- (b) Note that the result of part (a) may be written as  $E_R = \frac{k_e(Q)b}{(a^2 + b^2)^{3/2}}$  where  $Q = 2q$  is the total charge in the charge distribution generating the field.

In the case of a uniformly charged circular ring, consider the ring to consist of a very large number of pairs of charges uniformly spaced around the ring. Each pair consists of two identical charges located diametrically opposite each other on the ring. The total charge of pair number  $i$  is  $Q_i$ . At a point on the axis of the ring, this pair of charges generates an electric field contribution that is parallel to the axis and has magnitude  $E_i = \frac{k_e b Q_i}{(a^2 + b^2)^{3/2}}$ .

The resultant electric field of the ring is the summation of the contributions by all pairs of charges, or

$$E_R = \Sigma E_i = \left[ \frac{k_e b}{(a^2 + b^2)^{3/2}} \right] \Sigma Q_i = \frac{k_e b Q}{(a^2 + b^2)^{3/2}}$$

where  $Q = \Sigma Q_i$  is the total charge on the ring.

$$\vec{E}_R = \frac{k_e Q b}{(a^2 + b^2)^{3/2}} \text{ in the } +x \text{ direction}$$

$$15.52 \quad (a) \quad a_y = \frac{v_y^2 - v_{0y}^2}{2(\Delta y)} = \frac{(21.0 \text{ m/s})^2 - 0}{2(5.00 \text{ m})} = 44.1 \text{ m/s}^2 \quad (\text{downward})$$

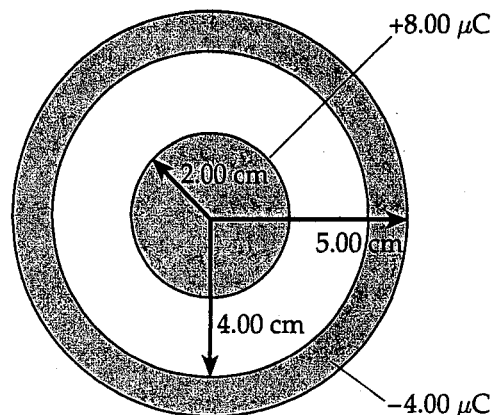
Since  $a_y > g$ , the electrical force must be directed downward, aiding the gravitational force in accelerating the bead. Because the bead is positively charged, the electrical force acting on it is in the direction of the electric field. Thus, the field is directed downward.

- (b) Taking downward as positive,  $\Sigma F_y = qE + mg = ma_y$ .

Therefore,

$$q = \frac{m(a_y - g)}{E} = \frac{(1.00 \times 10^{-3} \text{ kg})[(44.1 - 9.80) \text{ m/s}^2]}{1.00 \times 10^4 \text{ N/C}} = 3.43 \times 10^{-6} \text{ C} = \boxed{3.43 \mu\text{C}}$$

- 15.53 Because of the spherical symmetry of the charge distribution, any electric field present will be radial in direction. If a field does exist at distance  $R$  from the center, it is the same as if the net charge located within  $r \leq R$  were concentrated as a point charge at the center of the inner sphere. Charge located at  $r > R$  does not contribute to the field at  $r = R$ .



- (a) At  $r = 1.00$  cm,  $E = 0$  since static electric fields cannot exist within conducting materials.
- (b) The net charge located at  $r \leq 3.00$  cm is  $Q = +8.00 \mu\text{C}$ .

Thus, at  $r = 3.00$  cm,

$$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(8.00 \times 10^{-6} \text{ C})}{(3.00 \times 10^{-2} \text{ m})^2} = 7.99 \times 10^7 \text{ N/C (outward)}$$

- (c) At  $r = 4.50$  cm,  $E = 0$  since this is located within conducting materials.
- (d) The net charge located at  $r \leq 7.00$  cm is  $Q = +4.00 \mu\text{C}$ .

Thus, at  $r = 7.00$  cm,

$$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(4.00 \times 10^{-6} \text{ C})}{(7.00 \times 10^{-2} \text{ m})^2} = 7.34 \times 10^6 \text{ N/C (outward)}$$

- 15.54 The charges on the spheres will be equal in magnitude and opposite in sign. From  $F = k_e q^2 / r^2$ , this charge must be

$$q = \sqrt{\frac{F \cdot r^2}{k_e}} = \sqrt{\frac{(1.00 \times 10^4 \text{ N})(1.00 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = 1.05 \times 10^{-3} \text{ C}$$

The number of electrons transferred is

$$n = \frac{q}{e} = \frac{1.05 \times 10^{-3} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 6.59 \times 10^{15}$$

The total number of electrons in 100-g of silver is

$$N = \left( 47 \frac{\text{electrons}}{\text{atom}} \right) \left( 6.02 \times 10^{23} \frac{\text{atoms}}{\text{mole}} \right) \left( \frac{1 \text{ mole}}{107.87 \text{ g}} \right) (100 \text{ g}) = 2.62 \times 10^{25}$$

Thus, the fraction transferred is

$$\frac{n}{N} = \frac{6.59 \times 10^{15}}{2.62 \times 10^{25}} = \boxed{2.51 \times 10^{-10}} \text{ (that is, 2.51 out of every 10 billion).}$$

15.55  $\Phi_E = EA \cos \theta$

$$= (2.00 \times 10^4 \text{ N/C}) [(6.00 \text{ m})(3.00 \text{ m})] \cos 10.0^\circ = \boxed{3.55 \times 10^5 \text{ N} \cdot \text{m}^2 / \text{C}}$$

- 15.56 (a) The downward electrical force acting on the ball is

$$F_e = qE = (2.00 \times 10^{-6} \text{ C})(1.00 \times 10^5 \text{ N/C}) = 0.200 \text{ N}$$

The total downward force acting on the ball is then

$$F = F_e + mg = 0.200 \text{ N} + (1.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) = 0.210 \text{ N}$$

Thus, the ball will behave as if it was in a modified gravitational field where the effective free-fall acceleration is

$$"g" = \frac{F}{m} = \frac{0.210 \text{ N}}{1.00 \times 10^{-3} \text{ kg}} = 210 \text{ m/s}^2$$

The period of the pendulum will be

$$T = 2\pi \sqrt{\frac{L}{"g"}} = 2\pi \sqrt{\frac{0.500 \text{ m}}{210 \text{ m/s}^2}} = \boxed{0.307 \text{ s}}$$

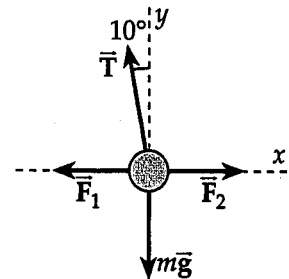
- (b) **Yes**. The force of gravity is a significant portion of the total downward force acting on the ball. Without gravity, the effective acceleration would be

$$"g" = \frac{F_e}{m} = \frac{0.200 \text{ N}}{1.00 \times 10^{-3} \text{ kg}} = 200 \text{ m/s}^2$$

$$\text{giving } T = 2\pi \sqrt{\frac{0.500 \text{ m}}{200 \text{ m/s}^2}} = 0.314 \text{ s}$$

a 2.28% difference from the correct value with gravity included.

- 15.57 The sketch at the right gives a free-body diagram of the positively charged sphere. Here,  $F_1 = k_e |q|^2 / r^2$  is the attractive force exerted by the negatively charged sphere and  $F_2 = qE$  is exerted by the electric field.



$$\Sigma F_y = 0 \Rightarrow T \cos 10^\circ = mg \quad \text{or} \quad T = \frac{mg}{\cos 10^\circ}$$

$$\Sigma F_x = 0 \Rightarrow F_2 = F_1 + T \sin 10^\circ \quad \text{or} \quad qE = \frac{k_e |q|^2}{r^2} + mg \tan 10^\circ$$

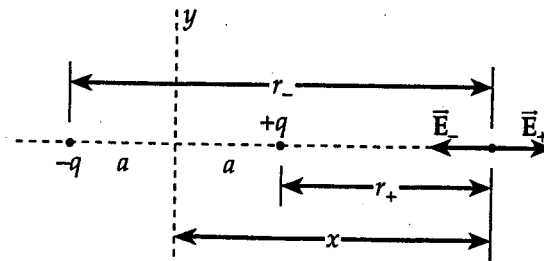
At equilibrium, the distance between the two spheres is  $r = 2(L \sin 10^\circ)$ . Thus,

$$\begin{aligned} E &= \frac{k_e |q|}{4(L \sin 10^\circ)^2} + \frac{mg \tan 10^\circ}{q} \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.0 \times 10^{-8} \text{ C})}{4[(0.100 \text{ m}) \sin 10^\circ]^2} + \frac{(2.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan 10^\circ}{(5.0 \times 10^{-8} \text{ C})} \end{aligned}$$

or the needed electric field strength is

$$E = \boxed{4.4 \times 10^5 \text{ N/C}}$$

- 15.58 As shown in the sketch, the electric field at any point on the  $x$ -axis consists of two parts, one due to each of the charges in the dipole.



$$E = E_+ - E_- = \frac{k_e |q|}{r_+^2} - \frac{k_e |q|}{r_-^2}$$

$$E = \frac{k_e |q|}{(x-a)^2} - \frac{k_e |q|}{(x+a)^2} = k_e |q| \left[ \frac{(x+a)^2 - (x-a)^2}{(x-a)^2 (x+a)^2} \right] = k_e q \left[ \frac{4ax}{(x^2 - a^2)^2} \right]$$

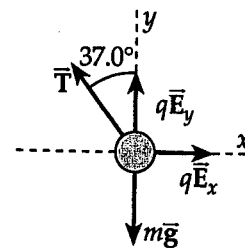
Thus, if  $x^2 \gg a^2$ , this gives  $E \approx k_e q \left[ \frac{4ax}{x^4} \right] = \boxed{\frac{4k_e qa}{x^3}}$

- 15.59 (a) Consider the free-body diagram for the ball given in the sketch.

$$\Sigma F_x = 0 \Rightarrow T \sin 37.0^\circ = qE_x \quad \text{or} \quad T = \frac{qE_x}{\sin 37.0^\circ}$$

and

$$\Sigma F_y = 0 \Rightarrow qE_y + T \cos 37.0^\circ = mg \quad \text{or} \quad qE_y + qE_x \cot 37.0^\circ = mg$$



$$\text{Thus, } q = \frac{mg}{E_y + E_x \cot 37.0^\circ} = \frac{(1.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{[5.00 + (3.00) \cot 37.0^\circ] \times 10^5 \text{ N/C}}$$

$$= 1.09 \times 10^{-8} \text{ C} = \boxed{10.9 \text{ nC}}$$

- (b) From  $\Sigma F_x = 0$ , we found that  $T = \frac{qE_x}{\sin 37.0^\circ}$ .

$$\text{Hence, } T = \frac{(1.09 \times 10^{-8} \text{ C})(3.00 \times 10^5 \text{ N/C})}{\sin 37.0^\circ} = \boxed{5.44 \times 10^{-3} \text{ N}}$$

- 15.60 (a) At any point on the  $x$ -axis in the range  $0 < x < 1.00 \text{ m}$ , the contributions made to the resultant electric field by the two charges are both in the positive  $x$  direction. Thus, it is not possible for these contributions to cancel each other and yield a zero field.



- (b) Any point on the  $x$ -axis in the range  $x < 0$  is located closer to the larger magnitude charge ( $q = 5.00 \mu\text{C}$ ) than the smaller magnitude charge ( $|q| = 4.00 \mu\text{C}$ ). Thus, the contribution to the resultant electric field by the larger charge will always have a greater magnitude than the contribution made by the smaller charge. It is not possible for these contributions to cancel to give a zero resultant field.
- (c) If a point is on the  $x$ -axis in the region  $x > 1.00 \text{ m}$ , the contributions made by the two charges are in opposite directions. Also, a point in this region is closer to the smaller magnitude charge than it is to the larger charge. Thus, there is a location in this region where the contributions of these charges to the total field will have equal magnitudes and cancel each other.
- (d) When the contributions by the two charges cancel each other, their magnitudes must be equal. That is,

$$k_e \frac{(5.00 \mu\text{C})}{x^2} = k_e \frac{(4.00 \mu\text{C})}{(x - 1.00 \text{ m})^2} \text{ or } x - 1.00 \text{ m} = +\sqrt{\frac{4}{5}} x$$

Thus, the resultant field is zero at  $x = \frac{1.00 \text{ m}}{1 - \sqrt{4/5}} = \boxed{+9.47 \text{ m}}$

- 15.61 We assume that the two spheres have equal charges, so the repulsive force that one exerts on the other has magnitude  $F_e = k_e q^2 / r^2$ .

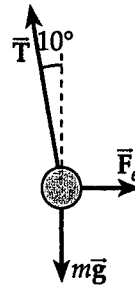
From Figure P15.61 in the textbook, observe that the distance separating the two spheres is

$$r = 3.0 \text{ cm} + 2[(5.0 \text{ cm}) \sin 10^\circ] = 4.7 \text{ cm} = 0.047 \text{ m}$$

From the free-body diagram of one sphere given above, observe that

$$\Sigma F_y = 0 \Rightarrow T \cos 10^\circ = mg \text{ or } T = mg / \cos 10^\circ$$

and  $\Sigma F_x = 0 \Rightarrow F_e = T \sin 10^\circ = \left( \frac{mg}{\cos 10^\circ} \right) \sin 10^\circ = mg \tan 10^\circ$



$$\text{Thus, } k_e q^2 / r^2 = mg \tan 10^\circ$$

$$\text{or } q = \sqrt{\frac{mgr^2 \tan 10^\circ}{k_e}} = \sqrt{\frac{(0.015 \text{ kg})(9.8 \text{ m/s}^2)(0.047 \text{ m})^2 \tan 10^\circ}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}}$$

$$\text{giving } q = 8.0 \times 10^{-8} \text{ C or } \boxed{q \sim 10^{-7} \text{ C}}$$

15.62 Consider the free-body diagram of the rightmost charge given below.

$$\Sigma F_y = 0 \Rightarrow T \cos \theta = mg \quad \text{or} \quad T = mg / \cos \theta$$

$$\text{and } \Sigma F_x = 0 \Rightarrow F_e = T \sin \theta = (mg / \cos \theta) \sin \theta = mg \tan \theta$$

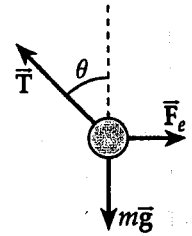
$$\text{But, } F_e = \frac{k_e q^2}{r_1^2} + \frac{k_e q^2}{r_2^2} = \frac{k_e q^2}{(L \sin \theta)^2} + \frac{k_e q^2}{(2L \sin \theta)^2} = \frac{5k_e q^2}{4L^2 \sin^2 \theta}$$

$$\text{Thus, } \frac{5k_e q^2}{4L^2 \sin^2 \theta} = mg \tan \theta \quad \text{or} \quad q = \sqrt{\frac{4L^2 mg \sin^2 \theta \tan \theta}{5k_e}}$$

If  $\theta = 45^\circ$ ,  $m = 0.10 \text{ kg}$ , and  $L = 0.300 \text{ m}$  then

$$q = \sqrt{\frac{4(0.300 \text{ m})^2 (0.10 \text{ kg})(9.80 \text{ m/s}^2) \sin^2(45^\circ) \tan(45^\circ)}{5(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)}}$$

$$\text{or } q = 2.0 \times 10^{-6} \text{ C} = \boxed{2.0 \mu\text{C}}$$



15.63 (a) When an electron (negative charge) moves distance  $\Delta x$  in the direction of an electric field, the work done on it is

$$W = F_e(\Delta x) \cos \theta = eE(\Delta x) \cos 180^\circ = -eE(\Delta x)$$

From the work-energy theorem ( $W_{\text{net}} = KE_f - KE_i$ ) with  $KE_f = 0$ , we have

$$-eE(\Delta x) = -KE_i, \text{ or } E = \frac{KE_i}{e(\Delta x)} = \frac{1.60 \times 10^{-17} \text{ J}}{(1.60 \times 10^{-19} \text{ C})(0.100 \text{ m})} = \boxed{1.00 \times 10^3 \text{ N/C}}$$

- (b) The magnitude of the retarding force acting on the electron is  $F_e = eE$ , and Newton's second law gives the acceleration as  $a = -F_e/m = -eE/m$ . Thus, the time required to bring the electron to rest is

$$t = \frac{v - v_0}{a} = \frac{0 - \sqrt{2(KE_i)/m}}{-eE/m} = \frac{\sqrt{2m(KE_i)}}{eE}$$

or

$$t = \frac{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-17} \text{ J})}}{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^3 \text{ N/C})} = 3.37 \times 10^{-8} \text{ s} = \boxed{33.7 \text{ ns}}$$

- (c) After bringing the electron to rest, the electric force continues to act on it causing the electron to accelerate in the direction opposite to the field at a rate of

$$|a| = \frac{eE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^3 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1.76 \times 10^{14} \text{ m/s}^2}$$

- 15.64 (a) The acceleration of the protons is downward (in the direction of the field) and

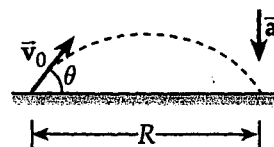
$$|a_y| = \frac{F_e}{m} = \frac{eE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(720 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 6.90 \times 10^{10} \text{ m/s}^2$$

The time of flight for the proton is twice the time required to reach the peak of the arc, or

$$t = 2t_{\text{peak}} = 2 \left( \frac{v_{0y}}{|a_y|} \right) = \frac{2v_0 \sin \theta}{|a_y|}$$

The horizontal distance traveled in this time is

$$R = v_{0x}t = (v_0 \cos \theta) \left( \frac{2v_0 \sin \theta}{|a_y|} \right) = \frac{v_0^2 \sin 2\theta}{|a_y|}$$



Thus, if  $R = 1.27 \times 10^{-3}$  m, we must have

$$\sin 2\theta = \frac{|a_y| R}{v_0^2} = \frac{(6.90 \times 10^{10} \text{ m/s}^2)(1.27 \times 10^{-3} \text{ m})}{(9550 \text{ m/s})^2} = 0.961$$

giving  $2\theta = 73.9^\circ$  or  $2\theta = 180^\circ - 73.9^\circ = 106.1^\circ$ . Hence,  $\theta = \boxed{37.0^\circ \text{ or } 53.0^\circ}$

(b) The time of flight for each possible angle of projection is:

$$\text{For } \theta = 37.0^\circ: \quad t = \frac{2v_0 \sin \theta}{|a_y|} = \frac{2(9550 \text{ m/s}) \sin 37.0^\circ}{6.90 \times 10^{10} \text{ m/s}^2} = \boxed{1.66 \times 10^{-7} \text{ s}}$$

$$\text{For } \theta = 53.0^\circ: \quad t = \frac{2v_0 \sin \theta}{|a_y|} = \frac{2(9550 \text{ m/s}) \sin 53.0^\circ}{6.90 \times 10^{10} \text{ m/s}^2} = \boxed{2.21 \times 10^{-7} \text{ s}}$$