Physics 140B, Winter 2010
Homework 4 --- due Feb 11

1. Consider a Carnot engine with 'black-body radiation' as its working substance. To start with, the state of the radiation is defined by volume V_0 and temperature T_0 (which determines the pressure P_0 as well). The radiation is now subjected to
(i) an isothermal expansion from volume V_0 to $2V_0$,
(ii) an adiabatic expansion from volume $2V_0$ to $4V_0$,
(iii) an isothermal compression from volume $4V_0$ to $2V_0$, and finally
(iv) an adiabatic compression from volume $2V_0$ to V_0 .

(a) Sketch this cycle in a properly labeled P-V diagram.

(b) Calculate the work done, the heat absorbed and the change in the internal energy of the system in each of these processes.
[You may express your results in terms of the product P_0V_0].

(c) Verify that the net work done in the cycle is equal to the net heat absorbed by the system.

(d) Calculate the efficiency of this cycle and verify that your result is in conformity with the Carnot theorem.

2. The "surface waves" in a low-temperature Bose liquid, on quantization, behave like a two-dimensional gas of non-interacting excitations called "ripplons". These excitations (like photons) are indefinite in number and obey Bose-Einstein statistics. Their energy-momentum relation, however, is $\varepsilon = a \cdot p^{3/2}$, where a is a constant.
Set up an expression for the total energy per unit area of these excitations and determine the manner in which this quantity depends on the temperature T of the system.

3. Solve Problem 19.3 of Carter.

4. Using Maxwell's relations, show that for a magnetic system

$$(\partial C_H / \partial H)_T = T(\partial^2 M / \partial T^2)_H ;$$

cf. Carter Problem 8.10 (a).

Apply this result to a paramagnetic material *in the Curie regime* and show that, in this case,

$$C_H = C\, H^2 / T^2 + f\,(T),$$

where C is the Curie constant and f (T) an unknown function of T .

5. Consider a paramagnetic solid composed of N magnetic dipoles with J = 1/2 and g = 2, so that $\mu_z = +\mu_B$ or $-\mu_B$. The system is in equilibrium at temperature T, the external applied field being H.

(a) Using Maxwell-Boltzmann statistics, write down the expectation values of the numbers, $N_\uparrow$ and $N_\downarrow$, of dipoles aligned parallel to (and anti-parallel to) the applied field.

(b) Using these numbers, evaluate the net magnetization M and the net energy U of the system. Check that, for $\mu_B H \ll kT$, you recover the Curie law --- with proper value of the Curie constant.

(c) With $N_\uparrow$ and $N_\downarrow$ as given in part (a), determine the total number of microstates W of the system. Using this value of W, evaluate the entropy S of the system. [Your result for S should agree with eqn. (17.29) of the text, though your derivation here is very different.]

(d) Using this expression of entropy, evaluate the specific heat C_H of the system. [Your result should now agree with eqn. (17.27) of the text.]

(e) Finally show that, in the Curie regime,

$$S \simeq Nk\,[\,\ln 2 - \varepsilon^2 /(2k^2 T^2)\,] \quad \text{and} \quad C_H \simeq Nk\,.\,\varepsilon^2 /(k^2 T^2),$$

where $\varepsilon = \mu_B H$.