Physics 161: Black Holes: Lecture 23: 3 Mar 2010

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24 Can anything escape a black hole? Hawking radiation

In 1974 Stephen Hawking startled the Physics community by proving that black holes are not black; they radiate and lose mass. We have always heard and seen proofs that nothing can get out of a black hole, but here was a proof that radiation must escape! This was not any mistake in previous work – General Relativity is a classical theory, and Hawking used quantum mechanics in his proof. Hawking was considered to be a very smart guy before this result, but this result is what made him famous.

We have considered photons classically following geodesics, but in quantum mechanics the uncertainty principle means that rays cannot be localized to arbitrary precesion. Near the horizon of a black hole this changes their behavior.

To do the Hawking calculation properly one needs to use Quantum Field Theory (QFT), the relativistically correct version of quantum mechanics, and this theory is beyond the scope of this course. So we will just describe things heuristically to give you a conceptual idea of what is going on here.

The uncertainty principle has several forms, one of which is

$$\Delta E \Delta t \geq \hbar,$$

where ΔE is the uncertainty in a particle's energy in a quantum state for a measurment lasting a time Δt. Thus on a very short time scale one can't know precisely the energy of any quantum state. This principle is preserved in Quantum Field Theory, and it means that the vacuum is filled with energy fluctuations that violate energy conservation. As long as the violation lasts for a time less than $\Delta t = \hbar/\Delta E$, everything is OK. Short times implies small distances since particles travel at speeds less than c, so on small scales energy does not have to be conserved. Only over larger times and larger distances is energy conservation valid. This is how particles tunnel under barriers that seem to not allow particles through. Thus in QFT we visualize the vacuum, that is empty space, as being a boiling caldron of particles and anti-particles being continually created and destroyed. These particles are living on borrowed energy and so have to disappear again in time to satisfy the uncertainty relation, unless they can get enough energy somehow to "go on mass shell" and survive. One can actually use this idea to calculate the probability of creating particles in particle accelerators. The particles in the accelerator transfer their energy to the **virtual particles** living on borrowed energy and allow them to become real.

Now consider such fluctuations near the horizon of a black hole. Suppose two photons are spontaneously created, one with energy E and the other with energy $-E$. The particle with $-E$ cannot propagate freely through space and would normally have to pay back its borrowed energy in a short time $\Delta t = \hbar/E$. However the particle can gain energy by falling into the black hole. If the negative energy particle falls in, then that negative energy will be forced down to the singularity and be added to the black hole. Then the positive energy photon can escape to infinity. Recall that inside the black hole

forward in time means decreasing r, and that the energies of particles can be either negative or positive depending on whether t is going forward or backward. Over long times the energy has to be conserved, so if $-E$ goes down the hole, $+E$ has to escape to infinity. Another way to say this is that the virtual particle can gain enough energy to live by falling into the hole. If it was originally a negative energy photon, then the hole mass will decrease and the excess energy radiated away.

I realize this is not a very satisfactory explanation. It is not easy to explain without QFT, and even then it is tricky, but careful consideration by many physicists have convinced most everyone that this effect in fact happens. However, it has not been observed experimentally, as we shall see. Also, since all types of particles are created by quantum fluctuations near the horizon, a black hole must emit all types of particles: photons, electrons, quarks, etc.

Their are other ways to get at Hawking radiation that are more general. I like a method that considers what space time looks like in a permanently accelerating frame (like the shell frame we discussed earlier, which is the frame of us sitting still on Earth.) If one uses Rindler coordinates one can show that someone moving with a constant acceleration through empty space, actually sees a thermal bath of particles and also an event horizon. This is called the Unruh effect and the radiation is called Unruh radiation. One can calculate the temperature that is seen using this method, say in the shell frame just above the horizon of a black hole and get Hawking's answer.

In any case, Hawking showed that the radiation coming out of a black hole is a nearly perfect blackbody, characterized by the **Hawking Temperature**

$$T = \frac{\hbar c^3}{8\pi k G M} = 10^{-7} \left(\frac{M_\odot}{M} \right) \ \mathrm{K},$$

where k is Boltzmann's constant and the temperature is given in degrees Kelvin. Recall that the typical energy of a photon radiated from a blackbody is given by

$$E = kT = \frac{\hbar c^3}{8\pi G M}.$$

Also the total energy coming off of a spherical object (like a star or black hole) is proportional to its surface area A and the temperature to the fourth power,

$$L = \sigma A T^4,$$

where $A = 4\pi R^2$, and the Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8} \ \mathrm{W \ m^{-2} \ K^{-4}}$.

We can use these formulas to find out the temperature of a black hole, and also how much energy is coming off it. But first we note above that the temperature of the smallest black hole expected to exist in nature, $(3M_\odot)$ is a few times 10^{-7} K. This is a very low temperature!

Of course the Hawking radiation causes the black hole to decrease in mass. One can ask how long will it take to reduce the mass of the black hole to zero, that is to completely evaporate a black hole? Well, currently with a cosmic microwave background radiation filling the Universe with a 2.7K radiation field, black holes are not losing any mass, but are gaining! So black holes would never evaporate in the current environment. However, the Universe is cooling down and eventually may get to below 10^{-7} K, and then black holes could start losing mass. Also if there were black holes with masses less than about the Moon's mass, then their Hawking temperature would be greater than 2.7K and they would be evaporating (as long as other stuff wasn't falling it).

Now the area of a black hole is $A = 4\pi r_S^2 = 16\pi G^2 M^2$, and the energy lost through Hawking radiation is equal to the change in the black hole mass:

$$\frac{dM}{dt} = -L_{\text{hawking}} = -\sigma A T^4 = -\sigma 16\pi G^2 M^2 T^4 = -16\pi\sigma G^2 (10^{-7} M_\odot K)^4 M^{-2}.$$

We integrate this from the hole's current mass to zero mass over the black hole's lifetime T_{life},

$$\int_0^{T_{\text{life}}} dt = -\int_M^0 \frac{M^2 dM}{B},$$

where $B = 16\pi\sigma G^2 (10^{-7} M_\odot K)^4$. Thus we find

$$T_{\text{life}} = \frac{M^3}{3B} \approx 10^{10}\text{years } \left(\frac{M}{10^{12}\text{kg}}\right)^3 \approx 10^{66}\text{years } \left(\frac{M}{M_\odot}\right)^3.$$

Since the Universe is only 1.37×10^{10} years old we see that for normal black holes, their lifetimes are way longer than the age of the Universe. These holes are going to be around for a very long time.

But since the temperature goes inversely with the mass, lighter holes evaporate faster and also have less mass to evaporate. Thus their lifetimes are much shorter. We can ask how light would a black hole have to be in order to just be evaporating today?

I give you this question as a homework problem! It is a value quite a bit smaller than black holes expected to form from normal stellar processes. I think the answer comes out to be around the mass of a mountain. While no known stellar processes could form such small black holes, Stephen Hawking proposed that the Big Bang itself could create a large number of these **primordial black holes**. These then could be floating around in space. Could you detect them? The Schwarzschild radius of the Earth is only 0.88 centimeter and these would be much smaller. Their gravity would be much less than a planet or moon or even an asteroid. So could you see it? Well, look again at the temperature formula. What happens as $M \to 0$? The temperature of the black hole increases; in fact it increases without limit! If you plug in a guess at the mass of such a primordial black hole into the Hawking Temperature formula you find that they are radiating at a temperature of billions of degrees K! Thus the shine very bright! However, they are small and can't be seen very far away until the very end when they explode in a burst of very high energy gamma rays.

We can be more precise. Consider the last second of the primordial black hole's life. Plugging $\tau = 1$ second into the lifetime formula you get a mass of about 10^6 kg which (using $E = Mc^2$) is about 10^{23} Joules. Thus the power output during the last second is about 10^{23} Joules/sec which can be compared to the solar luminosity of 4×10^{26} J/s. Thus this is a few thousand times dimmer than the Sun, about the energy of a very small star. This could be seen if it was fairly near. The energy spectrum would not be that of a star but would be peaked in the gamma rays. Thus the signature of these small primoridal black holes is an intense burst of gamma rays that last a very short time. When gamma ray bursts were discovered people considered this as a possible source, but the energy spectrum was wrong. For evaporating black holes we expect the energy of the emitted radiation to become more energic near the end since the temperature continually rises. The non-detection of these bursts mean that primordial black holes that have lifetimes equal to the age of the Universe can't exist. However, primordial black holes that are more massive and therefore live longer are still a possibility and could even be the dark matter. Most people don't think about these however, because to date, no one has thought of a good way to create the right number of these primordial black holes in the early Universe just after the Big Bang.

Note the final fate of an evaporating black hole is still not really known. All of Hawking's calculations break down when the quantum mechanical wavelength of the particles near the size of the black hole horizon $\lambda \sim r_S$. As the black holes evaporate and get smaller we have to eventually reach this limit. At that point we would need a real theory of quantum gravity, which does not yet exist. String theory is a candidate, but so far they can't calculate anything using string theory. We can estimate when the breakdown occurs using the above condition

$$\lambda \sim r_S.$$

Using

$$E = h\nu = hc/\lambda = Mc^2,$$

we find

$$\lambda = \frac{h}{Mc},$$

and the above condition becomes

$$\frac{h}{Mc} = \frac{GM}{c^2},$$

or $M^2 = hc/G$, or

$$M_{planck} = \sqrt{hc/G} \approx 2 \times 10^{-5} \text{gm},$$

where we introduced the standard name of this mass: **the Planck mass**. At this mass, and at the scales corresponding to this mass, we expect quantum effects to become so important that standard General Relativity breaks down. This is the scale string theory and any theory of quantum gravity works at.